作业三

1

(1)

$$
\begin{aligned}
\forall\, b \in R(AB) &\implies \exists\, x,\ ABx = b \\
&\implies \exists\, y = Bx,\ Ay = b \\
&\implies b \in R(A) \\
&\implies R(AB) \subseteq R(A)
\end{aligned}
$$

(2)

$$
\begin{aligned}
\forall\, x \in N(B) &\implies Bx = 0 \\
&\implies ABx = 0 \\
&\implies b \in N(AB) \\
&\implies N(B) \subseteq N(AB)
\end{aligned}
$$

2

$$
\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\} \iff \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\} \iff
$$

$$
\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}_{A_{4x4}}
$$

$rank(A) = 4$该集合线性无关。

3

$$A^TA = \begin{pmatrix} 6 & 18 & 4 & -20 \\ 18 & 54 & 12 & -60 \\ 4 & 12 & 6 & -20 \\ -20 & -60 & -20 & 80 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 3 & 1 & -4 \\ -1 & -3 & 1 & 0 \\ 2 & 6 & 2 & -8 \end{pmatrix}$$

$$AA^T = \begin{pmatrix} 27 & -9 & 54 \\ -9 & 11 & -18 \\ 54 & -18 & 108 \end{pmatrix}$$

$rank(A^TA) = rank(A) = rank(AA^T) = 2$