ChE 597 Computational Optimization

Homework 8

March 22nd 11:59 pm

1. Given is the integer programming problem

$$
\begin{aligned}
&\max Z = 1.2y_1 + y_2 \\
&\text{s.t. } y_1 + y_2 \leq 1 \\
&1.2y_1 + 0.5y_2 \leq 1 \\
&y_1, y_2 = 0, 1
\end{aligned}
$$

   (a) Plot the contours of the objective and the feasible region for the case when the binary variables are relaxed as continuous variables $y_1, y_2 \in [0, 1]$.
   (b) Determine from inspection the solution of the relaxed problem.
   (c) Enumerate the four 0-1 combinations in your plot to find the optimal solution.
   (d) Solve the relaxed LP problem by hand and derive Gomory mixed-integer cuts based on the LP relaxation (from the optimal simplex tableau) and verify that they cut-off the relaxed LP solution.

2. For each of the three sets below, find a missing valid inequality and verify graphically that its addition to the formulation gives conv(X).

   (a) $X = \left\{x \in \{0,1\}^2 : 3x_1 - 4x_2 \leq 1\right\}$
   (b) $X = \left\{(x,y) \in \{0,1\} \times \mathbb{R}^1_+ : y \leq 20x, y \leq 7\right\}$
   (c) $X = \left\{(x,y) \in \mathbb{Z}^1 \times \mathbb{R}^1_+ : y \leq 6x, y \leq 16\right\}$.

3. Consider the Haverly's pooling problem (reference: `http://www.ii.uib.no/~lennart/drgrad/Adhya1999.pdf`) Formulate this problem using the P-formulation, Q-formulation, and PQ-formulation in pyomo and solve them using Gurobi.

Table 1: Summary

Category	**Quality**	**Unit Cost**	
Pool Sources	1: 3% sulfur	$6	
	2: 1% sulfur	$16	
Direct Supply	3: 2% sulfur	$10	
Category	**Max Quality**	**Unit Price**	**Max Demand**
Products	1: 2.5% sulfur	$9	100
	2: 1.5% sulfur	$15	200

You don't need to consider the availability of raw materials and the pool capacity.

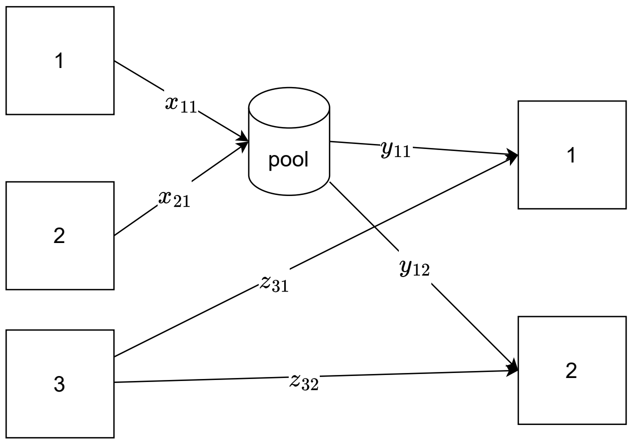

Figure 1: Haverly's pooling problem

4. Consider a k-means clustering problem. Each data point has dimension of 10. We have 20 data points, $k = 3$. Formulate the MIQCP and solve with gurobi

   The data set given in `https://github.com/li-group/ChE-597-Computational-Optimization/blob/main/HW%208/data_HW8_Q4.csv`

5. Consider the following set of squares with lengths as shown below:

Square	Length
1	2
2	3
3	6
4	9
5	10
6	12

Try to pack these squares into a rectangle whose height and width are at least 10 and at most 25. Suppose we do not allow rotating the squares. What is optimum dimensions of the rectangle and how are the squares packed?