

ChE 597 Computational Optimization**Homework 12**

April 26th 11:59 pm

1. Implement the ADMM algorithm to solve the farmer's problem in python.

2. The Lasso problem is commonly solved in machine learning for linear regression problems. The motivation is to obtain sparse regression coefficients x . See [https://en.wikipedia.org/wiki/Lasso_\(statistics\)](https://en.wikipedia.org/wiki/Lasso_(statistics)) if you are curious. The Lasso regression can be formulated as,

$$\min_{x \in \mathbb{R}^n} \frac{1}{2} \|Ax - b\|_2^2 + \lambda \|x\|_1$$

where parameters $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$. λ is a scalar. Apply ADMM to the following reformulation of the Lasso problem by dualizing the constraint $x = z$. Derive a closed-form update formula for each iteration of the ADMM algorithm.

$$\min_{x, z \in \mathbb{R}^n} \frac{1}{2} \|Ax - b\|_2^2 + \lambda \|z\|_1 \quad \text{s.t. } x = z$$

3. Show that for the semi-infinite programming formulation of the robust optimization problem with polyhedral uncertainty sets, it suffices to enforce the constraints at the extreme points of the polyhedra.

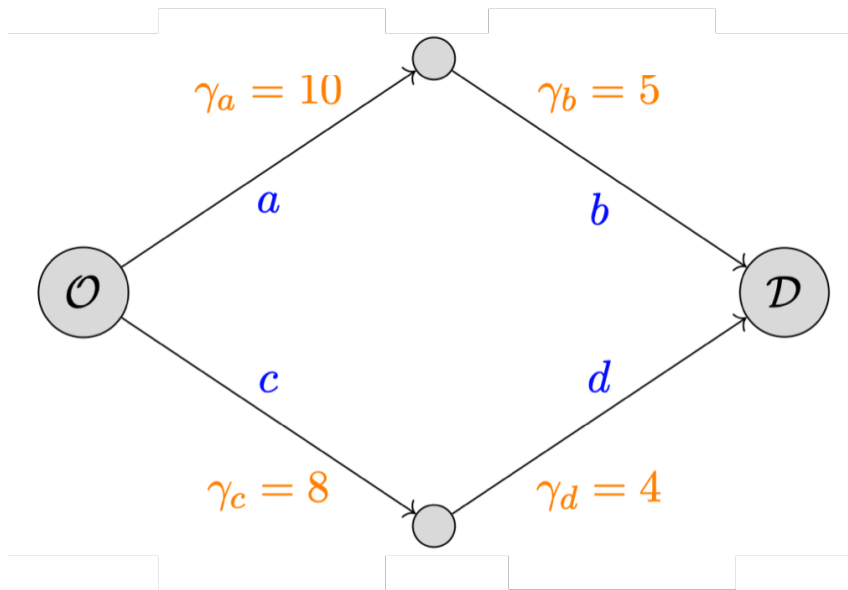
More precisely, consider

$$\begin{aligned} \min_{x, \alpha} \quad & \alpha \\ \text{s.t.} \quad & c^\top x \leq \alpha, \quad \forall c \in U_c \\ & a_i^\top x \leq b_i, \quad \forall a_i \in U_{a_i}, \forall b_i \in U_{b_i}, \quad i = 1, \dots, m. \end{aligned}$$

where $U_c, U_{a_i}, U_{b_i}, \forall i = 1, \dots, m$ are bounded polyhedra.

If the constraints at the extreme points of $U_c, U_{a_i}, U_{b_i}, \forall i = 1, \dots, m$ are added, then the solution must be feasible for the semi-infinite program.

4. Solve the toll setting problem with demand $D = 20$. The upper bound of the flow through route ab is 15, while the upper bound of the flow through route cd is 20. The maximum toll for each edge is 5. Solve the bilevel optimization using the KKT reformulation in pyomo.



5. Try playing with the Gurobi lazy callback and cut callback. List a small example with your code.

An example can be found

<https://stackoverflow.com/questions/58200552/pyomo-and-gurobi-does-pyomo-support-solve>