ChE 597 Computational Optimization

Homework 8

March 22nd 11:59 pm

1. Given is the integer programming problem

$$\begin{aligned} &\max Z = 1.2y_1 + y_2 \\ &\text{s.t. } y_1 + y_2 \leq 1 \\ &1.2y_1 + 0.5y_2 \leq 1 \\ &y_1, y_2 = 0, 1 \end{aligned}$$

   (a) Plot the contours of the objective and the feasible region for the case when the binary variables are relaxed as continuous variables $y_1, y_2 \in [0, 1]$.

   (b) Determine from inspection the solution of the relaxed problem.

   (c) Enumerate the four 0-1 combinations in your plot to find the optimal solution.

   (d) Solve the relaxed LP problem by hand and derive Gomory mixed-integer cuts based on the LP relaxation (from the optimal simplex tableau) and verify that they cut-off the relaxed LP solution.

2. For each of the three sets below, find a missing valid inequality and verify graphically that its addition to the formulation gives $\text{conv}(X)$.

   (a) $X = \left\{x \in \{0,1\}^2 : 3x_1 - 4x_2 \leq 1\right\}$

   (b) $X = \left\{(x,y) \in \{0,1\} \times \mathbb{R}^1_+ : y \leq 20x, y \leq 7\right\}$

   (c) $X = \left\{(x,y) \in \mathbb{Z}^1 \times \mathbb{R}^1_+ : y \leq 6x, y \leq 16\right\}$.

3. Consider the Haverly's pooling problem (reference: `http://www.ii.uib.no/~lennart/drgrad/Adhya1999.pdf`) Formulate this problem using the P-formulation, Q-formulation, and PQ-formulation in pyomo and solve them using Gurobi.

Table 1: Summary

Category	**Quality**	**Unit Cost**	
Pool Sources	1: 3% sulfur	\$6	
	2: 1% sulfur	\$16	
Direct Supply	3: 2% sulfur	\$10	
Category	**Max Quality**	**Unit Price**	**Max Demand**
Products	1: 2.5% sulfur	\$9	100
	2: 1.5% sulfur	\$15	200

You don't need to consider the availability of raw materials and the pool capacity.

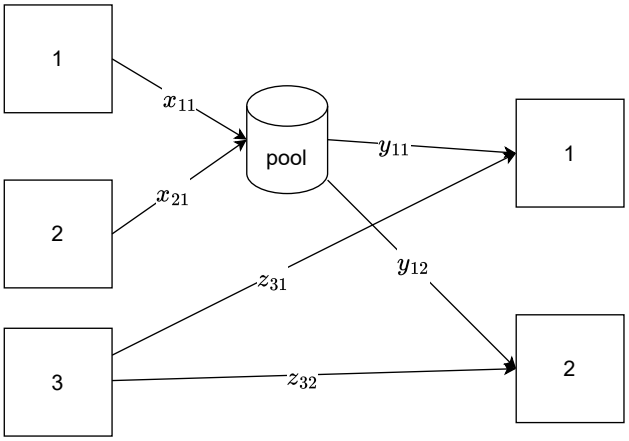

Figure 1: Haverly's pooling problem

4. Consider a k-means clustering problem. Each data point has dimension of 10. We have 20 data points, $k = 3$. Formulate the MIQCP and solve with gurobi

   The data set given in `https://github.com/li-group/ChE-597-Computational-Optimization/blob/main/HW%208/data_HW8_Q4.csv`

5. Consider the following set of squares with lengths as shown below:

Square	Length
1	2
2	3
3	6
4	9
5	10
6	12

Try to pack these squares into a rectangle whose height and width are at least 10 and at most 25. What is optimum dimensions of the rectangle and how are the squares packed?