ChE 597 Computational Optimization

Homework 7

March 8th 11:59 pm

1. A company is considering to produce a chemical C which can be manufactured with either process II or process III, both of which use as raw material chemical B. B can be purchased from another company or else manufactured with process I which uses A as a raw material. The superstructure of the problem is show below

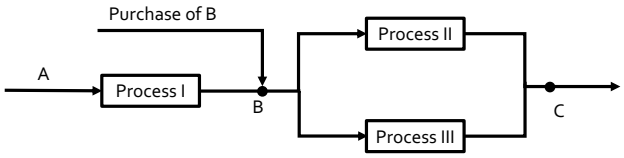

Figure 1: Superstructure

Given the specifications below, formulate an MILP model and solve it with Pyomo to decide:

(a) Which process to build (II and III are exclusive)?

(b) How to obtain chemical B?

(c) How much should be produced of product C? The objective is to maximize profit.

Consider the two following cases:

1. Maximum demand of C is 10 tons/hr with a selling price of $1800/ton.

2. Maximum demand of C is 15 tons/hr; the selling price for the first 10 ton/hr is $1800/ton, and $1500/ton for the excess.

Data:

Fixed the variable costs		
	Fixed ($/hr)	Variable ($/ton raw mat)
Process I	1000	250
Process II	1500	400
Process III	2000	550

Prices:

- A: $500/ton
- B: $950/ton

Conversions:

- Process I: 90% of A to B
- Process II: 82% of B to C
- Process III: 95% of B to C

Maximum supply of A: 16 tons/hr

NOTE: You may want to scale your cost coefficients (e.g., divide them by 100).

2. Solving the following lot sizing problem using pyomo

	Period, t					
	1	**2**	**3**	**4**	**5**	**6**
Demand, d_t	10	40	20	5	5	15
Fixed cost, f_t	50	50	50	50	50	50
Production cost, p_t	1	3	3	1	1	1
Holding cost, h_t	2	2	2	2	2	2

Assume the production limit $C = 25$

3. Consider the following scheduling problem with release and due dates. Note there are two sets of durations. All the data can be accessible directly from this jupyter notebook `https://github.com/li-group/ChE-597-Computational-Optimization/blob/main/HW%207/HW7%20Q3.ipynb`

   (a) Solve problem set 1 in pyomo. What is the assignment of jobs to machines; what about the precedence of jobs and the total cost?

   (b) Solve problem set 2 in pyomo. What is the assignment of jobs to machines; what about the precedence of jobs and the total cost?

Table 1: Data

Order (i)	r_i	d_i	Cost on Machine c_{im}		
			1	2	3
1	2	16	10	6	8
2	3	13	8	5	6
3	4	21	12	7	10
4	5	28	10	6	8
5	10	24	8	5	7
6	1	28	12	7	10
7	2	23	12	7	10

Order (i)	Machine	Durations (p_{im})	
		Set 1	Set 2
1	1	10	5
	2	14	7
	3	12	6
2	1	6	3
	2	8	4
	3	7	3
3	1	11	2
	2	16	4
	3	13	3
4	1	6	3
	2	12	6
	3	8	4
5	1	10	2
	2	16	4
	3	12	3
6	1	7	1
	2	12	3
	3	10	2
7	1	10	1
	2	8	2
	3	10	1

4. Implement your branch and bound algorithm to solve the scheduling problem in 3. You can use Gurobi to solve the relaxation at each node. To obtain an upper bound, you can use the rounding heuristic, i.e., round the binary variables to the nearest integer and solve the rest of the LP. Feel free to design your own heuristic based on your understanding of the problem. For node selection rule, use the best bound first rule.

   (a) Implement a branch and bound algorithm with the "most fractional variable branching rule". Compare it with the solution you got from 3. How many branch and bound nodes are there in total for problem set 1 and 2, respectively?

   (b) Implement a branch and bound algorithm with the "strong branching". Compare it with the solution you got from 3. How many branch and bound nodes are there in total for problem set 1 and 2, respectively?

5. Solve the following problems related to the branch and bound algorithm.

   (a) Show that the number of nodes in a tree where we represent all possible combinations of m $0-1$ binary variables is $2^{m+1}-1$.

   (b) If a complete enumeration of all the nodes in the tree were required, by what factor would this enumeration increase with respect to the direct enumeration of all 0-1 combinations.