ChE 597 Computational Optimization

Homework 11

April 19th 11:59 pm

1. Prove that the Lagrangian master problem and the Dantzig Wolfe master problem are LP dual of each other. Without loss of generality, you can assume the original problem is of the form

$$
\begin{aligned}
v(P) := \min \quad & c^T x \\
\text{s.t. } & Ax \leq b \\
& Ex \leq d \\
& x \in X \\
& X = \left\{ x_j \in \{0,1\}, \forall j = 1 \ldots p,\ x_j \in \mathbb{R}^+, \forall j = p+1, \ldots n \right\}
\end{aligned}
$$

where $Ax \leq b$ represents the complicating constraints.

2. Solve the cutting stock problem using the classical formulation and the column generation algorithm respectively. Start the first iteration of the column generation with patterns that only produce a single type of width.

   - Given $K = 20$ rolls of width $W = 100$ inches.
   - Widths w_i (inches) and Demand n_i (rolls) are given as:

Width w_i (inches)	25	35	40
Demand n_i (rolls)	7	5	3

3. Implement the Lagrangian decomposition algorithm using cutting plane and subgradient method to solve the farmer's problem.

4. Consider the following MILP problem:

$$\begin{array}{ll} \min & c^T y + \sum_{i=1}^{n} a_i^T x_i \\ \text{s.t.} & Cy + Ax_i = b \quad i = 1,2,\dots n \\ & y \geq 0, x_i \in X, i = 1,2,\dots n \end{array}$$

where X is a mixed-integer set.

(a) Reformulate the problem so that the MILP can be solved by Lagrangean decomposition.

(b) Write down the value of the Lagrangian decomposition $v(LD)$ using the theorem of the primal characterization discussed in class.