ChE 597 Computational Optimization

Homework 9

March 29th 11:59 pm

1. For the pooling problem you implemented in Homework 8, replace all the bilinear terms with the McCormick envelopes. Solve the McCormick relaxation using Gurobi. Compare the McCormick relaxations of the P formulation, Q formulation, and the PQ formulation.

2. Consider the following nonconvex quadratic constraint

$$-x_1^2 + x_2^2 + 4x_1x_2 \leq 7$$

Let

$$f(x) := -x_1^2 + x_2^2 + 4x_1x_2$$

(a) Write this nonconvex constraint as a difference of convex function, $f(x) = p(x) - q(x)$ where both p and q are convex. Hint: using uniform perturbation of the Hessian.

(b) Relax the concave function $-q(x)$ by McCormick envelopes. You can assume $0 \leq x_1 \leq 3$, $0 \leq x_2 \leq 5$

(c) Take the relaxation you derived, solve the problem with Gurobi using the objective $\min x_1 - 2x_2$

3. Assume the following constraint is in a convex relaxation of an optimization problem

$$x_1 = 2x_2 - x_3$$

where $0 \leq x_2 \leq 2$, $-1 \leq x_3 \leq 1$.

(a) Use the bounds of x_2 and x_3 to derive valid upper and lower bounds for x_1.

(b) Given the bounds of x_1 you derived in (a), try using FBBT to tighten the bounds of x_2 and x_3.

4. Derive a valid convex relaxation of the following nonconvex optimization problem using factorization and convex envelopes of univariate functions.

$$\min x_1 + x_2$$

$$\text{s.t.} \quad \exp(x_2\sqrt{x_1 x_2} + \log(x_1)) \leq x_1^2$$

$$1 \leq x_1 \leq 2, 0 \leq x_2 \leq 1$$

5. Consider the following convex relaxation,

$$
\begin{aligned}
2x_1 + x_2 - x_3 - 2x_4 &= 1 \\
3x_2 + x_4 &= 5 \\
0 \leq x_1 &\leq 4 \\
-1 \leq x_2 &\leq 2 \\
0 \leq x_3 &\leq 3 \\
-1 \leq x_4 &\leq 1
\end{aligned}
$$

Use OBBT to tighten the bounds of all the variables. You can solve the LPs using Gurobi.