ChE 597 Computational Optimization

Homework 6

March 1st 11:59 pm

1. Convert the logic expression below into a system of inequalities with $0, 1$ variables:

   (a)
   $$(P_1 \vee \neg P_2) \Rightarrow (P_3 \vee P_4)$$

   (b)
   $$\Big((P_1 \wedge P_2) \Rightarrow P_3\Big) \Rightarrow \Big(P_5 \vee P_6\Big)$$

2. Formulate linear constraints in terms of binary variables for the following cases:

   (a) If A is true and B is true then C is true or D is true. (inclusive OR)

   (b) The choice of all 0-1 combinations for $y_j, j \in J$ is feasible, except the one for which $y_j = 0, j \in N, y_j = 1, j \in B$, where N and B are specified partitions of J.

   (c) If power must be generated in any time period 1,2 or 3, then install a gas turbine. Represent whether power is generated at the three time periods as three separate binary variables.

3. Formulate mixed-integer linear constraints for the following disjunctions, using both big-M and convex-hull formulations:

   (a) Either $0 \le x \le 10$ or $20 \le x \le 30$

   (b) The temperature approach constraint for a heat exchanger

   $$T_{in} - t_{out} \geq DTmin$$

   should only hold only if the exchanger is actually selected.

4. Consider the following knapsack sets represented by mixed-integer linear constraints

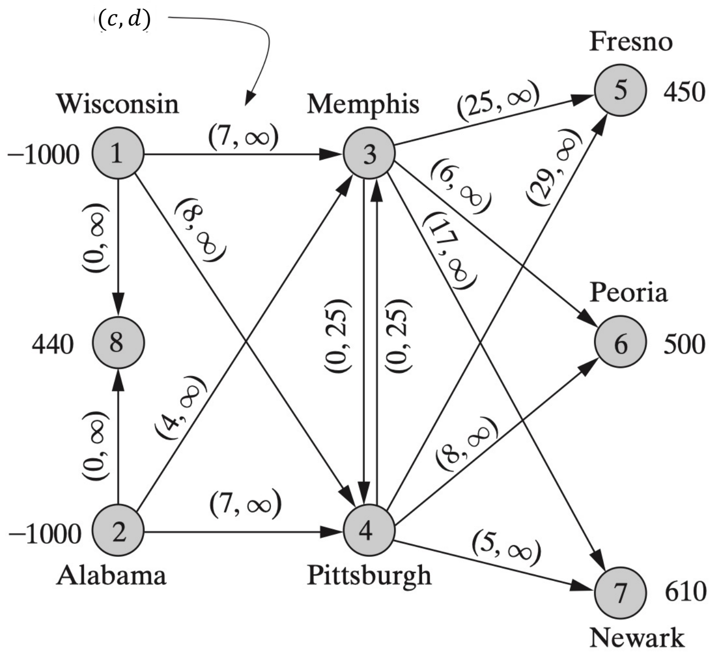
$$K := \left\{x \in \{0,1\}^3 : 2x_1 + 3x_2 + 4x_3 \leq 5\right\}.$$

   (a) Reformulate the knapsack problem using the minimal cover inequalities.

   (b) Denote the feasible region of the minimal cover inequalities as K^C. Show that $K = K^C$

   (c) Denote the LP relaxation of K and K^C as P and P^C, respectively. Show $P^C \subsetneq P$

5. *Minimum Cost Network Flows:* Consider the following Minimum Cost Network Flows problem. The demand of each node are shown next to the node, e.g., the demand at Wisconsin is -1000, meaning that we need to send out 1000 units of flow from Wisconsin. The cost and capacity of each edge are also shown.



(a) Formulate the Minimum Cost Network Flows in pyomo and solve it with Gurobi using the primal simplex algorithm. Check if the solution is integral, explain why.

Hint: to use Gurobi's primal simplex algorithm the following option should be specified.

```
opt = SolverFactory('gurobi')
opt.options['Method'] = 0
opt.solve(model, tee=True)
```

(b) Solve the model using the barrier's algorithm without crossover, check if the solution is integral, explain why.

Hint: to use Gurobi's barrier algorithm without crossover, the following solver option should be used.

```
opt = SolverFactory('gurobi')
opt.options['Method'] = 2
opt.options['Crossover'] = 0
opt.solve(model, tee=True)
```

(c) Read the Gurobi user manual to understand these solver options.

LP methods: `https://www.gurobi.com/documentation/current/refman/method.html`

Crossover: `https://www.gurobi.com/documentation/current/refman/crossover.html`