Long Duration Energy Storage Writeup

Sai Madhukiran Kompalli

1 Problem Description

We model a transmission grid composed of buses, transmission lines, conventional generators, and fleets of battery-storage units [1]. Over a finite horizon (for example, 24 hourly periods) the controller must specify, at each hour, how much power every generator produces, how much each battery charges or discharges, and—if necessary—how much *slack generation* or *load-shedding* is used to keep the system feasible. Given these continuous controls the environment

- solves a DC power-flow system to enforce network constraints,
- updates each battery's state of charge (SOC) with a linear carry-over and efficiency model, and
- computes operating cost (fuel, load-shedding penalty, slack-generation penalty, plus any violation charges), returning the negative of this cost as the reward.

2 Observation Space

The flattened observation vector at time step t contains:

- **Demand** at each bus: $\left[d_{1,t}, \ldots, d_{N,t}\right] \in \left[0, d_{\max}\right]^N$,
- **Normalised SOC**: $\hat{E}_{n,t} = E_{n,t}/E_n^{\max} \in [0,1]$ for all buses,
- **Wild-fire risk** on each line, normalised to $[0,1]$,
- **Flow ratio** $f_{\ell,t}/p_\ell^{\max} \in [-1,1]$ for each line,
- **Scaled time index** $t/T \in [0,1]$.

Dimension: $2N + 2L + 1$.

3 Action Space

At each step the controller sends the vector

$$\left[g_{1,t}, \ldots, g_{G,t},\ p^c_{1,t}, \ldots, p^c_{N,t},\ p^d_{1,t}, \ldots, p^d_{N,t},\ g^{\text{slack}}_{1,t}, \ldots, g^{\text{slack}}_{N,t},\ p^{\text{ls}}_{1,t}, \ldots, p^{\text{ls}}_{N,t}\right].$$

Component bounds:

- Generator outputs $g_{g,t} \in [g_g^{\min},\ g_g^{\max}]$,
- Charge rates $p^c_{n,t} \in [0,\ p_n^{c,\max}]$,
- Discharge rates $p^d_{n,t} \in [0,\ p_n^{d,\max}]$,
- Slack generation $g^{\text{slack}}_{n,t} \in [0,\ g^{\text{slack}}_{\max}]$,
- Load-shedding $p^{\text{ls}}_{n,t} \in [0,\ d_{n,t}]$.

Dimension: $G + 4N$.

4 Reward

$$r_t = -\Big(\underbrace{\textstyle\sum_{g=1}^{G}\sum_{j=0}^{J-1} c_{g,j}\, g_{g,t}^{j}}_{\text{fuel}} + K_{\text{ls}} \sum_{n=1}^{N} p_{n,t}^{\text{ls}} + K_{\text{slack}} \sum_{n=1}^{N} g_{n,t}^{\text{slack}} + \text{penalties}_t\Big).$$

5 Termination

The episode terminates when the internal clock advances past the horizon:

$$\texttt{terminated} = (t > T).$$

6 Penalty Costs

- *Action bounds:* each out-of-range component incurs $(\text{diff})^2 + P$ before it is clipped.
- *Line-flow overloads:* any excess $|f_{\ell,t}| - p_\ell^{\max}$ is penalised by $\big(|f_{\ell,t}| - p_\ell^{\max}\big)^2 + P$.
- *Observation bounds:* the same quadratic+constant penalty is applied if an observation element lies outside its Box bounds.

7 Transition Functions

1. **SOC update** $E_{n,t} = h\, E_{n,t-1} + \eta\, p_{n,t}^{c} - \dfrac{1}{\eta}\, p_{n,t}^{d}$.
2. **Load-shedding bound** $p_{n,t}^{\text{ls}} \leftarrow \min\big(p_{n,t}^{\text{ls}}, d_{n,t}\big)$.
3. **Net bus injection** $P_{n,t} = \sum_{g\in G(n)} g_{g,t} + g_{n,t}^{\text{slack}} - d_{n,t} + p_{n,t}^{\text{ls}} - p_{n,t}^{c} + p_{n,t}^{d}$.
4. **Voltage angles** Solve $B_{\text{red}}\theta_{\text{red}} = P_{\text{red}}$ with $\theta_{\text{ref}} = 0$.
5. **Line flows** $f_{\ell,t} = -b_\ell\big(\theta_{\text{fr}(\ell)} - \theta_{\text{to}(\ell)}\big)$.
6. **Penalty on overloads** Apply quadratic penalty if $|f_{\ell,t}| > p_\ell^{\max}$.
7. **Cost & reward** Combine fuel, shedding, slack, and penalty costs, then negate.
8. **Observation rebuild** Produce the vector $\big[d, \hat{E}, r, f/p^{\max}, t/T\big]$ and clip with penalties as needed.
9. **Advance time** $t \leftarrow t+1$; if $t > T$ flag `terminated`.

References

[1] Ryan Piansky, Georgia Stinchfield, Alyssa Kody, Daniel K Molzahn, and Jean-Paul Watson. Long duration battery sizing, siting, and operation under wildfire risk using progressive hedging. *Electric Power Systems Research*, 235:110785, 2024.