COVID-19

By Fan Mo, Haolin Li, Jindou Li

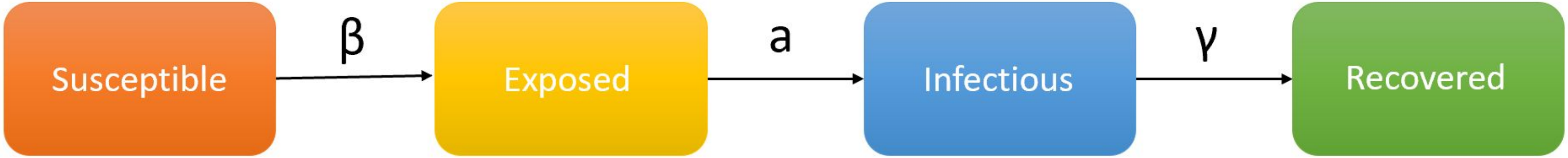
Introduction

- Our main goal is to use mathematical modelling of infectious disease to study the spread of COVID-19 and related policies and behaviors in order to increase public awareness.
- Establish mathematical modeling to predict the development trend of Covid-19.
- Make the prediction of the epidemic development trend under two situations:
    - keeping social distance policy
    - testing and quarantining

3 Models

- A basic SEIR model
- The SEIR model with new assumption and the keeping social distance policy
- The SEIR model with behaviors of testing and quartaning

Model 1



- S - susceptible
- E - Exposed
- I - Infectious
- R - Recovered
- N - Total population

- β - represents the chance of get infected
- a - the chance a exposed person turned into infected
- γ -represents the recovery rate

Assumptions:

- The total population remains unchanged
- Focus on the person to person transmission (respiratory transmission)
- Assume that once a patient recovers, the person will be immune and will not be infected again

Parameters :

- $T = 100$ (total time want to predict)
- $t = 1$ (unit of time change: 1 day)
- $S = 331883985$ (initial number of susceptible $S = N-I$)
- $E = 0$ (initial number of exposed)
- $I = 1$ (initial number of infected)
- $R = 0$ (initial number of recovered)
- $N = S + E + I + R$
- $\beta = 0.21$ (possibility of the virus infect susceptible)
- $a = 1/5.2$ (the chance an exposed person turned into infected)
- $\gamma = 0.81$ (the recovery rate)
- $r = 18$(number of people one person can meet per day)

Equations in Model 1

$$\frac{dS}{dt} = -r\beta\frac{I}{N}S$$

$$\frac{dE}{dt} = r\beta\frac{I}{N}S - aE$$

$$\frac{dI}{dt} = aE - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

Model 2

About the Model 2:

- Added social distancing policy
- Assumed the exposed population has the ability to infect others

Parameters:

- S = Susceptible
- E = Exposed
- I = Infectious
- R = Recovered
- r = number of people one person can meet per day
- β = possibility of the virus infect susceptible
- σ = the chance an exposed person turned into infected
- γ = the recovery rate
- T = total time want to predict
- k = number of people the exposed can meet per day

Model 2

- Assumption of the Exposed

Model 1

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{r\beta IS}{N} - \sigma E$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{-r\beta IS}{N}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \gamma I$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \sigma E - \gamma I$$

Model 2

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{r\beta IS}{N} + \frac{k\beta ES}{N} - \sigma E$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{-r\beta IS}{N} - \frac{k\beta ES}{N}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \gamma I$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \sigma E - \gamma I$$

Model 2

- Adding Social Distancing Policy

Before Social Distancing

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{r\beta IS}{N} + \frac{k\beta ES}{N} - \sigma E$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} - \frac{-r\beta IS}{N} - \frac{k\beta ES}{N}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \gamma I$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \sigma E - \gamma I$$

During Social Distancing

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{r2\beta IS}{N} + \frac{k2\beta ES}{N} - \sigma E$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{-r2\beta IS}{N} - \frac{k2\beta ES}{N}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \gamma I$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \sigma E - \gamma I$$

$$\text{change of r and k during quarantine} \begin{cases} r2 < r \\ k2 < k \end{cases}$$

Model 3

About the Model 3:

- Added the behavior of testing covid-19 for each group of people in the population

Assumption:

- The distribution of the population: Susceptible = 0.8, Exposed = 0.1 and Infectious = 0.1.
- Test result for the susceptible is always negative (test positive rate = 0)
- Test result for the infectious is always positive (test positive rate = 1)

Model 3

- Test behavior rate

$$\text{From the data}\begin{cases}\text{test performed rate} = 0.0496\%(0.000496)\\ \text{test positive rate} = 0.0064\%(0.000064)\end{cases}$$

$$\text{Susceptible}\begin{cases}t = 0.8 * 0.000496 = 0.0003968\\ nt = 1 - t = 0.9996032\end{cases}$$

$$\text{Infectious}\begin{cases}tp = 0.1 * 0.000496 = 0.0000496\\ nt = 1 - tp = 0.9999504\end{cases}$$

$$\text{Exposed}\begin{cases}t = 0.1 * 0.000496 = 0.0000496\\ tn = (1 - 0.29) * 0.0000496 = 0.000035216\\ tp = 1 * 0.0000496 + E_{tp} * 0.0000496 = 0.000064 \rightarrow 0.29 * 0.0000496 = 0.000014384\\ nt = 1 - tn - tp = 0.9999504\end{cases}$$

$$\begin{cases}t = \text{tested rate}\\ nt = \text{non-tested rate}\\ tp = \text{test positive rate}\\ tn = \text{test negative rate}\end{cases}$$

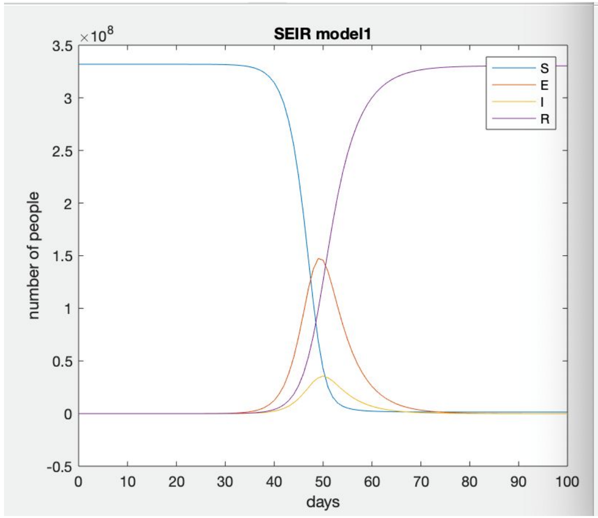
Model 3

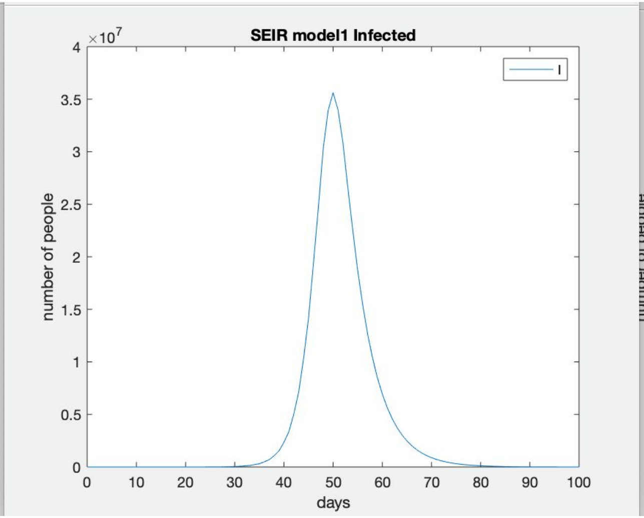
$$
\begin{aligned}
\frac{\mathrm{d}E}{\mathrm{d}t} &= 1 * \beta * I_{tp} * \frac{S}{N} * I \\
&+ (S_t * rt + S_{nt} * r) * \beta * I_{nt} * \frac{S}{N} * I \\
&+ 1 * \beta * E_{tp} * \frac{S}{N} * E \\
&+ 2 * \beta * E_{tn} * \frac{S}{N} * E \\
&+ (S_t * kt + S_{nt} * k) * \beta * E_{nt} * \frac{S}{N} * E \\
&- a * E
\end{aligned}
$$

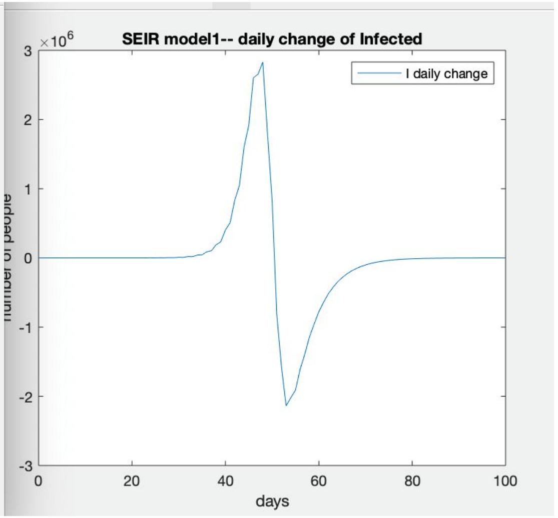
$$
\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t} &= -1 * \beta * I_{tp} * \frac{S}{N} * I \\
&- (S_t * rt + S_{nt} * r) * \beta * I_{nt} * \frac{S}{N} * I \\
&- 1 * \beta * E_{tp} * \frac{S}{N} * E \\
&- 2 * \beta * E_{tn} * \frac{S}{N} * E \\
&- (S_t * kt + S_{nt} * k) * \beta * E_{nt} * \frac{S}{N} * E
\end{aligned}
$$

$$
\text{isolation} \begin{cases} 1, \text{Tested positive individuals} \\ 2, \text{Tested negative individuals} \end{cases}
$$

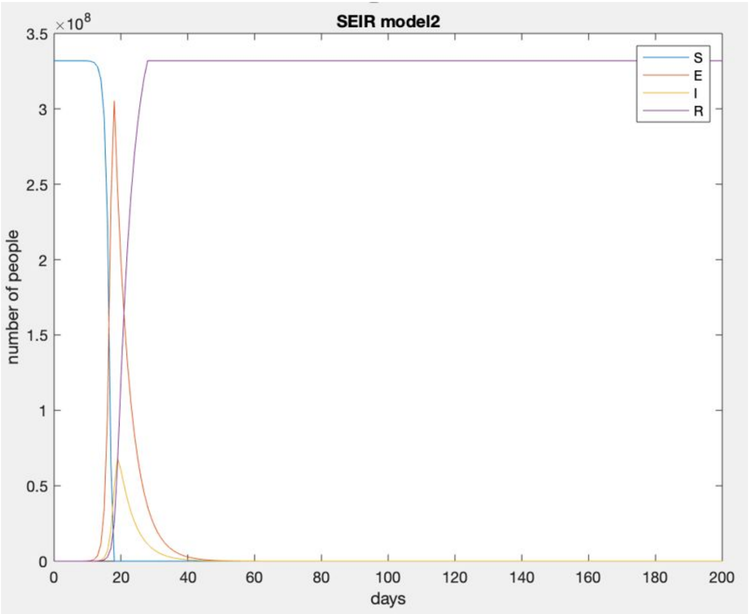
$$
\text{number of contacts for the non-tested individuals} \begin{cases} rt < r \\ kt < k \end{cases}
$$

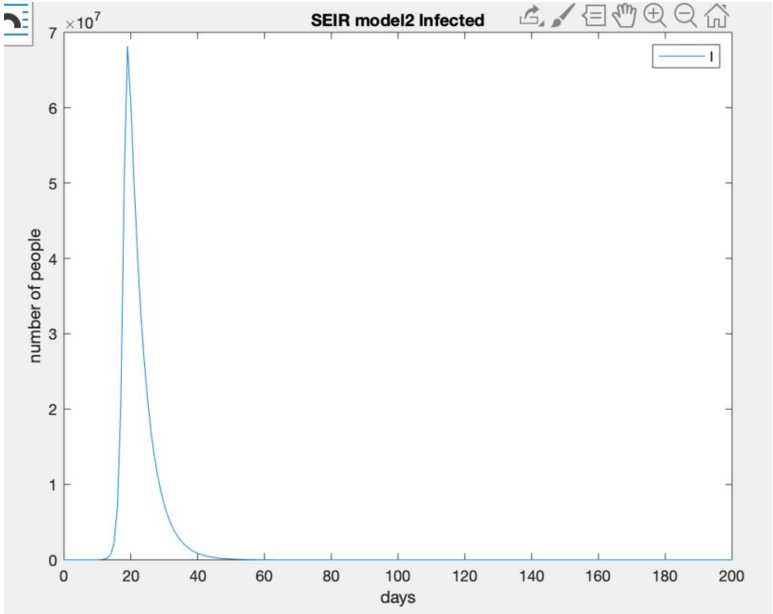




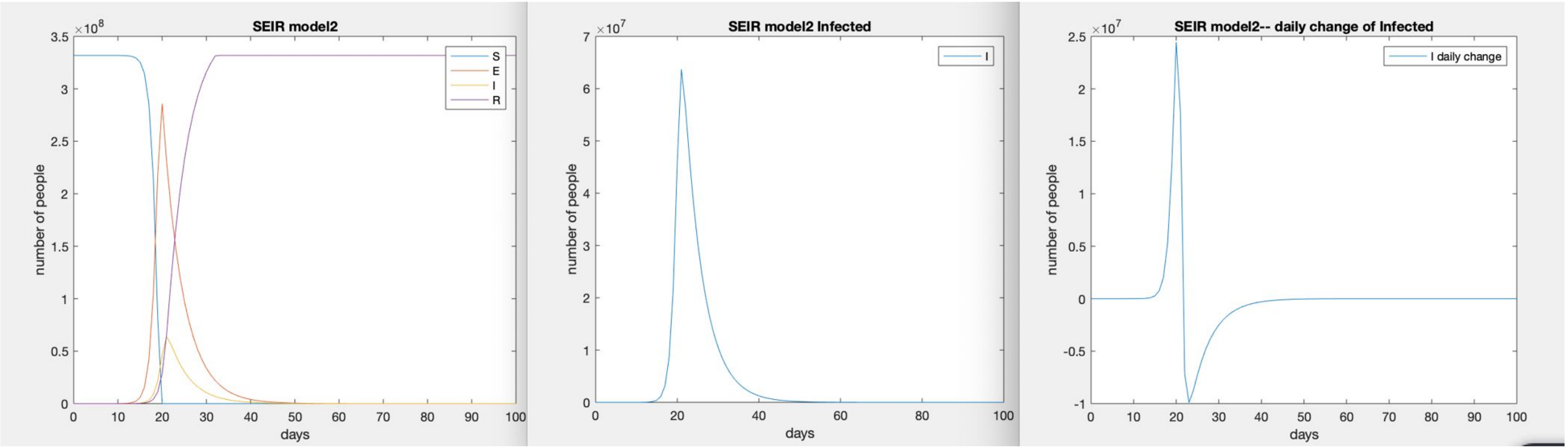


- From t = 40 to 50, the increasing speed of the number of the infected is very fast
- At t = 51, the maximum of the infected occurs
    - about 35 million (35618735) will get infected
    - 10.7% of the whole population
- At t = 80, the epidemic is about to end



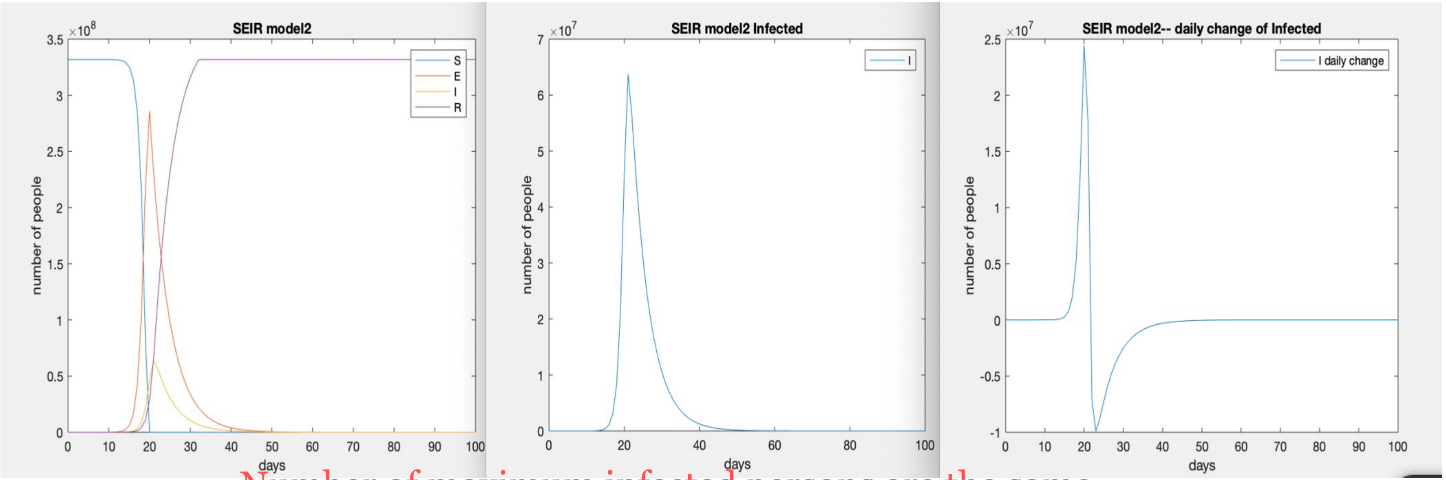


- Assume the exposed also have the ability to infect the susceptible
- At t = 20, the maximum occurs
    - 68 millions infected
    - 20.5% of the whole population and is about 1.91 times of the maximum infected in the first model.
    - The time of the maximum number is almost two-fifths of that of the first model. This time, almost the whole population can get exposed.



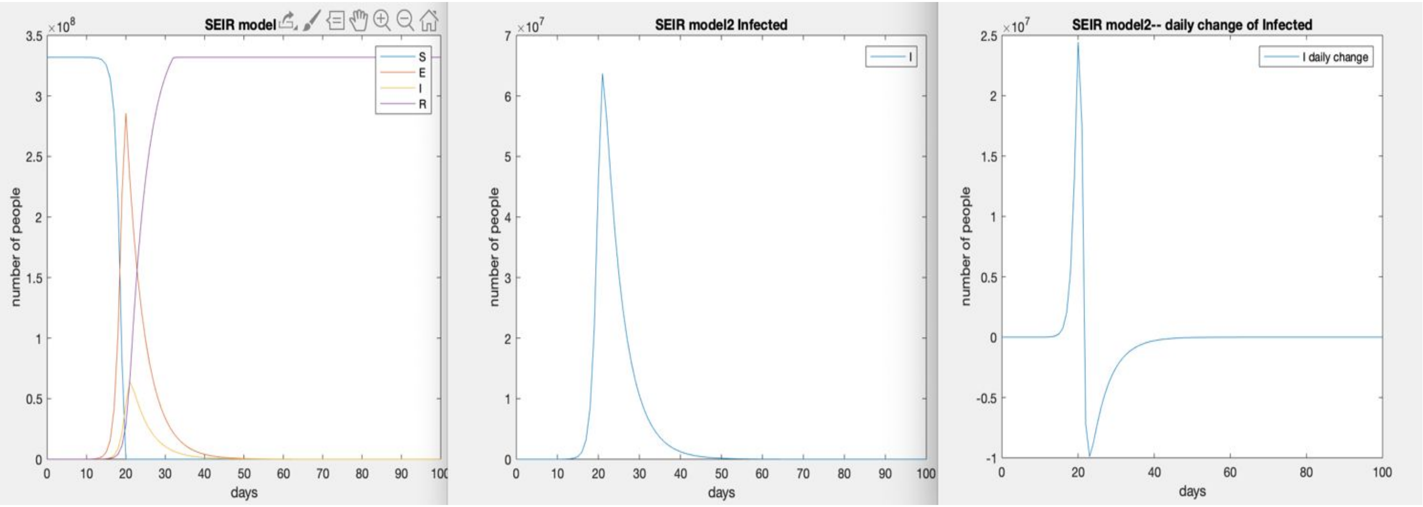
- Add the policy of keeping social distance that started at t = 55
- At t = 22, the maximum occurs
    - About 4 millions (4461619) get infected
    - 6.5% less than the second model that without the policy.

Keeping social distance policy started at t = 55

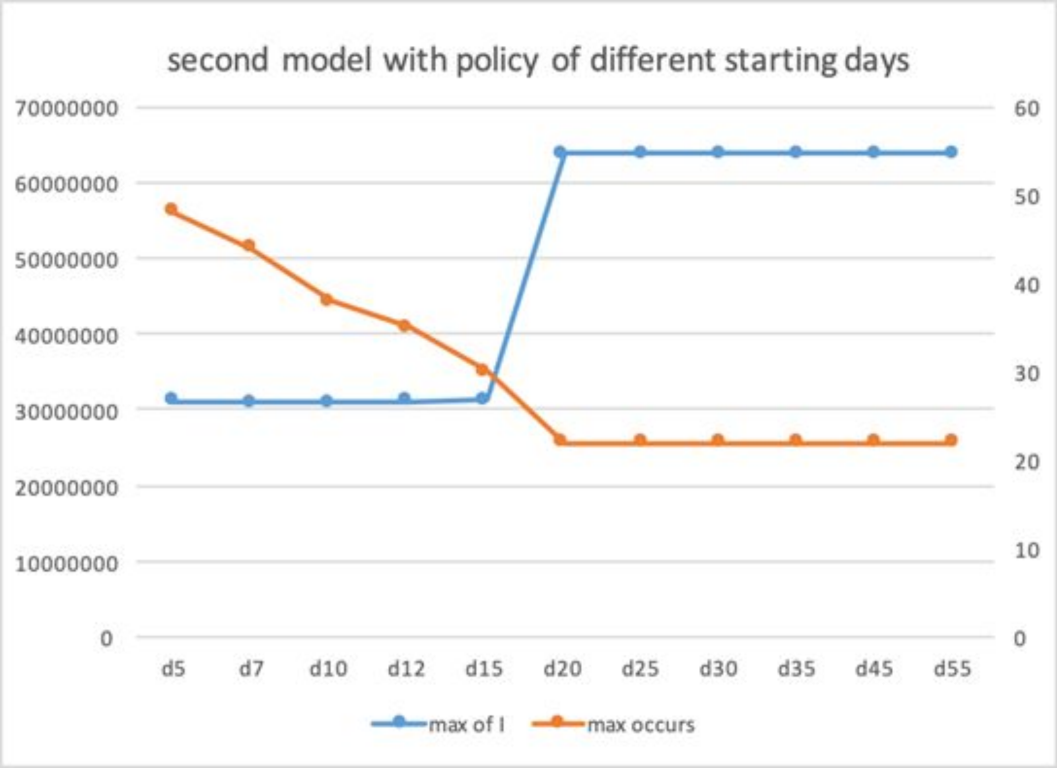


Number of maximum infected persons are the same

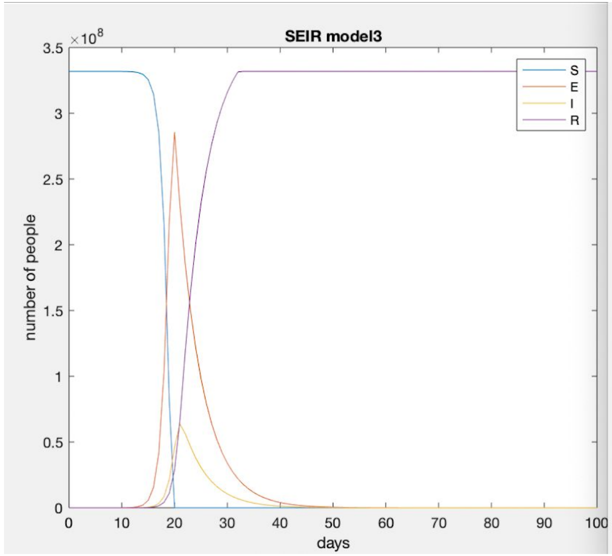
Keeping social distance policy started at t = 35

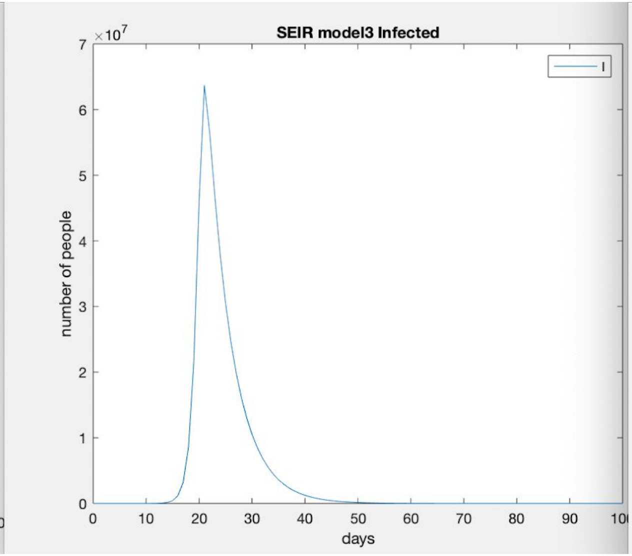


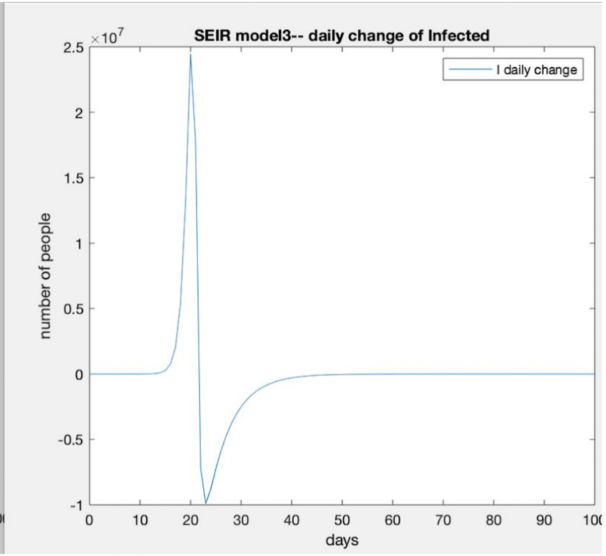
We want to try other starting days at t = 5, 7, 10, 12, 15, 20, 25, 30, 45 to see the differences



- ➢ At $t < 20$, the maximum number of the infected greatly decrease
- ➢ almost half of the original number that without policy.
- ➢ At $t \geq 20$, the maximum numbers of the infected are all the same.
- ➢ We should take action before the maximum arriving (as soon as possible)







- In the third model, we add the behavior of testing and isolation with the keeping social distance policy that start at 55 days and can get:
  - At t = 55, the maximum number of the infected decrease only 340 people
  - Reason: the test rate itself is low and the policy starting day is too late.
- We tried an earlier starting date and comparing with the data from the second model
  - At t = 7,the maximum of the infected decreased 1045
  - At t = 12, the maximum of the infected decreased 836