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2018 Interdisciplinary Contest in Modeling (ICM) Summary Sheet

Summary

In recent years, many people have realized the danger of fossil fuels and started to use electric vehicles to reduce the emission of harmful gas. However, the transition from fuel vehicles to electric vehicles is not a simple thing for some countries. The crucial problem they need to solve is how to build a network of charging stations which allow people to use electric vehicles conveniently and efficiently.

In order to address the problem above and provide a handout for the leaders of many countries who are attending an international energy summit, we conclude **three sub-problems and its solution** in our paper: 1)The optimal amount of charging stations;2) Model building of a growth model of charging stations; 3)Model building of a growth model of electric vehicles.

In first model, we choose the most appropriate ***Fitting Function***. Using ***hierarchical clustering***, we divide the region into three categories. Then we successfully figure out the convenience coefficient of each categories and get the optimal amount of charging stations by the model. In our second model, we collect the data and make a scatter plot. Then, we fit the trend to find three stages of the charging station's development. In the third model, we build a set of parameters to describe the sales of electric vehicles and successfully get the amount of electric vehicles.

In the whole modeling process, we give full consideration to validity, feasibility and cost-efficiency of our model.

Three Models for electric vehicles and charging stations

Contents

1 Introduction

Environmental problems have been main concerns for many countries, such as air pollution. It is true that cars and other transportations which use fossil fuels have been one of the reasons since the industrial revolution. Fortunately, many scientists and governors have realized the importance of new energy and electric vehicles are invented and put on the market, such as the all-electric Tesla in the US and other kinds of electric vehicles in other countries.

However, it is difficult for a country to build a charging grid considering the differences between different areas. For example, the amount of electric vehicles stations, locations, the amount of chargers and the different demand for electric vehicles between rural areas and urban areas and the development of the network of charging stations all should be considered.

In order to address problems above, we conclude three sub-problems to tackle in our paper.

- The optimal amount of charging stations
- Model building of a growth model of charging stations
- Model building of a growth model of electric vehicles

In the whole modeling process, we give full consideration to validity, feasibility and cost-efficiency of our models.

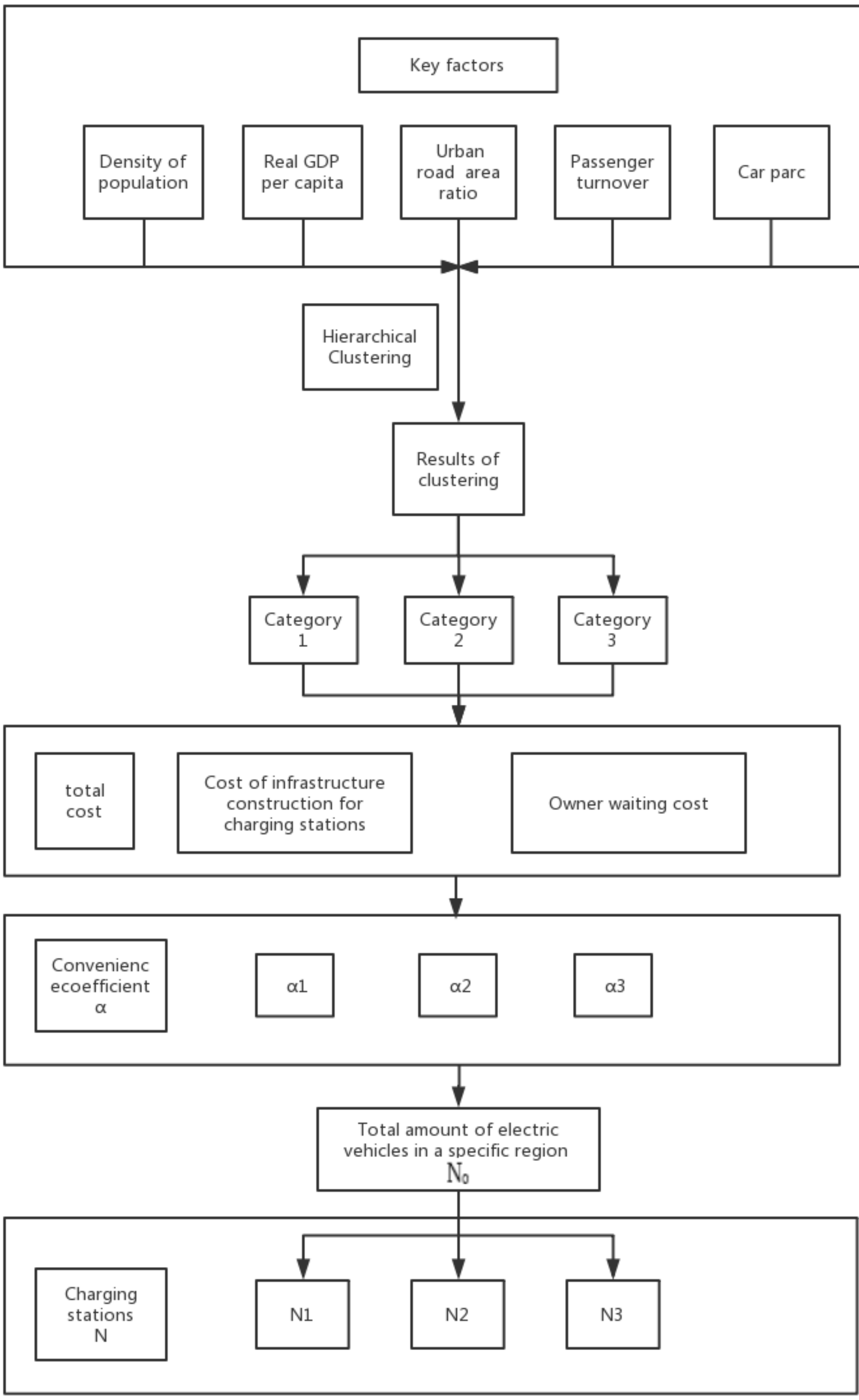
2 Nomenclatures

f_1	The cost of construction
c_1	The cost of charging station construction
c_2	The cost due to driving to the charging station and waiting in line to charge cars
Q	The amount of conventional charging stations
C_{total}	The total cost of construction
a_1	The fixed investment of construction
a_2	The relative cost that is proportional to the cost of chargers and the amount of purchase
a_3	The relative cost that is proportional to the amount of cables and the square of the number of transformers
M	The depreciation life of charging stations
R	The discount rate
N	The total amount of charging stations
f_2	The loss due to driving to the charging station
w_{qi}	The average waiting time
γ	Time value coefficient
α	The charging convenience coefficient
D	The maximum distance that drivers can find charging stations
D	The driving range of electric vehicles
$\bar{V}$	The average speed of driving to the charging station

3 Model one: Optimal amount of charging stations model

3.1 Introduction

In order to work out the number of charging stations, we firstly divide the region into three categories by hierarchical clustering. These categories have different convenience coefficient so that we can get the number of charging stations by our model.



3.2 Assumptions

- Economic person assumption: assume that people tend to drive to the nearest charging station to charge their cars.
- Assume that the area can be divided by Voronoi diagram.
- Assume that all of the electric vehicles and charging stations are homogeneous.
- Assume that population are distributed equally.

3.3 Driving range of electric vehicles D

The farthest distance an electric car can travel in a fully charged state. In the location of the charging station, we assume that the distance between two charging stations must be less than the electric vehicle's driving range D[1].

3.4 Charging convenience coefficient α of electric cars

Taking into account the rationality of the number of charging stations and charging piles in different regions of the city, we introduce charging convenience coefficient into the model. The driving cost and queuing cost of the car owner are large when the charging convenience coefficient σ is large. On the contrary, the driving cost and queuing cost of the owner are small when the convenience coefficient α is relatively small

Correspondingly, there is a relationship between driving range and the charging convenience coefficient:

$$d = D \times \alpha$$

Considering the sustainable development of charging stations, we include the cost of the infrastructure construction, driving cost and operating cost of charging stations into the model. The total construction cost of charging stations is:

$$C_{total} = C_{1(\alpha)} + C_{2(\alpha)}$$

3.4.1 The infrastructure construction cost of charging stations

The function form of the construction cost of charging stations in each day is:

$$C_{1(\sigma)} = \sum_{i=1}^{N} \left[f_{1(\alpha)} \frac{r(1+r)^m}{(1+r)^m - 1} + f_{2(\alpha)} \right]$$

The relationship between f_1 and the convenience coefficient α and the relationship between f_2 and the convenience coefficient α are:

$$f_{1(\alpha)} = (a_1 + a_2 Q + a_3 Q^2)\alpha$$

$$f_{2(\alpha)} = 0.5\alpha(a_1 + a_2 Q + a_3 Q^2)$$

3.4.2 The cost of charging

For those people who choose the first conventional charging method, they do not have waiting cost.

However, for those drivers who choose the second super charging method, they need to queue for the completion of the charging. Thus, they have waiting cost and electric vehicles are related to the time in the process of waiting, the function is:

$$C_{2(\alpha)} = \sum_{i=1}^{N} \left[W_{qi}\gamma + d / \bar{v} \right] \gamma$$

3.4.3 M/M/S queuing model

In order to simplify the model, we assume that the time interval of the electric car charging on the charging stations is negative exponential distributed[2] with the parameter λ. Also, we assume that the charging duration of electric vehicles is independent and identically distributed with a negative exponential distribution (parameter $\boldsymbol{\mu}$).

It can be seen from the journal that when the system tends to stabilize in the queuing model, the average waiting time of the owner is:

$$W_q = \frac{L_q}{\lambda} = \frac{p_0 \rho^s \sum_{n=s}^{\infty} (n-s) \rho_s^{n-s}}{\lambda}$$

$$\rho_s = \frac{\lambda}{\mu s}, \quad \rho_0 = \left[\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s!(1-\rho_s)} \right]^{-1}$$

λ—the arrive rate of electric vehicles

$\boldsymbol{\mu}$—the average service rate of charging stations

$\boldsymbol{\rho}$—the intensity of the service

$\boldsymbol{\rho^s}$—the intensity of the service when the number of service stations is s

According to the data about American city, the relationship between total cost and charging convenience coefficient $\boldsymbol{\alpha}$ is shown in the figure 1[3]:

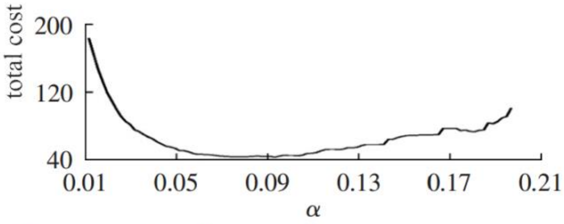


Fig.1 Curve of total cost vs. convenience coefficient

We can see from the figure that when the range of the charging convenience coefficient is from 0.05 to 0.13, the total cost is small and stable. We divide the city into three regions[4] according to the transportation convenience, economic development and population characteristics. For a particular region, the region has the same charging convenience coefficient α and we determine the locations of charging stations and their sizes in each specific area of the charging station according to the principle of minimum total cost in the region.

Then, we collect the data about real GDP per capita, density of population, urban road area ratio, car parc and passenger turnover and we get the correlation between different factors:

Table 1 Correlation between different factors

Factors	Real GDP per capita	Density of population	Car parc	Passenger turnover	Urban road area ratio
Real GDP per capita	1.00	-0.05	0.41	0.09	0.54
Density of population	-0.05	1.00	0.77	0.90	0.49
Car parc	0.41	0.77	1.00	0.82	0.55
Passenger turnover	0.09	0.90	0.82	1.00	0.51
Urban road area ratio	0.54	0.49	0.55	0.51	1.00

Next, we classify three categories based on cluster analysis of principal components in order to provide a reference for α value in different religions.

The calculated results of the initial load matrix are shown in table 2:

Table 2 Results of clustering

state	Category1	state	Category2	state	Category3
Alabama	2	Louisiana	2	Ohio	1
Alaska	3	Maine	3	Oklahoma	3
Arizona	2	Maryland	2	Oregon	2
Arkansas	3	Massachusetts	2	Pennsylvania	1
California	1	Michigan	2	Rhode Island	3

Colorado	2	Minnesota	2	South Carolina	2
Connecticut	2	Mississippi	3	South Dakota	3
Delaware	1	Missouri	2	Tennessee	2
Florida	1	Montana	3	Texas	3
Georgia	1	Nebraska	3	Utah	3
Hawaii	2	Nevada	3	Vermont	2
Idaho	3	New Hampshire	3	Virginia	2
Illinois	1	New Jersey	1	Washington	3
Indiana	2	New Mexico	3	West Virginia	3
Iowa	3	New York	1	Wisconsin	3
Kansas	3	North Carolina	1	Wyoming	3
Kentucky	2	North Dakota	3		

Category 1: The highway traffic and economy are developed and the density of population is very large. Moreover, the average car parc is large and the demand for the electric vehicles and convenience of charging is high. Therefore, the convenience coefficient is smaller.

Category 2: The highway traffic is moderately developed with a large amount of civil cars. The density of population is large and most of drivers drive from the first region to the second region, so the convenient coefficient is moderate.

Category 3: The highway traffic is generally developed and the density of population is small, so it just needs to meet the demand for fixed industrial production capacity. Therefore, the demand for charging piles is lower than the other two categories and convenience coefficient is relatively higher.

3.5 Calculation of the number of charging stations

Calculating the number of charging stations that each region requires and the area of each Voronoi diagram, we can get:

$$S_{total} = \sum_{i=1}^{k} S_i$$

$\boldsymbol{S_{total}}$—the total area of region

$\boldsymbol{S_i}$—the ith Voronoi area

Assuming that the total amount of electric vehicles in a specific region is $\boldsymbol{N_0}$, the amount of electric vehicles in this region is:

$$N_0 = \frac{N_0 \times e \times 50}{\beta \times 24}$$

$$e = f(\alpha)$$

e—the proportion of electric vehicles that need charge every day

$\boldsymbol{\beta}$—the average power of chargers

Therefore, we can get the amount of charging stations and electric vehicles in the region. Also, charging stations are uniformly distributed and the layout of the charging station in different regions[5] is:

Finally, we can get the amount of charging stations in the city, even in the whole country, by calculating the amount of electric vehicles and convenience coefficient in the regions.

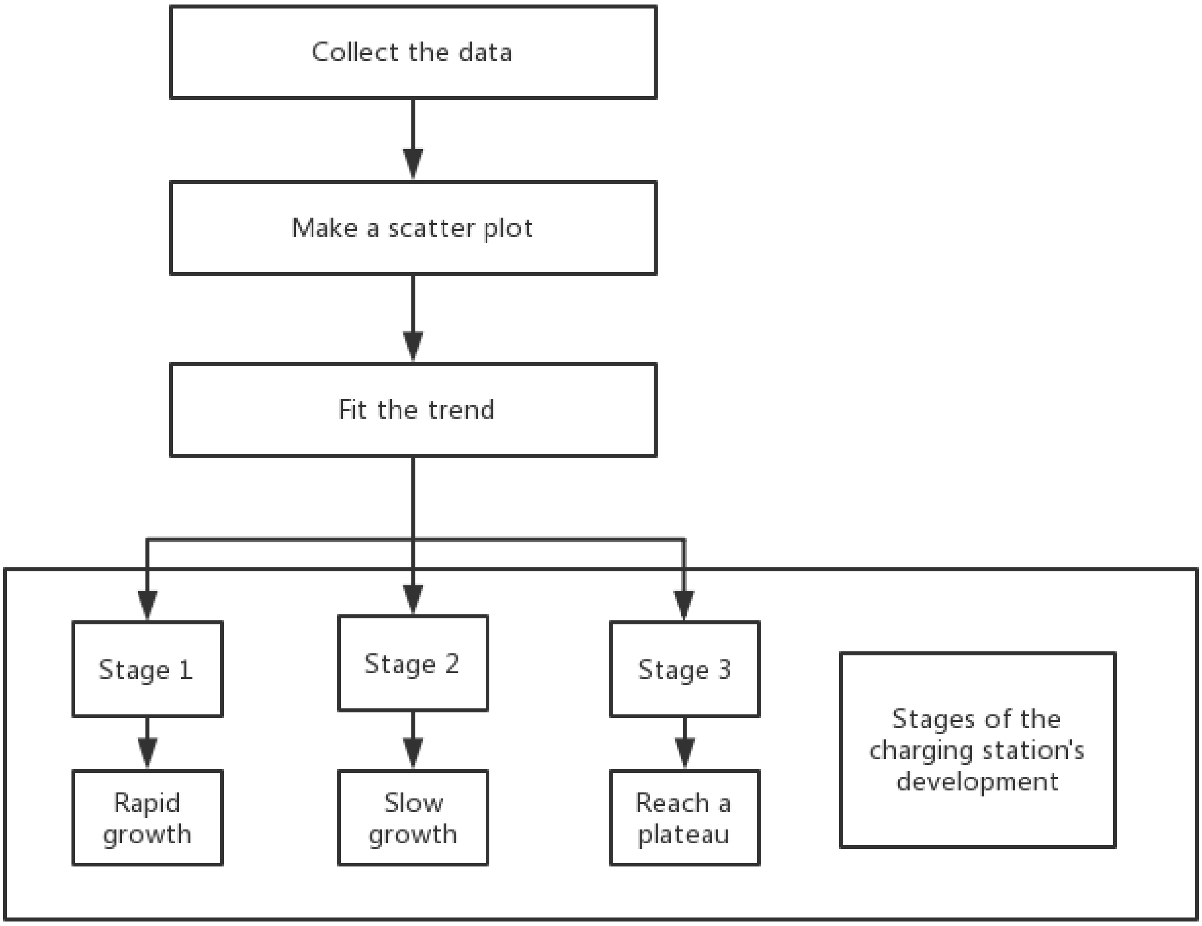
3.6 Conclusion

The key factor of the model is convenience coefficient α. We choose the density of population, GDP per capita, the urban road area ratio, passenger turnover and the number of cars as the standards to divide three categories by using hierarchical clustering. Also, we can get corresponding α of each category so that we can get the optimal amount of electric vehicles.

4 Model Two: Growth Model of charging stations

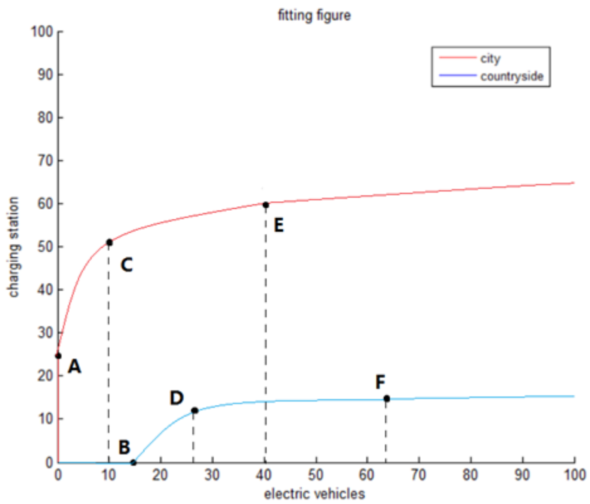
4.1 Introduction

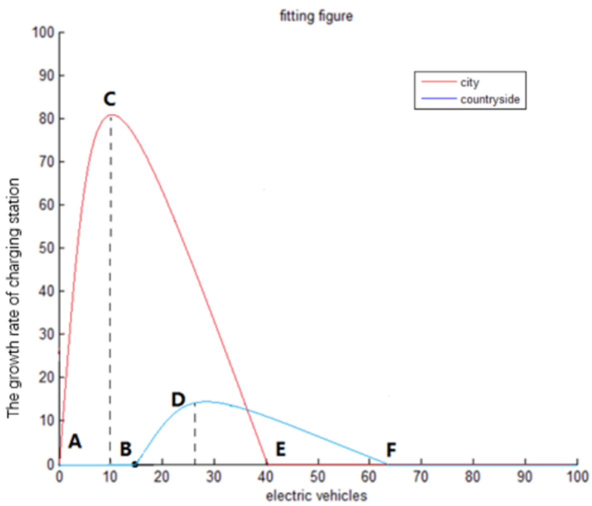
Considering the development status of electric vehicles in South Korea, we take South Korea for an example. We study a specific city (Seoul) to analyze the growth model of electric vehicles based on the development status of charging stations. To simplify the model, we assume the development of electric vehicles industry is from none to presence and all gas cars need convert to all-electric vehicles.



4.2 Growth model of charging stations

We collect the data about the cities and rural areas of South Korea and do a scatter plot analysis of the data and fit trend. The fitting results are:





It can be seen from the figures that the number of charging stations increases before it reaches to the saturation value no matter in rural areas or urban areas. However, people start to buy electric vehicles after appearing a certain number of charging stations in urban areas while the charging stations are built when the number of electric cars rises to a certain amount in rural areas.

The reason is that the convenience coefficients of cities and countryside are different. In urban areas, the convenient transportation and complete infrastructures can meet the demand for daily travelling. Also, people will not choose the electric vehicles that cannot be charged at home because of imperfect equipment and a quick pace of lifestyle in urban areas. On the contrary, people in rural areas can charge at home and the price of electricity is relatively lower than that of gas. Thus, they tend to choose electric vehicles.

Therefore, we connect the model 1 and conclude that we should consider the convenience coefficient of charging stations to decide the order of the promotion of electric vehicles and charging stations. If the convenience coefficient of charging stations is high, that is to say, the economic level in the area is quite low, so the priority of promotion goes to electric vehicles. On the other hand, if the convenience coefficient of charging stations is low, which means the economic level in the area is high, so the priority of promotion goes to charging stations.

4.3 Analysis

Comparing the model 1 and model 2, we analyze the model in three stages based on the different growth trend of urban areas and rural areas:

- First stage: rapid growth

  For the development of urban areas and rural areas, the number of charging stations grows rapidly at the beginning of the construction of charging station. The growth rate is largest when the number of electric vehicles rises to point C in urban areas and the number of electric vehicles rises to point D in rural areas

- Second stage: slow growth

  The number of electric vehicles rises slowly in this stage.

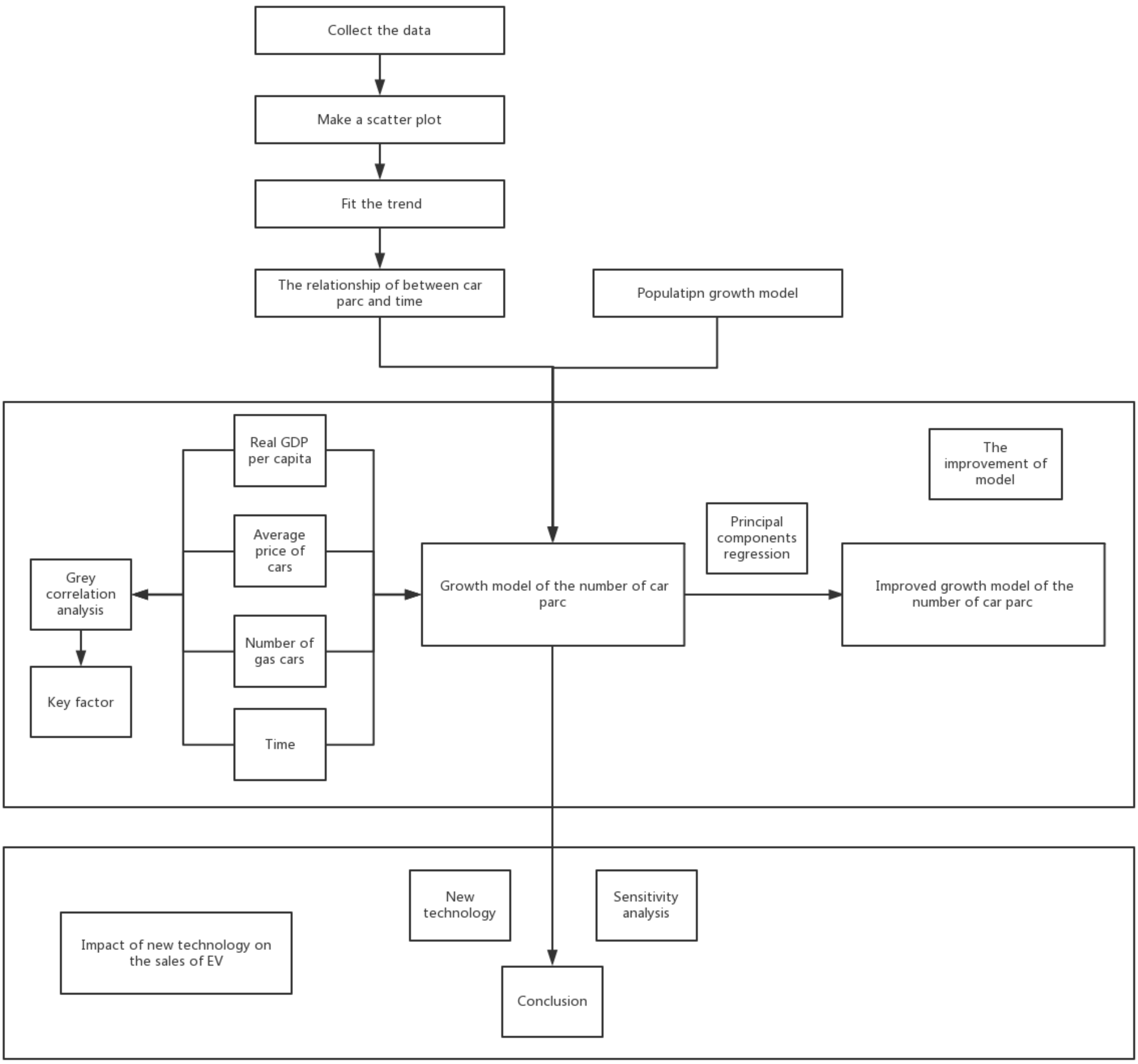
- Third stage: Reach a plateau

  The number of charging stations remains unchanged when it reaches at the saturation capacity E and F (in urban areas and rural areas respectively).

5 Model Three: Growth model of the number of electric vehicles

5.1 Introduction

In order to figure out the growth process of the electric vehicles in other countries and the key factors that influence the model, we fit the trend and compare the population growth model to get the growth model of the number of cars parc. Then, we improve the model and do a sensitivity analysis to work out the impact of the new technology.

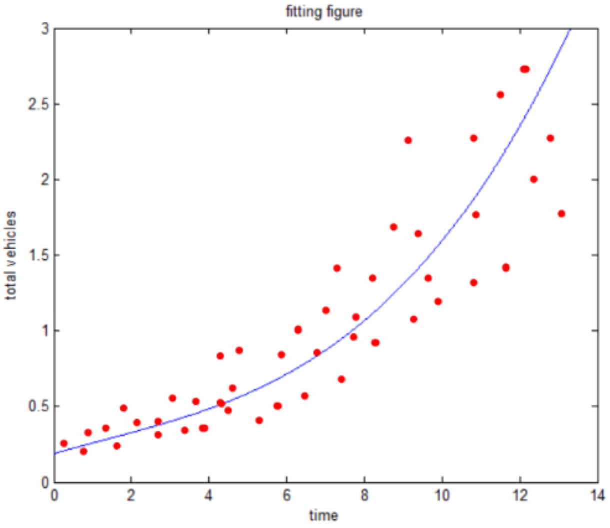


5.2 Assumptions

We assume that the increased number of cars is fixed at any time. The increased number of electric vehicles and gas vehicles is growing in opposite directions according to our assumption. To simplify the model, we also assume that the increased number of N(t) is continuous.

5.3 Growth Model

We collect the increased number of cars in South Korea in recent 10 years and fit them:



Analyzing the trend that the figure shows, we fit the relationship[6] between Y(t) and t approximately:

$$Y(t)=0.0011t^3-0.0043t^2+0.0738t+0.1876$$

Assuming the increased amount of electric vehicles fit Malthus's population exponential growth model, we can get:

$$Y(t+\Delta t)=Y(t)+aY(t)\Delta t-bY(t)\Delta t$$

t—time

Δt—variation of time

a—growth coefficient of electric vehicles

b—growth coefficient of gas vehicles

Then, we can get:

$$\frac{\Delta y}{\Delta t}=ay(t)-by(t)$$

Then, we can get:

$$y(t)=y_0e^{(a-b)(t-t_0)}$$

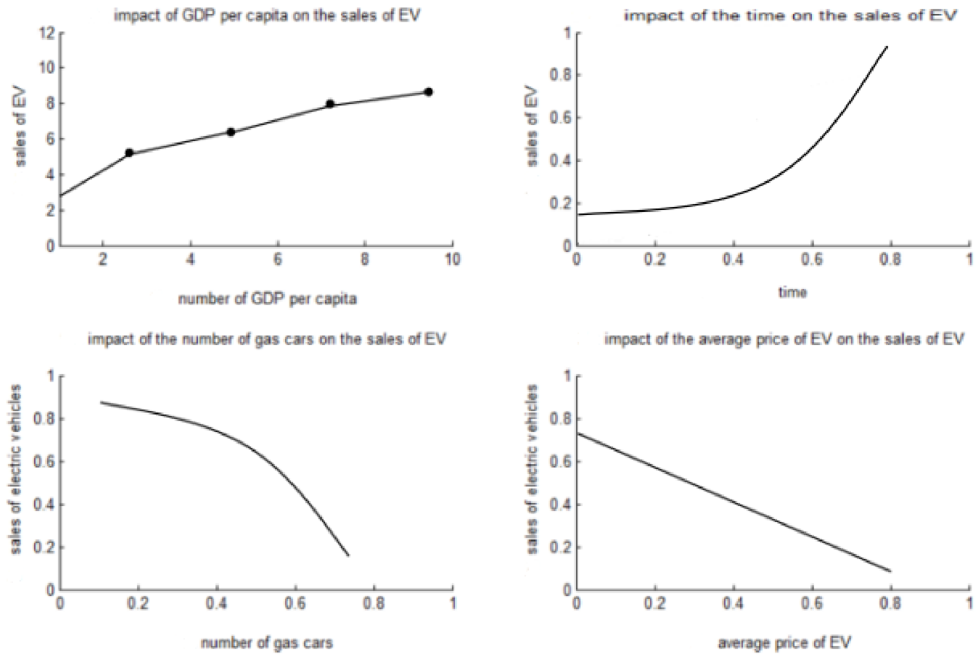
Thus, the proportion of electric vehicles:

$$w = \frac{y(t)}{Y(t)} = \frac{y_0 e^{(a-b)(t-t_0)}}{Y(t)}$$

5.4 Improvement of model

Considering the differences between regions and the impact of different wealth distribution on the usage of electric vehicles, we introduce GDP per capita, the number of gas cars, the average price of electric vehicles and other factors to figure out the key factors that affect the usage of electric vehicles based on the model 3.

We collect the data about the region to analyze the impact of the factors above on the sales of electric vehicles:



Next, we use grey correlation analysis to analyze these factors and get the grey correlation coefficient between variables:

Correlation factor	Sales of electric vehicles
GDP per capita X_1	0.6643
Average price of vehicles X_2	0.6520
Number of gas cars X_3	0.6509
Time t	0.6656

It can be seen from the table that the impact of these factors is different, that is to say, t>X_1>X_2> X_3.

Obviously, time affects the sales of electric vehicles the most and GDP per capita is the second. Therefore, we can make sure that GDP per capita is another key factor that affects the sales of electric vehicles.

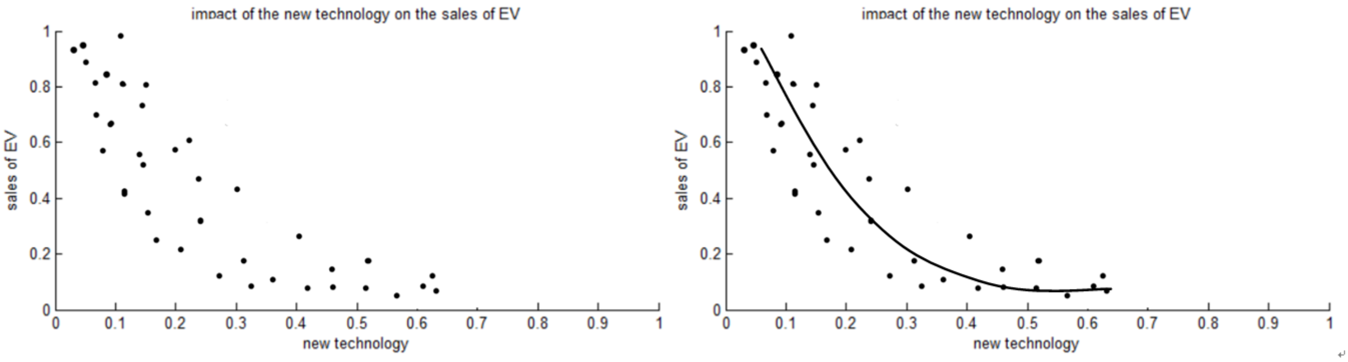
To avoid the multicollinearity, we use principal component regression to analyze the four variables above. And we get the model:

$$y = 0.2183 X_1^{0.2794} X_2^{0.2795} X_3^{0.2778} t^{0.2795}$$

The power exponent of variables does not represent the impact on the sales of electric vehicles because the standardization of the results above.

5.5 Sensitivity analysis

The new transportation can change the frequency of utilization and market share of the existing transportation. The new transportation is the complement of electric vehicles, which can slow down the growth of electric vehicles. The impact is:



From the model above, we conclude:

- The impact on the increased number of electric vehicles is small when the automatic driving, flying cars and other new transportation come out

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- It can be seen from the figure that the impact on the sales of electric vehicles is quite large when new technology comes out. However, the sales of electric vehicles would gradually remain stable when the new technology increases.

6 Conclusion

We build three models to work out the problems of the distribution of charging stations in different regions. Then, we make an investment plan for developing cities and countryside by comparing the development trend of charging stations and electric vehicles between urban areas and rural areas. Finally, we figure out the different geographical factors that influence the usage of electric vehicles by analyzing the factors that influence the sales of electric vehicles by the principal components analysis.

7 Strengths and Weaknesses

7.1 Strengths

- In Model 1, we divide a region into several categories in terms of geographical factors and take the economic level, the density of population, the urban road area ratio and passenger turnover into account so that we can get a admirable convenience coefficient.

- Our Models have robustness, the models adapt to other regions when we introduce other factors into models.

- In Model 3, we successfully use grey correlation analysis to analyze the impact of the key factors.

7.2 Weaknesses

- The assumptions are too many to predict the accurate location of charging stations in real life.

- The data collected is not enough to get precise models.

- The conclusions may be not accurate because of the different level of development and different attitude to electric vehicles.

8 Handout for the leaders who are attending an international energy summit

Energy problem is a frequently discussed topic in recent years and there have been some electric vehicles that can help reduce the emission of the harmful gas. Tesla is a successful example in this new area and charging system is developed in the United States, but it is also a fact that some countries do not know how to make a successful transformation of energy use. To help you figure out the problem, we provide some key factors that can help you make a plan to convert personal cars to all-electric cars and suggest a date for you to ban gas vehicles.

Key factors:

- The convenience coefficient α of the areas.
- The density of population
- The passenger turnover
- Real GDP per capita
- The urban road area ratio
- Car parc

A suggested date to ban gas vehicles:

We find that when the amount of electric vehicles reaches to 80% of all the cars, the growth rate of charging stations is almost 0 and the quantity remains unchanged. Therefore, people's life cannot be affected when you stop the supply of gasoline. At the same time, it is a good policy to promote the electric vehicles and encourage people to buy electric vehicles. Also, it can save the waste of energy and help protect the environment.

9 References

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[5] Kou L.F, Liu Z.F & Zhou H. Model and algorithm of regional charging station planning [J] modern electric power 2010.8, 27(4):44-48.

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10 Appendix and Supporting Datas

Grey correlation analysis:

```
function output=grayrela(x0)
for i=2:length(x0(:,1))
x1(i,:)=x0(i,:)-x0(i-1,:);
end
m=length(x1(1,:));
for i=1:m
x2(:,i)=x1(:,i)/std(x1(:,i));
end
```