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Asymmetric competition among nation states: A differential game approach☆

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ABSTRACT

This paper analyzes the impact of foreign investments on a small country's economy in the context of international competition. To that end, we model tax and public input competition within a differential game framework between two unequally sized countries. The model accounts for the widely recognized characteristic that small states are more flexible in their political decision-making than larger countries. However, we also acknowledge that small size is associated with limited institutional capacity in the provision of public services. The model shows that the long-term outcome of international competition crucially depends on the degree of capital mobility. In particular, we show that flexibility mitigates against – but does not eliminate – the likelihood of collapse in a small economy. Finally, we note that the beneficial effect of flexibility in a small state increases with its inefficiency in providing public services and with the degree of international openness.

1. Introduction

Small states generally suffer from limited access to capital and labor resources, both in amount and in variety. Then, foreign direct investments (in short, FDI, hereafter) can contribute significantly to their development (Read, 2008). In fact, small economies tend to have high level of access to private foreign capital as a ratio of total capital formation (Streeten, 1993). Using data from the World Bank, Fig. 1 suggests that the ratio of FDI flows to the gross fixed capital formation is higher in small countries (i.e., population less than two million)[1] than in large countries (i.e., population in excess of 30 million).[2] Moreover, the economic well-being of small countries is positively correlated with the ratio of FDIs. The data in Fig. 1 indicate that small countries above the average line, such as Luxembourg, Malta, Cyprus or Estonia, exhibit a high level of per capita GDP, whereas small countries below this threshold have a lower level of per capita GDP. This is confirmed in Fig. 2, which suggests that a direct relationship exists between the level of GDP per capita and foreign investments in small economies. In the cluster of larger countries, however, this relationship is hardly apparent.[3] Countries, such as Poland, Italy, Turkey, India and Spain appear above the threshold in Fig. 1, whereas the USA, Ukraine, Nepal, and Greece among others, are situated below it.[4]

Given these facts, this paper analyzes the impact of foreign investment flows on the economic performance of a small country competing internationally for mobile production factors. In this context, we investigate the conditions by which the economies of such countries can be viable, or even expand, in the long term. To that end, we develop a

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[1] Our data set contains 51 countries with population less than 2 million. This represents 72% of all the existing “small” countries. An exhaustive description cannot be provided due to a lack of relevant information.

[2] Our data set of countries with population in excess of 30 million is exhaustive. It contains 41 countries.

[3] Note that, we have not controlled for other determinants of per capita GDP; for example, the availability of natural resources. Taking into account oil reserves and the recent increase in oil prices would explain the position of Qatar or Brunei in our figures.

[4] The ambiguous role of FDIs on the economic performance of countries is documented in the literature (see, for example Alfaro et al., 2004).

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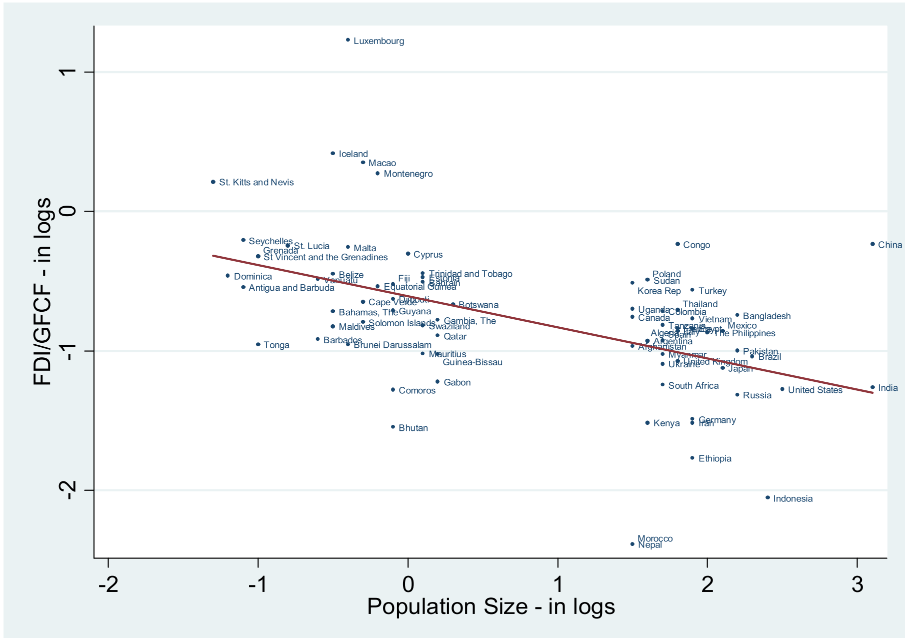

Fig. 1. Relationship between the ratio of FDIs to Gross Fixed Capital Formation and population from 2000 to 2010.
Source: World Bank.

dynamic framework to study how a small country attracts foreign capital through two policy instruments, namely taxes and public services.[5]

For the sake of simplicity, we focus on two competing countries of uneven size. In this study, size is defined as number of capital-owners in a respective country and these capital owners are simultaneously entrepreneurs and workers. By adopting this approach, our model focuses on the economic size of a country.

The dynamic aspect of international competition is addressed by a differential game framework in which the strategic behavior of the small country differs from that of its larger rival. We account for the widely recognized characteristic that small states are more flexible in their political decision making than much larger countries (see, in particular, Streeten, 1993).

We thus assume that the small country adopts a Markovian feedback behavior (i.e., the policy variables are continuously reset in response to the dynamics of the states of the world), whereas the larger country chooses an open-loop rule (i.e., the policy variables are set only once at the initial time). We also acknowledge that small size is associated with handicaps, as small economies are generally characterized by limited institutional capacity in the provision of public goods (Commonwealth Secretariat, 2000) relative to large countries. Finally, we assume that the capital owners living in both countries have heterogenous attitudes toward their attachment to home. Thus, they incur costs related to moving abroad. The extent of these costs depends on their attitudes toward their countries. Additionally, their decision to relocate their capital is affected by capital taxation and by productivity-enhancing public services.

The main results of the paper can be summarized as follows. First, the model shows that GDP, in particular the GDP per capita, of the small country increases with the flow of FDIs, which is consistent with the facts presented above. Moreover, the long-run solutions show that the economy of the small country can expand, shrink or even collapse. In this context, two cases can be distinguished: one exhibits high international openness and another exhibits low international openness. The fundamental difference between these cases is that the small country will only experience economic collapse if capital mobility is high (i.e., high international openness). However, higher efficiency in the provision of public services can partially countervail this effect by decreasing the likelihood of collapse. In the second case, when capital mobility is low, international competition for capital can eventually reduce the size of the small economy without provoking its collapse. If capital mobility is very low, the model shows that international competition tends to expand the economy of the small country. We also assess the extent to which flexibility is beneficial to the small country, given that it suffers from limited institutional capacity. By comparing the Markovian and open-loop outcomes, we find that flexibility mitigates against – but does not eliminate – the likelihood of a small economy collapse. Finally, we show that the benefit of flexibility increases in tandem with the inefficiency of public service provision and with the degree of international openness in the small country.

Our paper contributes to the existing literature in the following ways. First, we provide a dynamic counterpart to previous static papers in which countries compete with two instruments.[6] Following the Zodrow and Mieszkowsky (1986) model, there has been a growing body of literature on the joint role of taxes and public inputs in attracting mobile production factors. For example, Zissimos and Wooders (2008) analyze how the provision of public goods designed to reduce the production cost of private firms is able to relax international tax competition between governments of equal size. Benassy-Quere et al. (2007) provide an empirical analysis of the impact of taxes and public goods on the allocation of private capital. They find that both corporate taxes and public capital contribute significantly to inward FDIs. Pieretti and Zanaj (2011) propose a two-stage game in which both small and large jurisdictions compete for capital

[5] These public services contribute to the domestic attractiveness of private capital, as they are supposed to enhance private productivity. Examples of this are spending for the operation and maintenance of power and transportation infrastructures, operating costs of universities, and also the enforcement of property rights and the provision of capital market, labor and environmental regulations. It follows that countries' attractiveness may also be due to the quality of their institutions. In the Oxford Handbook of Entrepreneurship (Casson et al. 2006), it is argued that the abundance of entrepreneurs in a country depends on the existence of regulations, property rights, accounting standards and disclosure requirements, among other factors. Furthermore, in recent years, there has been a surge of national and cross-country studies relating economic development to institutions, especially institutions affecting capital market development and functionality (see, for example, La Porta et al., 1997).

[6] A exception is Wildasin (2003, 2011) who studies tax competition within an explicitly dynamic framework. In addition to other differences to our paper, he does not consider competition in a non-tax instrument.

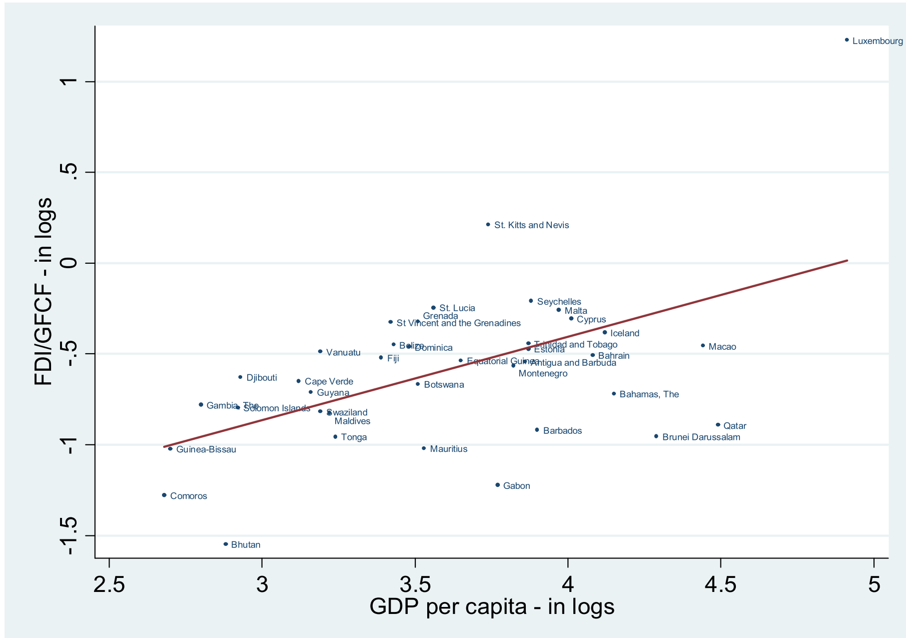

Fig. 2. Relationship between GDP per capita of small countries and the ratio of FDIs to Gross Fixed Capital Formation from 2000 to 2010. Source: World Bank.

using taxes and public goods as policy variables. These contributions are, however, static and thus unable to provide insights into dynamic outcomes. Differential games have already been applied to oligopolistic competition (Dockner and Jorgensen, 1984; Karp and Perloff, 1993; Cellini and Lambertini, 2004); however, few studies have applied differential games to tax competition. For example, Coates (1993) deals with the issue of property tax competition and partially analyzes the open-loop equilibrium of a dynamic game.[7] Secondly, by assuming that small countries are more flexible in taking decisions than their larger rivals but at a higher institutional cost as explained above, we account for behavioral and institutional asymmetries which, to the best of our knowledge, are not considered in the traditional tax competition literature.

We assume that the economic magnitude expressed in terms of productive resources can vary endogenously as a consequence of public policy and international competition, while the political size is fixed. Similar to our model, the contribution of de la Croix and Dottori (2008) is also concerned with the collapse of a community. To explain the tragedy of Easter Island, these authors show how a closed system can collapse as a result of non-cooperative bargaining between clans. The context and the methodology of their paper are, however, different from ours, given that they use an overlapping generations model in which people live for two periods and compete in fertility rates.

The paper is organized as follows. The next section models the dynamic competition between two countries of asymmetric size. In Section 3, we derive long-run solutions and Section 4 analyzes the long-run conditions of a small country. The importance of flexibility in small economies is assessed in Section 5 and Section 6 presents the conclusion.

2. The model

Suppose that the world is composed of two countries (regions) with unequal populations. Country size may be defined by population, area, or national income (Streeten, 1993). In this study, population, rather than area, is used to define country size. More precisely, size is defined with respect to the number of capital owners who populate the country and these capital owners are simultaneously entrepreneurs and workers. By adopting this approach, our model identifies a country by the size of its economy. Furthermore, capital owners (and their associated activities) are free to relocate to the neighbor country at any point in time. At time $t = 0$, capital flows have not yet taken place, so the population size in each country coincides with its native population.

At $t = 0$, the population of jurisdictions is evenly distributed with unit density on the interval $[-S_1(0), S_2(0)]$. The small country extends from $-S_1(0)$ to the origin 0, and the rest of the world extends from 0 to $S_2(0)$. It follows that the small economy has a size of $S_1(0)$, and the rest of the world has a size of $S_2(0)$, with $S_1(0) < S_2(0)$. We assume that the total number of firms is constant over time and is normalized to one. Thus, for any future time $t \geq 0$, $S_1(t) = S(t)$ and $S_2(t) = 1 - S(t)$.

2.1. Entrepreneurs

Each citizen is endowed with one unit of capital which is combined with her labor to establish a firm. Therefore, all citizens are self-employed entrepreneurs. Throughout the rest of the paper, we thus use firms and entrepreneurs interchangeably. The firms are distributed at their respective sub-interval according to their disposition to establish a firm outside of their home location. As in Ogura (2006), we assume that this population of entrepreneurs is heterogenous in the degree of their attachment to the home country.[8] Within the model, we dictate that the closer entrepreneurs are located to extremes of the interval, the more they are attached to their current location. Conversely, the closer that firms are to border 0, the less they are attached to their territory, and the easier it will be for them to relocate abroad.[9] This means that a firm of type $\alpha_1 \in [-S_1(0), 0]$ located at $t = 0$ in the home country incurs a disutility of relocating abroad equal to kd, where d is the distance between 0 and α_1.[10] The coefficient k represents the unit cost of moving capital abroad and can also be interpreted as the degree of international openness.

[7] As mentioned by Cardarelli et al. (2002).

[8] This characteristic was first considered in the fiscal competition research of Mansoorian and Myers (1993).

[9] For simplicity, we assume that firms can only relocate to their neighboring jurisdiction.

[10] It follows that $d = -\alpha_1$.

As in Pieretti and Zanaj (2011), we assume that each firm produces $q + a_i$ ($i = 1, 2$) units of a final good, where q is the private component of (gross) productivity.[11]

The fraction a_i of the produced good depends on the public input supplied by the home (foreign) jurisdiction.[12] Note that the product $S_i \cdot (q + a_i)$ represents the total output or GDP produced in country $i = 1, 2$. This implies that $q + a_i$ is the per capita output in a respective country. The total output is sold in a competitive (world) market at a given price normalized to one. Thus, we suppose that both countries have equal access to a common market.[13] This also implies that the smaller jurisdiction does not suffer from a reduced home market. We further consider that the unit production cost is constant and equal to zero without loss of generality. Each entrepreneur pays a tax on capital which is denoted by T_i ($i = 1, 2$) and levied in the country $i = 1, 2$.[14]

The temporal perspective of the setting described above is as follows. For each period $t \in [\Delta t, +\infty)$ and for any $\Delta t > 0$, governments update their choices in terms of the public services and taxes offered. In addition, we have to make more precise how firms update their location attachment after a subset of them decides to relocate their activity in another country. In other words, we need to consider how the ranking at date t is affected[15] by relocations at date $t - \Delta t$.

We now apply the following updating rule.

Assume first that $x(t) > 0$ ($x(t) < 0$) firms move at date t from country 1 (country 2) to country 2 (country 1). All the firms of type $\alpha(t) \in [-S_1(t), S_2(t)]$ at date t will be of type $\alpha(t + \Delta t) = \alpha(t) + u \cdot x(t)$ at date $t + \Delta t$, where u is an updating parameter with[16] $u \in [-1, 1]$. Three cases can be distinguished. First, we can assume that $u = 0$, which means that there will be no preference updating. This implies in particular that the firms which move to another location keep their initial attachment ranking to their country of origin. Second, we can assume that $0 < u \leq 1$. In this case firms value positively the presence of other firms in the same location. The industry size can be viewed as a source of positive externality which increases the attachment to the local site. A possible justification is that the feeling to be located at the right place is linked to the economic prosperity of the current location, which is reflected by the size of the economy in our model. If the updating parameter is positive but strictly smaller than 1, only a subset of the firms which relocate their activity will keep an attachment to their country of origin. This number will be $(1 - u)\, x(t)$ at time t, while $u \cdot x(t)$ firms which moved at t will now have an attachment to the destination country.

Finally, we can suppose that $-1 \leq u < 0$, in which case firms value negatively the presence of other firms in the same location. This situation can be viewed as the perception of a congestion effect.

More generally, the parameter u introduces an inter-temporal "contagion effect", which can be on the upside ($u > 0$), on the downside ($u < 0$) or nonexistent ($u = 0$).

Suppose now that an entrepreneur of type $\alpha_1(t)$ is located in country 1 at date t and considers staying at home or investing her physical capital abroad.[17] If she decides not to move, her profit is given by[18]

$$\pi_1(t) = q + a_1(t) - T_1(t). \tag{1}$$

If she invests abroad, her profit becomes

$$\pi_2(t) = q + a_2(t) - T_2(t) - kd(t),$$

where the distance $d(t)$ is the location attachment of the indifferent firm among those which are still located in country 1. Among the firms located in country 2 that are attached to country 1, the most attached will be of type $(1 - u)(S_1(0) - S_1(t))$ (where $u \in [-1, 1]$) in the preference space. This augments the distance from the origin of the indifferent firm. It follows that $d(t) = (1 - u)(S_1(0) - S_1(t)) + x(t)$.

It follows that the marginal entrepreneur of country 1 who is indifferent between investing abroad and staying at home verifies the condition

$$a_1(t) - T_1(t) = a_2(t) - T_2(t) - kd(t).$$

Conversely, if firms move from country 2 to country 1 ($x(t) < 0$), the marginal condition becomes, $a_1(t) - T_1(t) - kd'(t) = a_2(t) - T_2(t)$, where $d'(t) = (1 - u)(S_2(0) - S_2(t)) - x(t)$.

Consequently, we obtain

$$x(t, a_1, a_2, T_1, T_2) = \frac{a_2(t) - T_2(t)}{k} - \frac{a_1(t) - T_1(t)}{k} + (1-u)[S(t) - S(0)]. \tag{2}$$

In other words, the large country attracts capital ($x(t) = d'(t) > 0$) from the smaller jurisdiction if the net gain of investing abroad, $a_2(t) - T_2(t)$, is higher than the net gain of staying at home, $a_1(t) - T_1(t)$ after taking into account the mobility cost $kx(t)$. If $x(t) = -d'(t) < 0$, capital moves from the large jurisdiction to the smaller one.

The motion equation of the size of the small country's economy $S(t)$ is given by

$$\dot{S}(t) = -x(t) = \frac{a_1(t) - T_1(t)}{k} - \frac{a_2(t) - T_2(t)}{k} - (1-u)[S(t) - S(0)]. \tag{3}$$

For reasons of simplicity and without loss of generality[19] we consider in the following that $u = 1$.

2.2. Governments

Adopting a public-choice perspective, we posit that the governments maximize tax revenue.[20] To this end, countries compete simultaneously by using taxes and public services to attract entrepreneurs, and firms decide where to locate based on these government policies. We

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11 Because each unit of capital is indissolubly bound to a unit of labor, we assume that labor and capital are used in fixed proportions (i.e., Leontief production technology).

12 A public input satisfies the local public good characteristics; that is, it is jointly used without rivalry by firms located within the same jurisdiction. It follows that the benefits and costs of these goods only accrue at the jurisdictional level. As in Zissimos and Wooders (2008), we abstract from congestion costs. Incorporating congestion into the model would complicate our framework without qualitatively improving the results. Moreover, if public input represents immaterial goods as laws and regulations (e.g., protecting intellectual property and, specifying accurate rules for dispute resolution), the lack of congestion in our model is justified by the particular nature of these goods.

13 Recent empirical work (Guerin, 2006) demonstrates that the distance is economically more significant for FDI than trade, indicating that there are significant information costs to FDI in particular. Consequently, it is likely that trade, being less sensitive to distance, occurs among more countries than FDIs. In our model we account for this fact by assuming that foreign direct investments take place among two jurisdictions but trade occurs among many countries.

14 Given that each entrepreneur invests exactly one unit of capital in our model, the total tax will be T_i ($i = 1, 2$).

15 For a more technical discussion see Han et al. (2013).

16 More generally we can assume $u \in [\underline{u}, \overline{u}]$ with $\underline{u} < 0$ and $\overline{u} > 0$.

17 At each period t, the firms decide whether it is in their best interest to move their business. More exactly, they choose their best location given that tax rates and infrastructure expenditures are time-variant. We could assume that firms are forward-looking and able to anticipate the future choices of the other agents (firms and governments). However, it is possible to show, already in a three-period version of our model, that multiple solutions are very likely to appear (the details of this model are available on request). If this is the case, firms' decisions will be time inconsistent. This problem is drastically emphasized in continuous time. To be able to solve the model anyhow, we have to impose limiting conditions. Therefore, we assume that firms are myopic in their location choices.

18 For the sake of simplicity, we consider that q is such that the profit of each firm is positive for all equilibrium levels of public goods and taxes.

19 We show that the results of the paper are qualitatively not changed when we consider the general case $u \in [-1, 1]$. The calculations are provided in a supplement on the JPE website.

20 This assumption should not be interpreted in the classical sense given by Brennan and Buchanan (1980) and applied to Leviathan governments. We do not consider here that regulators are self-interested governments. We simply assume that collected taxes are used to finance not only productive public services but also public goods that do not directly affect the productivity of firms, such as green spaces, swimming pools, and security bodies.

suppose that the effective (net) tax revenue collected by the governments does not coincide with the gross amount of tax revenue collected. Following Vaillancourt (1989) and Blumenthal and Slemrod (1992), tax collection is costly due to the administration, monitoring and enforcing procedures associated with it (Kenny and Winer, 2006). If the marginal cost of collecting taxes rises, then the net tax revenue $R(t)$ at time t is a concave function of the collected taxes. For tractability reasons, the net tax revenue will be given by $R_i = \sqrt{S_i T_i}$.

The instantaneous objective function of government $i (i = 1, 2)$ is thus given by the following:

$$w_i(T_i, a_i) = \sqrt{S_i T_i} - \frac{\beta_i}{2} a_i^2, \tag{4}$$

where the second term is the cost of providing public inputs,[21] which is assumed quadratic, whereas β_i is a country specific efficiency parameter. Indeed, the higher the value of β_i, the higher the unit and marginal costs of providing public service.

The key focus of this paper is the long-run behavior of small states. To this end, we highlight two opposing features of small open economies.

First, according to the Commonwealth Secretariat (2000), the public sector of mini-states generally suffers from limited institutional capacity.[22] Moreover, it may be difficult for small states to recruit high-quality civil servants given their limited pool of candidates (Streeten, 1993). These factors can reduce the efficiency and increase the unit costs for the provision of public services (Briguglio, 1998). To account for these facts, we assume that $\beta_1 \equiv \beta > \beta_2$. Normalizing β_2 to 1, we impose $\beta > 1$. It follows that β represents the inefficiency of the small country relatively to the large one.[23]

However, small size is a source of more responsive decision-making in a changing economic environment. This can be the case for different reasons. First, small communities are intrinsically more able to reach a consensus on policy issues. This idea has long been put forward by philosophers and political scientists as acknowledged in Alesina (2003). Several economists (for example, Kuznets, 1960; Alesina and Spolaore, 1997; Streeten, 1993) recognize that small-sized communities display a high degree of political homogeneity. In particular, Streeten (1993) suggests that problems related to collective action can be solved more easily in small countries, whereas larger jurisdictions are not able or not willing to attain this degree of flexibility in their decision making. It follows that, mobilization around a common effort should be easier. For example, Kuznets (1960) notes that one advantage of small states is to have small and more cohesive populations, which allows them to adapt better to change. In the same vein, Armstrong and Read (1995) recognize that smallness facilitates greater single-mindedness and focus on economic policy-making and a more rapid and effective response to exogenous change. Indeed, reforming existing laws or passing new ones takes much longer in large and diversified economies, where any change in the status quo requires long negotiations involving a large variety of interest groups.

Another reason of higher adaptability in the decision-making is that small countries are specialized in a handful of sectors. Thus the absence of a wide range of lobbyists makes the parliament and the entire administrative body much more responsive.

Finally, small developed economies have to adapt more quickly to a changing environment because they are highly open to the rest of the world and thereby, subject to more volatile business cycles than larger countries. Consequently, responsiveness to external shocks is a question of economic survival. Rodrik (1998) demonstrates that highly open countries and thus small countries have proportionally larger governments in order to mitigate the exposure to the insecurities generated by extreme openness. According to Katzenstein (2003) what really matters politically regarding small economies is their perceived (external) vulnerability. He notes that, "Perceived vulnerability generated an ideology of social partnership that acted like a glue for the corporatist politics of the small European states" (Katzenstein, 2003, p. 11). Moreover, because of their high exposure to international shocks, they created relatively robust welfare states in order to reach political bargains. In other words, small economies achieve social cohesion through redistribution policies.

To capture the just highlighted concept of flexibility, we assume that the large jurisdiction commits to a policy path that was adopted at the beginning of the game (i.e., open-loop strategy), whereas policymakers in the small jurisdiction adopt a Markovian feed-back strategy.

This mixed representation offers a convenient way of modeling differences in flexibility of decision making (Dockner et al., 2000). Although small in a political sense, the mini-state can grow larger as a result of sustained capital inflows. The small country's size could thus exceed a critical threshold that would cause the large country to react more aggressively by also adopting a Markovian strategy. To rule out such a behavioral change, we assume that the size of the small economy will remain tiny enough. In other words, we assume that the size $S(t)$ is bounded from above and impose $S(t) \leq \bar{S} < \frac{1}{2}$, for any $t \geq 0$.

The dynamic objective-functions of the competing jurisdictions are respectively[24]

$$J_1 = \max_{a_1, T_1} \int_0^{+\infty} e^{-rt} w_1(T_1(S,t), a_1(S,t)) dt, \tag{5}$$

$$J_2 = \max_{a_2, T_2} \int_0^{+\infty} e^{-rt} w_2(T_2(t), a_2(t)) dt, \tag{6}$$

where r is the discount rate of the public decision-makers, which should reflect the degree of impatience of the population. Given that there is no evidence that this rate is dependent on the size of a population, we accept that r is common to both jurisdictions.

3. Steady states and the long-run policy mix

As explained above, we assume that the small jurisdiction adopts a Markovian policy setting, and its larger rival chooses open-loop strategies when designing its optimal decision path. The long run solutions of the above dynamic system are highlighted in the following proposition. The proof is given in Appendix A.1.

Proposition 1. *For any given parameters k, r, β, there exists a Nash equilibrium characterized by the following interior steady state*

$$\hat{S} = \frac{(kr)^{-\frac{3}{2}}}{6\sqrt{2}} \left(\frac{\sqrt{2}}{\beta} - 1 \right) + \frac{2}{3}, \tag{7}$$

$$\hat{a}_1 = \frac{1}{2\beta} \left(\frac{1}{kr} \right)^{\frac{1}{2}}, \hat{T}_1 = kr\ \hat{S},\ \ \hat{a}_2 = \frac{1}{2} \left(\frac{1}{2kr} \right)^{\frac{1}{2}}, \hat{T}_2 = 2kr \left(1 - \hat{S} \right), \tag{8}$$

with the costate variables $\hat{\lambda}_1 = \frac{1}{2}\left(\frac{k}{r}\right)^{\frac{1}{2}}, \hat{\lambda}_2 = -\frac{1}{2}\left(\frac{k}{2r}\right)^{\frac{1}{2}}$. *In the state space of*

[21] In this paper we stress the importance of public expenditures which improve private productivity. In reality public expenditures are used for many other policy variables. So the policy relevance of the paper is only to stimulate further thoughts.

[22] In small states, the median wage bill of the public sector as a proportion of GDP is 31%, whereas the ratio is 21% in large developing countries (Commonwealth Secretariat and World Bank, 2000).

[23] To be consistent, the parameter β should be inversely correlated with the size of the small country. Taking into account this feature would however complicate the analysis without important additional insight. Therefore, we shall assume that the small country is tiny enough to consider β as given. For that reason we assume that the size S_1 is bounded from above by $\bar{S}$ where $\bar{S} < \frac{1}{2}$.

[24] Similar to Barro (1990), we consider that the government provides flows of public services. It follows that the public service provision will be treated as a control variable.

the dynamic system, it is locally asymptotically stable.[25] *It follows that the long run policy mix of country i* ($i = 1, 2$) *is given by the pair* $(\hat{a}_i, \hat{T}_i)$.

We now impose that the steady state size[26] of the small country's economy remains smaller than $\frac{1}{2}$. This means that $\hat{S}<\frac{1}{2}$ requires that $k<k^* = \left(\frac{1}{2}\right)^{\frac{1}{3}}\frac{1}{r}$ and $\beta>\underline{\beta} = \frac{\sqrt{2}}{1-\sqrt{2}(kr)^{\frac{3}{2}}}$.

It is convenient to show that the long-run per capita GDP $(\hat{a}_1 + q)$ of the small country increases with $\hat{S}$.[27] Given that $\hat{S}$ is positively related to the FDI inflows, our model is consistent with the stylized fact highlighted in Fig. 2, which shows that the per capita output of small economies improves with inward foreign investments. This positive relationship results from infrastructure expenditures that impact the productivity of firms and, thus, improves the attractiveness of the location to foreign investments.

Proposition 2. *The smaller economy always undercuts the rival's tax rate but provides less infrastructure services.*

Indeed, it is readily verified that $\hat{a}_2-\hat{a}_1 = \frac{1}{4\beta}\left(\sqrt{2}\beta-2\right)\sqrt{\frac{1}{kr}}>0$ for $\beta>\sqrt{2}$ and $\hat{T}_2-\hat{T}_1 = kr\left(2-3\hat{S}\right)>0$, given that $\hat{S}<\frac{1}{2}$. In other words, the small economy will always be tax competitive but the public services it provides will never be attractive to investors. This result is consistent with the literature on tax competition among economies of uneven size (Bucovetsky, 1991; Wilson, 1991; Kanbur and Keen, 1993; Trandel, 1994), which demonstrates that the benefit of smallness translates into the ability to undercut the tax rates of larger countries. This means that the small country will never be able to tax more than its large rival, which contrasts with some static models on competition in taxes and infrastructure (Hindriks et al., 2008; Pieretti and Zanaj, 2011). The reason is that the small country has a relative disadvantage in providing infrastructure services due to its limited public capacity.

It appears that, the less efficient the small country is in providing public services, the more it will lower its tax rate. Indeed, it is easy to see that $\hat{a}_2-\hat{a}_1$ and $\hat{T}_2-\hat{T}_1$ rise with β. It is interesting to note that increasing international openness (lower k) increases the expenditure gap and the tax gap between the competing countries. Thus, the higher the capital mobility, the more the small country will be inclined to undercut the tax rate of its rival.

Finally, if the long-run solutions have to guarantee non-negative net budget constraints of both economies, the following two conditions must hold.[28] Either (a) $k^*>k\geq\overline{k}$ with $\overline{k} = (\frac{1}{32})^{\frac{1}{3}}\frac{1}{r}$, or (b) k verifies $\underline{k}<k\leq\overline{k}$, with $\underline{k} = \left(\frac{1}{50}\right)^{\frac{1}{3}}\frac{1}{r}$ and β satisfies $\underline{\beta}<\beta\leq\overline{\beta}$, with $\overline{\beta} = \frac{1}{2\sqrt{2}-16(kr)^{\frac{3}{2}}}$. The long run budget constraint of the large country will be satisfied if $\hat{w}_1\geq 0$, because there are relatively less stringent conditions on the parameters of the large country.[29] It is not excluded that the competing countries may issue temporarily debt to fund their infrastructure expenditures. This raises the question whether the intertemporal budget constraint is satisfied in both countries. In Appendix B we prove that the present value of net revenues in country i is positive if the steady state net revenue in country i is positive $(\hat{w}_i>0, i = 1, 2)$.

4. Will small states survive in the long run?

In this section, we focus our attention on the conditions under which the production potential of the small economy will expand $(\frac{1}{2}>\hat{S}>S(0))$, shrink $(\hat{S}<S(0))$ or even collapse $(\hat{S} = 0)$.[30] Two cases can be considered according to the degree of capital mobility.

Case 1. High degree of international openness: $\underline{k}<k<\overline{k}$.

In this case, the survival of the small economy depends on its relative efficiency in providing public services. Two sub-cases can be distinguished: one in which capital mobility is very high, i.e., $\underline{k}<k<k^s$ with[31] $k^s = \frac{1}{2[2+S(0)])^{\frac{2}{3}}}\frac{1}{r}$, and a second one in which capital mobility is moderately high, i.e., $k^s<k<\overline{k}$. In the first sub-case, it is readily verified that the small economy expands in the long run, $\hat{S}>S(0)$, if $\beta<\overline{\beta}$. However, if the relative efficiency of provision of public services in the small economy is too low (i.e., if $\beta>\overline{\beta}$), it will collapse. Furthermore, as the mobility cost approaches its lower bound $\underline{k}$, the small country is more likely to collapse. This occurs because the small economy has to lower its taxes to such an extent that it can no longer sustain its public expenditures $(\hat{w}_1<0)$. There are two extreme outcomes in the long-run. Either the small economy expands, or collapses. Therefore, if it shrinks, it must collapse.

This extreme scenario changes in the second sub-case (see Fig. 3). According to the values taken by β, the small economy can expand, collapse and shrink without collapsing. If $\beta < \beta^s$ with $\beta^s = \frac{\sqrt{2}}{1-6\sqrt{2}\left[\frac{2}{3}-S(0)\right](kr)^{\frac{3}{2}}}$, it will expand, and if $\beta>\overline{\beta}$, it will collapse. For an intermediate efficiency value, i.e., $\beta^s<\beta<\overline{\beta}$, the small country will shrink but still survive.

The following proposition can then be stated[32]:

Proposition 3. *Assume that international openness is high. The small country's economy can expand if it is relatively efficient in providing public services. Otherwise, its economy will shrink or even collapse in the long run.*

In a world of mobile capital, a small economy may have difficulty surviving even if it is able to adapt to change more quickly than larger countries. This can occur because the efficient provision of public services and capital mobility are crucial to generating the resources necessary to afford further public amenities. In fact, the model shows that below a given threshold, rising capital mobility causes the small economy to cut its taxes to such an extent that its budgetary resources vanish. It follows that small states, but especially micro-states, can secure their status in a global economy if their public sectors provide public services with sufficient efficiency and if their tax rates are more favorable than those of larger countries. At best, this is a necessary condition for attracting foreign capital, or at least, surviving.

Case 2. Low degree of international openness: $k^*>k>\overline{k}$.

In this case, the relative inefficiency of the provision of public services can no longer lead to the collapse of an economy because budget resources are not constrained. Formally, the limit value $\overline{\beta}$ tends to ∞ if k approaches $\overline{k}$. This is in marked contrast with the first case, as – in this case – a low degree of financial openness makes capital more captive and provides sufficient tax revenues to cover public expenditures.

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[25] We present the convergence path in Appendix A.2.

[26] More generally, for $u\in[-1, 1]$, it is possible to demonstrate that the steady state size of the small economy is $\hat{S}(u) = \frac{\left(\frac{1}{kr+k(1-u)}\right)^{\frac{1}{2}}\frac{1}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\beta}-1\right)}{3kr+4k(1-u)} + \frac{2r+(1-u)(2+S(0))}{3r+4(1-u)}$.

[27] The steady-state value $\hat{a}_1$ written as a function of $\hat{S}$ is $\hat{a}_1 = 3kr\left(\hat{S}-\frac{2}{3}\right) + \frac{1}{2}\left(\frac{1}{2kr}\right)^{\frac{1}{2}}$. It follows that $\frac{\partial\hat{a}_1}{\partial\hat{S}}>0$ is always true.

[28] The long run net budgets of the small and the large countries are $\hat{w}_1\left(\hat{T}_1, \hat{a}_1\right) = \frac{16\beta\left((kr)^{\frac{3}{2}}-\frac{1}{8}\sqrt{2}\right)+1}{24kr\beta}$ and $\hat{w}_2\left(\hat{T}_2, \hat{a}_2\right) = \frac{5\beta-8\sqrt{2}+16\sqrt{2}\beta(kr)^{\frac{3}{2}}}{3\beta}$, respectively.

[29] It also appears that $\hat{S}\in(0, \frac{1}{2})$ in cases (a) and (b).

[30] We impose (see proof in Appendix A.3) that $S(t)\leq\overline{S}<\frac{1}{2}$. If so, $\underline{\beta}$ would depend on the upper bound of $\overline{S}$. Thus, $\underline{\beta}\left(\overline{S}\right) = \frac{\sqrt{2}}{1+6\sqrt{2}\left(\overline{S}-\frac{2}{3}\right)(kr)^{\frac{3}{2}}}$, in which $\overline{S}$ is decreasing.

[31] It is readily verified that $k^s<\overline{k}$ if $0<S(0)<\frac{1}{2}$.

[32] Considering the general case where $u\in[-1, 1]$ does not alter the basic messages of the paper. In particular, we can show (the proof is available as a supplement on the JPE website) that the uniqueness of the steady state and asymptotic stability remain. The speed of adjustment to the steady state changes with u, but monotonic convergence to the steady state is warranted. Our calculations also demonstrate that, for any $u \in [-1, 1]$, the small economy can expand, decline or collapse according to the parameters of the model.

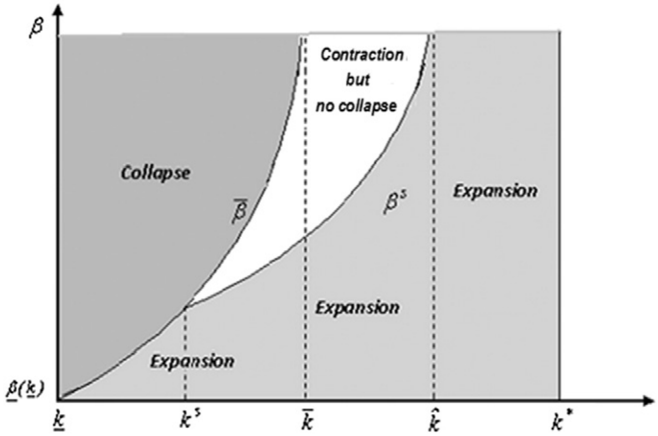

Fig. 3. The evolution of the small country's economic potential according to the mobility cost (k) and the degree of public inefficiency (β).

At worst, the economy of the small country can contract ($0<\hat{S}<S(0)$). This occurs if $\hat{k}>k>\overline{k}$ and $\beta>\beta^s$, with $\hat{k}=\left(\frac{1}{8[2-3S(0)])^2}\right)^{\frac{1}{3}}\frac{1}{r}$. However, if mobility is very low, i.e., $k^*>k>\hat{k}$, the small economy will attract foreign capital and thus expand. Surprisingly, this scenario occurs independently of the level of inefficiency.

We conclude with the following proposition:

Proposition 4. *Assume that international openness is low ($k^*>k>\overline{k}$). The small country's economic size never collapses but may shrink if the degree of international openness is not sufficiently low. In this case, the survival of the economy is independent of the efficiency of public service provision (β).*

We provide a summary illustration of the different cases with respect to the parameter values of k and β in Fig. 3.

5. How important is flexibility to the small economy?

To assess how beneficial flexibility is to the small country, we first calculate the long-run production potential $\widetilde{S}$ of the small country if it chooses an open-loop behavior identical to its larger rival. We thus obtain

$$\widetilde{S}=\frac{(kr)^{-\frac{3}{2}}}{4}\left(\frac{1}{\beta}-1\right)+\frac{1}{2}.$$

The benefit of flexibility can be represented by the difference $\hat{S}-\widetilde{S}$, which is obtained by comparing the Markovian and open-loop outcomes. It is easy to verify that this difference is always non-negative. Therefore, given the same parameters, the Markovian behavior adopted by the small country is preferable to the open-loop behavior. However, flexibility does not completely eliminate the potential for collapse; it only makes its occurrence less likely.

Given that $\frac{\partial(\hat{S}-\widetilde{S})}{\partial\beta}>0$, the advantage of the small country's flexibility increases with its inefficiency to provide public services. In other words, the economic size of the small country is more sensitive to an increase in efficiency (β decreases) in the Markovian scenario.[33] Consequently, flexibility counterbalances inefficiency, and the more inefficient a small country is in providing public inputs, the more valuable flexibility is to its long-run survival.

Furthermore, higher capital mobility increases the relative advantage of flexibility, given that $\frac{\partial(\hat{S}-\widetilde{S})}{\partial k}<0$. Note that increased capital mobility reduces (k increases) the long-term economic potential of the small economy; however, this occurs to a lesser extent in the Markovian scenario. It follows that flexibility countervails the negative effect of high capital mobility, and flexibility brings greater benefits to the small country when capital mobility is low. So, we can conclude by the following proposition.

Proposition 5. *The benefit of flexibility decreases with the small country's efficiency to provide public services and increases with capital mobility.*

We finally observe that similar to the Markovian scenario, the small country never collapses by adopting an open-loop behavior when capital mobility is sufficiently low. However, this condition becomes more restrictive in the open-loop scenario. Indeed, the absence of flexibility in policy making requires now that the mobility cost is higher than $\overline{\overline{k}}$, which exceeds the threshold $\overline{k}$ corresponding to the Markovian case.[34]

6. Conclusion

In this paper, we investigate whether a small open economy can survive in the long-run when facing global competition. To this end, we model the dynamic competition between two unequally sized economies. The policy makers of these two countries compete simultaneously by taxing mobile capital and offering public services. Firms choose to locate their capital in the country where their profits are maximized. We characterize the heterogenous behaviors of the two governments within a differential game framework, in which the small state adopts Markovian (i.e., flexible) behavior, and its larger rival commits to a strategy developed at the initial time point (i.e., open-loop behavior).

The results show that under conditions of high capital mobility, the small economy will risk economic collapse if it provides public services inefficiently. When capital mobility is very low, the economy of the small state always expands despite its limited institutional capacity.

However, further research is needed. In the present study, countries are treated solely as maximizers of tax revenue, and this overemphasizes the role of tax rates in the long-run outcomes. Therefore, it would be interesting to analyze a scenario in which governments are welfare maximizers and take into account the well-being of their populations. The present paper also models the private sector in an elementary way. Countries are undifferentiated in their ability to produce private goods and the production process is static. Future research should thus consider how international competition is able to impact the growth process of these competing economies when private productivity differs between jurisdictions.

Appendix A

A.1. Solution of the differential game in Section 3

We define as follows the notion of heterogenous strategic behavior that is used in Dockner et al. (2000, pages 87–92).[35]

Definition 1. A 2-tuple (Ψ_1,Ψ_2) of functions $\Psi_1:[0,1]\times[0,+\infty)\rightarrow R_+^2$ and $\Psi_2:[0,+\infty)\rightarrow R_+^2$, with $\Psi_1=(\Psi_{11}(S,t),\Psi_{12}(S,t)),\forall\,(S,t)\in[0,1]\times[0,+\infty)$ and $\Psi_2=(\Psi_{21}(t),\Psi_{22}(t))$, constitutes a heterogenous Strategic Nash Equilibrium if an optimal control path exists and is given by the Markovian Strategy $(a_1(t),T_1(t))=(\Psi_{11}(S(t),t),\Psi_{12}(S(t),t))=\Psi_1(S(t),\,t)$ of player 1, and an open-loop strategy $(a_2(t),\,T_2(t))=(\Psi_{21}(t),\Psi_{22}(t))=\Psi_2(t)$ of player 2.

[33] In fact, it is convenient to verify that $\left|\frac{\partial\hat{S}}{\partial\beta}\right|<\left|\frac{\partial\widetilde{S}}{\partial\beta}\right|$.

[34] It is convenient to show that $\overline{\overline{k}}=(\frac{1}{4})^{\frac{1}{3}}\frac{1}{r}$.

[35] A different but similar idea of guessing *symmetric* strategies via the Pontryagin maximum principle is also used in Cellini and Lambertini (2004 and the references therein). For a recent and detailed survey see Long (2010).

The small open economy (the Markovian strategic player) takes the large country's (open loop) strategy $\Psi_2(t)$ as given, and hence, faces the following optimization problem

$$\begin{cases} \max\limits_{a_1,T_1} \int_0^\infty e^{-rt}\left[(S(t)T_1(S,t))^{\frac{1}{2}} - \frac{\beta}{2}a_1^2(S,t)\right], \\ \text{subject to } \dot{S}(t) = \frac{a_1(S,t)-T_1(S,t)}{k} - \frac{\Psi_{21}(t)-\Psi_{22}(t)}{k}. \end{cases} \quad (9)$$

The corresponding current-value Hamiltonian is

$$\mathcal{H}_1(T_1,S,a_1,\lambda_1) = \left[S^{\frac{1}{2}}(t)T_1^{\frac{1}{2}}(S,t) - \frac{\beta}{2}a_1^2(S,t)\right] + \lambda_1\left(\frac{a_1(S,t)-T_1(S,t)}{k} - \frac{\Psi_{21}(t)-\Psi_{22}(t)}{k}\right)$$

with the costate variable λ_1.

The large economy faces the following problem

$$\begin{cases} \max\limits_{a_2,T_2} \int_0^\infty e^{-rt}\left[((1-S(t))T_2(t))^{\frac{1}{2}} - \frac{1}{2}a_2^2(t)\right], \\ \text{subject to } \dot{S}(t) = \frac{\Psi_{11}(S,t)-\Psi_{12}(S,t)}{k} - \frac{a_2(t)-T_2(t)}{k}. \end{cases} \quad (10)$$

The large country conjectures that the small economy's strategies[36] are $\Psi_{11}(S,t) = \frac{1}{\beta k}\lambda_1(t)$ and $\Psi_{12}(S,t) = \left(\frac{k}{2\lambda_1(t)}\right)^2 S$, $\forall\, S \in [0,1]$ and $t \geq 0$. The current-value Hamiltonian of the large economy is defined by

$$\mathcal{H}_2(T_2,S,a_2,\lambda_2) = \left[(1-S(t))^{\frac{1}{2}}T_2^{\frac{1}{2}}(t) - \frac{1}{2}a_2^2(t)\right] + \lambda_2\left(\frac{\Psi_{11}(S,t)-\Psi_{12}(S,t)}{k} - \frac{a_2(t)-T_2(t)}{k}\right)$$

with the costate variable λ_2.

The first order conditions yield the small economy's equilibrium choices $T_1(S,t) = \left(\frac{k}{2\lambda_1}\right)^2 S$ and $a_1(S,t) = \frac{\lambda_1}{k\beta}$. The costate variable verifies the equation $\dot{\lambda}_1(t) = r\lambda_1 - \frac{k}{4\lambda_1}$ with the transversality condition $\lim_{t\to\infty} e^{-rt}\lambda_1(t)S(t) = 0$.

The optimal choices of the large economy are $a_2(t) = -\frac{\lambda_2(t)}{k}$ and $T_2(t) = \left(\frac{k}{2\lambda_2(t)}\right)^2(1-S(t))$ with the costate equation

$$\dot{\lambda}_2(t) = r\lambda_2 - \frac{k}{4\lambda_2} + \frac{k}{4}\frac{\lambda_2}{\lambda_1^2}. \quad (11)$$

The associated transversality condition is $\lim_{t\to\infty} e^{-rt}\lambda_2(t)S(t) = 0$.

We can readily check that the maximized Hamiltonians $H_1^*(S,\lambda_1)$ and $H_2^*(S,\lambda_2)$ are

$$\mathcal{H}_1^*(S,\lambda_1,t) = \left[\frac{k}{2\lambda_1}S - \frac{\beta}{2}\left(\frac{\lambda_1}{k\beta}\right)^2\right] + \lambda_1\left(\frac{\frac{\lambda_1}{k\beta} - \left(\frac{k}{2\lambda_1}\right)^2 S}{k} - \frac{\left(-\frac{\lambda_2}{k}\right) - \left(\frac{k}{2\lambda_2}\right)^2(1-S)}{k}\right)$$

and

$$\mathcal{H}_2^*(S,\lambda_2,t) = \left[-\frac{k}{2\lambda_2}(1-S) - \frac{1}{2}\left(-\frac{\lambda_2}{k}\right)^2\right] + \lambda_2\left(\frac{\frac{\lambda_1}{k\beta} - \left(\frac{k}{2\lambda_1}\right)^2 S}{k} - \frac{\left(-\frac{\lambda_2}{k}\right) - \left(\frac{k}{2\lambda_2}\right)^2(1-S)}{k}\right).$$

It is straightforward to see that the equilibrium Hamiltonians are concave with respect to the state variable S. Hence, $a_i(t)$, $T_i(t)$ $(i = 1, 2)$ are optimal paths. It follows that the large country's conjecture about the rival's strategy is optimal. Consequently, the solutions $\Psi_1(S, t) = (a_1(S,t), T_1(S,t))$ and $\Psi_2(t) = (a_1(t), T_2(t))$ for $S \in [0, 1]$ and $t \geq 0$ constitute a non-degenerate Markovian Nash Equilibrium. QED.

A.2. Trajectories

The above analysis shows that there exists a stable trajectory associated to the dynamic system. Taking into account the initial and transversality conditions, the FOCs yield the explicit trajectories

$$\lambda_1(t) = \frac{1}{2}\left(\frac{k}{r}\right)^{\frac{1}{2}}, \quad \lambda_2(t) = -\frac{1}{2}\left(\frac{k}{2r}\right)^{\frac{1}{2}}.$$

The trajectory of the state variable is

$$S(t) = \left(S(0) - \hat{S}\right)e^{-3rt} + \hat{S}, \quad (12)$$

which is the optimal path converging to the steady state. The convergence speed is $3r$.

A.3. State variable constraint $S(t) \leq \bar{S} < \frac{1}{2}$

Recalling that the size of the small economy is constrained $\left(S(t) \leq \bar{S} \leq \frac{1}{2}\right)$, we adapt the Lagrangian function as follows

$$\mathcal{L}_1(T_1,S,a_1,\lambda_1) = \left[S^{\frac{1}{2}}(t)T_1^{\frac{1}{2}}(S,t) - \frac{\beta}{2}a_1^2(S,t)\right] + \lambda_1\left(\frac{a_1(S,t)-T_1(S,t)}{k} - \frac{a_2(t)-T_2(t)}{k}\right) + \mu\left(S - \bar{S}\right).$$

The above first order conditions still hold. The costate variable now verifies the equation $\dot{\lambda}_1(t) = r\lambda_1 - \frac{k}{4\lambda_1} + \mu$. Furthermore, we consider the Kuhn–Tucker condition

$$\mu\left(S - \bar{S}\right) = 0.$$

In other words, we have, either $S < \bar{S}$ with $\mu = 0$ or $S = \bar{S}$ with $\mu \geq 0$. Because the small economy's size is constrained by the upper-bound $\bar{S}$, we impose that $\mu = 0$ whenever $S = \bar{S}$.

Appendix B. Budget constraint

In this section, we prove that the present value of net tax revenues (the intertemporal budget constraint) is strictly positive for all $k \in [\underline{k}, k^*]$.

The FOCs relative to the Hamiltonians H_1 and H_2 yield the equilibrium choice variables of jurisdictions 1 and 2: $T_1(S,t) = \left(\frac{k}{2\lambda_1}\right)^2 S(t)$, $a_1(S,t) = \frac{\lambda_1}{k\beta}$, $a_2(t) = -\frac{\lambda_2(t)}{k}$ and $T_2(t) = \left(\frac{k}{2\lambda_2(t)}\right)^2(1-S(t))$. The FOCs also yield the motion equation of S: $S(t) = \left(S(0) - \hat{S}\right)e^{-3rt} + \hat{S}$, where $\hat{S} = \frac{1}{6\sqrt{2}}\left(\frac{1}{kr}\right)^{\frac{3}{2}}\left(\frac{\sqrt{2}}{\beta} - 1\right) + \frac{2}{3}$.

Case 1. The small country $(i = 1)$.

[36] To explain how players in a differential game guess each other's heterogenous strategy, first consider the case where both players make open-loop decisions. Then, after having solved the game, we would get the following solutions: $\Psi_1(t) = \Psi_1(S(t), \lambda_1(t), t)$ and $\Psi_2(t) = \Psi_1(S(t), \lambda_2(t), t)$, for any t. However, in our model, the small country makes Markovian decisions. Therefore, the large country, which is the open-loop player, conjectures that the small economy is a Markovian player. Consequently, $\Psi_1(t) = \Psi_1(S(t), t)$ has to be replaced by $\Psi_1(S, t)$ with any state variable S. Or, more precisely, the large economy guesses that the small economy's strategy is: $\Psi_1(S, t) = \Psi_1(S, \lambda(t), t)$, for any (S, t).

Consider first the case of the small country. The above results allow us to compute the equilibrium present value of net revenues

$$J_1 = \frac{1}{4}\sqrt{\frac{k}{r}}\left[S(0) + 3\hat{S}\right] - \frac{1}{8kr^2\beta}.$$

It is easy to check that $J_1 > 0$ if and only if $S(0) + 3\hat{S} > \frac{1}{2\beta}\left(\frac{1}{kr}\right)^{\frac{3}{2}}$, which is equivalent to $S(0) > \frac{1}{2\sqrt{2}}\left(\frac{1}{kr}\right)^{\frac{3}{2}} - 2$. The following conclusions can be derived.

(a) If $k > \bar{k} = \left(\frac{1}{32}\right)^{\frac{1}{3}}\frac{1}{r}$, we have $S(0) > 0 > \frac{1}{2\sqrt{2}}\left(\frac{1}{kr}\right)^{\frac{3}{2}} - 2$ and the above inequality is always true, that is, $J_1 > 0$, $\forall\, S(0)$. Considering that $\hat{S} > 0$, we claim that $k > \bar{k}$ is a sufficient condition under which the small economy never collapses.

(b) If $k < \bar{k}$, the sign of J_1 depends on the value of $S(0)$.

(b.1) If $\frac{1}{2\sqrt{2}}\left(\frac{1}{kr}\right)^{\frac{3}{2}} - 2 > S(0) > 0$, we have $J_1 < 0$. We also impose that $S(0) < \frac{1}{2}$. Consequently, if $\frac{1}{2\sqrt{2}}\left(\frac{1}{kr}\right)^{\frac{3}{2}} - 2 > \frac{1}{2}$, that is $k < \underline{k} = \left(\frac{1}{50}\right)^{\frac{1}{3}}\frac{1}{r}$, the small economy collapses for sure for any initial value $S(0)$.

(b.2) If $S(0) > \frac{1}{2\sqrt{2}}\left(\frac{1}{kr}\right)^{\frac{3}{2}} - 2 > 0$, we have $J_1 > 0$. Because $S(0) < \frac{1}{2}$, we must require that $\frac{1}{2} > \frac{1}{2\sqrt{2}}\left(\frac{1}{kr}\right)^{\frac{3}{2}} - 2$, or $k > \underline{k} = \left(\frac{1}{50}\right)^{\frac{1}{3}}\frac{1}{r}$ for guaranteeing $J_1 > 0$. However, this condition does not rule out that the small country may collapse in the long run. It may happen that the small economy keeps on shrinking infinitely and converges to 0, such that $\lim_{t\to\infty} S(t) \to \hat{S} \le 0$. In other words, stating that the small economy is able to payoff its public debt is not enough for eliminating collapse in the long run. Collapse is only excluded if the steady state net revenue is positive ($\hat{w}_1 > 0$).

From (a), (b.1) and (b.2) we conclude that $J_1 > 0$ for all $k \in [\underline{k}, k^*]$.

Case 2. The large country ($i = 2$).

Because $S(t) < \frac{1}{2}$, it is easy to show that $w_2(t) = (1 - S(t))\sqrt{2kr} - \frac{1}{16kr} > (1 - \frac{1}{2})\sqrt{2kr} - \frac{1}{16kr} > 0$ if $k > \underline{k} = \left(\frac{1}{50}\right)^{\frac{1}{3}}\frac{1}{r}$. Consequently, $w_2(t) > 0$, $\forall\, t \in (0, +\infty)$ and $\forall k \in [\underline{k}, k^*]$. Under the conditions of our model, the large country will never have to issue debt to fund its expenditures. It follows that the present value of net tax revenues is always positively signed. In other words, $J_2 = \int_0^\infty e^{-rt} w_2[T_2(t), a_2(t)]dt > 0, \forall k \in [\underline{k}, k^*]$.

After having analyzed cases 1 and 2, we finally conclude that $J_i > 0$, $(i = 1, 2)$ for all $k \in [\underline{k}, k^*]$. We also conclude that the present value of net revenues in country i is positive ($J_i > 0$, $i = 1, 2$) if the steady state net revenue is positive ($\hat{w}_i > 0, i = 1, 2$).That finishes the proof.

Appendix C. Supplementary data

Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.jpubeco.2014.07.008.

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