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Interfaces with Other Disciplines

A differential game model of the marketing-operations interface

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ABSTRACT

In the development of their dynamic strategies, the marketing and operations functions within a firm have differing objectives, and conflict between the two functions is common. The strategic interdependence involving marketing and operations decisions is modeled as a noncooperative differential game. Demand is assumed to be a function of price and advertising goodwill, and marketing controls price and advertising to maximize its discounted stream of revenue net of advertising costs. Backlogging is allowed, and operations controls production to minimize its discounted stream of production and backlog costs. A feedback Nash equilibrium is derived for the game, which allows a solution of the system of differential equations for goodwill and backlog, and is analyzed to study the nature of the dynamic strategies for price, advertising, and production.

1. Introduction

Marketing and operations management are both critical to an organization's success. Marketing seeks and creates demand for the firm's products, and operations' role is to manufacture the products efficiently, so each management function has a prominent effect on the bottom line, and, indeed, both are necessary for the firm's continued existence. Further, the two functions are crucially linked. Operations requires knowing how much demand is to be created through marketing, and marketing needs to be assured of the amount produced and the timing of product availability.

Marketing and operations are also inevitably in conflict, in that the two functional areas have differing objectives, marketing to enhance demand, and operations to minimize manufacturing costs. In a sense, marketing in its demand-creating role creates costs, along with revenue, while operations seeks to controls costs, so conflict is unavoidable. Shapiro (1977) discusses the conflict between marketing and manufacturing with respect to several problem areas, reproduced in Table 1. The basic causes of the conflict between the two are related to how the functional areas are evaluated and rewarded—marketing on the basis of profitable growth in terms of sales and market share, manufacturing on achieving smooth operation at minimum cost, the inherent complexity of the problems facing an organization that require effective management of both marketing and manufacturing, the differing career orientations and experiences of managers in the two functional areas, and cultural differences between managers of marketing and operations.

The operations research and management science literature involving the marketing-operations interface includes several studies that have combined production planning with price and/or advertising in overall profit-maximization models, in the attempt to provide normative aid in the coordination of the planning efforts of the two management functions. The present research presents an alternative perspective, and models the marketing-operations problem as a noncooperative game, more specifically a noncooperative differential game. In the differential game, marketing controls price and advertising to maximize its discounted flow of revenue net of advertising, and operations controls production to minimize its discounted flow of the sum of production and backlog costs. The noncooperative game structure allows the interpretation of the problem as involving both conflict and interconnectedness, while the differential game framework captures the dynamic nature of both marketing and operations management.

The motivation for the present paper is to find as explicitly as possible how the noncooperative nature of the dynamic game involving marketing and operations affects the nature of the strategies adopted by the two functional areas. To this end, the differential game model assumed allows the determination of feedback Nash equilibrium strategies for marketing and operations. The model and derived feedback strategies further permit an explicit solution for the dynamic evolution of price, advertising, and production. It is found that advertising and price follow monotonic time paths, and move in the same direction. The time path for production is either monotonic or changes direction once, and may move in the same or opposite direction as advertising and price.

The structure of the remainder of the paper is as follows. The next section reviews the literature on marketing-operations

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Table 1
Marketing/manufacturing areas of potential conflict.

Problem area	Typical marketing comment	Typical manufacturing comment
1. Capacity planning and long-range sales forecasting	"Why don't we have enough capacity?"	"Why didn't we have accurate sales forecasts?"
2. Production scheduling and short-range sales forecasting	"We need faster response. Our lead times are ridiculous"	"We need realistic customer commitments and sales forecasts that don't change like wind direction"
3. Delivery and physical distribution	"Why don't we ever have the right merchandise in in inventory"	"We can't keep everything inventory?"
4. Quality assurance	"Why can't we have reasonable quality at reasonable cost?"	"Why must we always offer options that are too hard to manufacture and that offer little customer utility?"
5. Breadth of product line	"Our customers demand variety"	"The product line is too broad—all we get are short, uneconomical runs"
6. Cost control	"Our costs are so high that we are not competitive in the marketplace"	"We can't provide fast delivery, broad variety, rapid response to change, and high quality at low cost"
7. New product introduction	"New products are our life blood"	"Unnecessary design changes are prohibitively expensive"
8. Adjunct services such as spare parts inventory support, installation, and repair	"Field service costs are too high"	"Products are being used in ways for which they weren't designed"

management models. The differential game model is introduced in Section 3. A feedback Nash equilibrium for the differential game is derived in Section 4. A possible Stackelberg game and feedback equilibrium is discussed in Section 5. In Section 6, the dynamic evolution of price, advertising, and production is analyzed theoretically, and Section 7 considers specific cases. Finally, Section 8 draws conclusions.

2. Literature review

There have been numerous studies that consider marketing and production decisions as a single-firm optimization problem. Thomas (1970) provides the first such effort, combining price and production decisions in a dynamic optimization model with deterministic demand as a function of time-varying price. (Gaimon (1998) provides a review of the price-production literature.) Thomas (1974) analyzes an extension with stochastic demand. Each of these models offers a dynamic programming algorithm for solving its problem in discrete time.

Pekelman (1974) provides a continuous-time model of the price-production optimization problem, and constructs an optimal control solution. In the model, demand is a linear function of price with time-varying parameters, production cost is strictly convex nonnegative increasing, and inventory is required to be nonnegative, i.e. backlogging is not allowed. Thompson et al. (1984) study a similar model, but with linear production cost, and production and warehouse constraints.

Feichtinger and Hartl (1985) analyze an optimal control model with a more general demand function, and allow backlogging. Feichtinger and Hartl, like Pekelman (1974), assume stricly convex nonnegative increasing production cost, and consider two types of inventory/backlog cost functions, one continuously differentiable and strictly convex, for which the value of the cost function and its first derivative are zero at an inventory/backlog level of zero, and the other nonsmooth piecewise linear. With the first, continuously differentiable and strictly convex, cost function, Feichtinger and Hartl find equilibrium inventory to be negative in the optimal control solution, that is, it is optimal in the long-run to have a backlog. Jørgensen et al. (1999) extend the Feichtinger and Hartl framework to include demand and cost learning effects. Jørgensen et al. do not allow backlogging in their optimal control solution, which leads to a characterization of the optimal solution according to regimes.

Several other studies consider advertising as an additional decision variable with price and production. Solution of the models involves a variety of mathematical programming methods, including linear programming (Thomas, 1971), nonlinear programming (Damon and Schramm, 1972; Welam, 1977), and mixed integer programming (Sogomonian and Tang, 1993; Ulusoy and Yazgaç, 1995). A discrete-time dynamic framework is assumed in these studies. (Freeland (1980) offers a static, single-period, model.) Numerical solution is needed in all cases.

Abad (1987) provides a continuous-time model that combines marketing, production, and finance decisions, with five control variables: advertising, price, production rate, rate of change of work-force level, and borrowing rate. Demand is assumed to be a general function of exogenous variables, price, and goodwill, with goodwill being formed through advertising according to the Nerlove and Arrow (1962) model. Abad offers an iterative decentralized procedure for solving the optimal control problem.

In addition, there is a large body of literature that is devoted to supply chain management, which is concerned with the integration of the activities of the various firms in a supply chain system; Li and Wang (2007) provide a review. Within the context of dynamic modeling, recent papers by De Giovanni (2010) and El Ouardighi et al. (2008) provide differential game contributions to this literature. Supply chain systems can be centralized, which means that there is a single entity that seeks to optimize system performance, or decentralized, in which the members of the supply chain are independent economic actors, each of which acts in its own best interest. Game-theoretic analyses involving the marketing-operations interface in this literature are conducted with the assumption that separate firms are the strategic players. The literature has not heretofore considered the strategic interaction of business functions within the firm as a noncooperative game.

Table 2 summarizes the contribution of the present paper to the literature on the interface involving marketing and operations management. There are several studies that consider marketing and production decisions as a single-firm optimization problem, as discussed above. The supply chain management literature considers centralized supply chain systems, in which a single entity wants to optimize the performance of the overall system, and decentralized supply chain systems, in which the several organizations are separate firms involved in a game setting. The present paper expands the literature by interpreting the marketing-operations interface within a single organization as a game, more

Table 2
Contribution of the present paper.

	Single decision maker	Two or more decision makers
Two or more firms	Centralized supply chain management	Decentralized supply chain management
Single firm	Optimization	*This paper*

specifically a noncooperative differential game. This interpretation allows an analysis of the dynamic implications of a decentralized management structure, in which marketing and operations are strategic players with the authority to develop their individual strategies in an dynamically interactive environment.

3. Model

In the model, specific functional forms are assumed in order to derive a feedback Nash equilibrium. Some generality is sacrificed in the process, but it allows the representation of certain key relationships, e.g. diminishing marginal advertising returns and convex manufacturing and backlog/inventory costs, that are emphasized in existing research, while achieving the critical goal of this research of obtaining a feedback equilibrium.

Marketing is presumed to use price and advertising both effectively and efficiently to create revenue for the firm's product. A goodwill model is used to capture the dynamic effects of advertising on demand. The Nerlove and Arrow (1962) goodwill model has been established as one of the basic frameworks for capturing continuous-time advertising dynamics. Jørgensen and Zaccour (2004) and Sethi and Thompson (2006) offer reviews of the literature regarding the Nerlove–Arrow model. Two contributions from the marketing and management science literature provide useful extensions of the model. Fershtman et al. (1990) use the model to explore theoretically market share pioneering advantages. Chintagunta (1993) employs the model to explore the validity of the "flat maximum principle"—the insensitivity of firm profit to changes in advertising from its optimal level, in monopolistic and duopolistic settings. Both studies adapt the Nerlove–Arrow model to allow for diminishing marginal effects of advertising, Fershtman et al. (1990) by assuming convex advertising costs, and Chintagunta (1993) with a square-root form for advertising in the Nerlove–Arrow advertising dynamics, the two approaches being equivalent mathematically with an appropriate variable transform.

Let G denote goodwill and a current advertising. Both G and a are dynamic variables, but the time notation is suppressed for the sake of exposition. Goodwill evolves according to the relationship

$$\dot{G} = a - \delta G, \tag{1}$$

where δ is a decay rate. In addition, demand is assumed to depend on goodwill and price according to the following function:

$$D = \alpha - \beta p + \gamma G. \tag{2}$$

In (2), α is an intercept in the linear demand function, β is a measure of the negative effect of price on demand, and γ a measure of the positive effect of goodwill on demand. Marketing's objective is to maximize the discounted flow of revenues minus quadratic advertising costs over an infinite time horizon:

$$\max_{p,a} \int_0^\infty e^{-rt}(pD - ca^2)dt. \tag{3}$$

As Dockner et al. (2000) indicate, the end of the planning period in a differential game is a point in time, call it T, so that the integral in (3) is over t in the interval $[0,T]$. If the end of the planning period is very far in the future or unknown, it is appropriate to set $T = \infty$ as an approximation. Discounting of returns is accomplished through the e^{-rt} discount factor, where r is a discount rate. It is especially important to discount returns if the time horizon is infinite, so that the integral in (3) can be finite. Also in (3), c is an advertising cost parameter.

Production is assumed to be responsible for the dynamic production rate u (time notation is suppressed) and the costs associated with the chosen production rate. Backlogging is allowed, that is, production and delivery to the customer can be delayed, at a cost to the firm. Backlogging is an interesting, and too little studied, aspect of inventory management, for it appears that when backlogging is allowed, backlogging is preferred in the long term, as shown by Feichtinger and Hartl (1985). The basic economic reason for this is that backlogging allows the deferring of production costs. Let B denote the dynamic backlog variable. Backlog evolves according to the dynamic difference between demand and production:

$$\dot{B} = D - u. \tag{4}$$

Backlog can be negative, which would indicate positive inventory. The cost of a current backlog (inventory) B is assumed to be hB^2, which is consistent with the general relationship suggested in Feichtinger and Hartl (1985) that both the value of the cost function and first derivative of the cost function be zero at a backlog (inventory) level of zero, and that the second derivative of the cost function be positive. Further, production cost is assumed to be quadratic in the production rate, gu^2, so that production cost is convex increasing in production rate. Production's objective is to minimize the discounted flow of production and backlog costs:

$$\min_u \int_0^\infty e^{-rt}(gu^2 + hB^2). \tag{5}$$

Taken together, the marketing and production problems form the differential game

$$\begin{aligned} &\text{marketing} \ \max_{p,a} \int_0^\infty e^{-rt}(p(\alpha - \beta p + \gamma G) - ca^2)dt, \\ &\text{operations} \ \max_u - \int_0^\infty e^{-rt}(gu^2 + hB^2)dt \end{aligned} \tag{6}$$

subject to the dynamic constraints

$$\begin{aligned} &\dot{G} = a - \delta G, \quad G(0) = G_0, \\ &\dot{B} = \alpha - \beta p + \gamma G - u, \quad B(0) = B_0. \end{aligned} \tag{7}$$

Other formulations of the differential game are possible. For instance, it could be reasoned that backlog B be entered into the dynamic goodwill relationship in (7) as a negative influence on goodwill formation. This is not done in the present study, so as to be consistent with modeling in existing literature, first by treating goodwill strictly as advertising goodwill—see, e.g., section 3.5 in Chapter 3 of Jørgensen and Zaccour (2004)—and second by modeling the negative aspect of backlog with a cost function, as in Feichtinger and Hartl (1985) and Ulusoy and Yazgaç (1995).

4. Feedback Nash equilibrium

A feedback Nash equilibrium is derived by solving the Hamilton–Jacobi–Bellman (HJB) equations

$$\begin{aligned} rV_M &= \max_{p,a} \left\{ p(\alpha - \beta p + \gamma G) - ca^2 + \frac{\partial V_M}{\partial G}(a - \delta G) \right. \\ &\quad \left. + \frac{\partial V_M}{\partial B}(\alpha - \beta p + \gamma G - u) \right\}, \\ rV_O &= \max_u \left\{ -gu^2 - hB^2 + \frac{\partial V_O}{\partial G}(a - \delta G) \right. \\ &\quad \left. + \frac{\partial V_O}{\partial B}(\alpha - \beta p + \gamma G - u) \right\}. \end{aligned} \tag{8}$$

The first-order conditions for the maximizations in (8) lead to

$$p = \frac{1}{2}\left(\frac{\alpha+\gamma G}{\beta} - \frac{\partial V_M}{\partial B}\right),$$
$$a = \frac{1}{2c}\frac{\partial V_M}{\partial G}, \tag{9}$$
$$u = -\frac{1}{2g}\frac{\partial V_O}{\partial B}$$

so that the HJB Eq. (8) become, through substitution from (9),

$$\begin{aligned} rV_M &= \frac{\alpha^2}{4\beta} + \frac{\alpha\gamma}{2\beta}G + \frac{\gamma^2}{4\beta}G^2 + \frac{1}{4c}\left(\frac{\partial V_M}{\partial G}\right)^2 - \delta G\frac{\partial V_M}{\partial G} \\ &\quad + \frac{\alpha}{2}\frac{\partial V_M}{\partial B} + \frac{\beta}{4}\left(\frac{\partial V_M}{\partial B}\right)^2 + \frac{\gamma}{2}G\frac{\partial V_M}{\partial B} + \frac{1}{2g}\frac{\partial V_M}{\partial B}\frac{\partial V_O}{\partial B}, \\ rV_O &= -hB^2 + \frac{\alpha}{2}\frac{\partial V_O}{\partial B} + \frac{1}{4g}\left(\frac{\partial V_O}{\partial B}\right)^2 + \frac{\gamma}{2}G\frac{\partial V_O}{\partial B} \\ &\quad - \delta G\frac{\partial V_O}{\partial G} + \frac{\beta}{2}\frac{\partial V_M}{\partial B}\frac{\partial V_O}{\partial B} + \frac{1}{2c}\frac{\partial V_M}{\partial G}\frac{\partial V_O}{\partial G}. \end{aligned} \tag{10}$$

Conjecture the following functional forms for the value functions:

$$\begin{aligned} V_M &= A_M + C_M G + D_M G^2, \\ V_O &= A_O + C_O G + D_O G^2 + E_O B + F_O B^2 + H_O GB, \end{aligned} \tag{11}$$

where A_M, C_M, D_M, A_O, C_O, D_O, E_O, F_O, H_O are determined through equating of coefficients when the relationships in (11) and their partial derivatives are substituted into (10). For the marketing value function, this results in the following relationships:

$$\begin{aligned} rA_M &= \frac{\alpha^2}{4\beta} + \frac{C_M^2}{4c}, \\ rC_M &= \frac{\alpha\gamma}{2\beta} + \frac{C_M D_M}{c} - \delta C_M, \\ rD_M &= \frac{\gamma^2}{4\beta} + \frac{D_M^2}{c} - 2\delta D_M \end{aligned} \tag{12}$$

from which, in particular,

$$D_M = \frac{c}{2}\left(2\delta + r \pm \sqrt{(2\delta+r)^2 - \frac{\gamma^2}{c\beta}}\right). \tag{13}$$

We assume the following constraint on the parameters of the problem:

$$4\delta(r+\delta) > \frac{\gamma^2}{c\beta} \tag{14}$$

so that the roots (13) are real. Also,

$$C_M = \frac{\alpha\gamma}{2\beta\left(r+\delta-\frac{D_M}{c}\right)}. \tag{15}$$

The smaller root in (13)

$$D_M = \frac{c}{2}\left(2\delta + r - \sqrt{(2\delta+r)^2 - \frac{\gamma^2}{c\beta}}\right) \tag{16}$$

guarantees a positive value for

$$C_M = \frac{\alpha\gamma}{\beta\left(r+\sqrt{(2\delta+r)^2-\gamma^2/c\beta}\right)} \tag{17}$$

and therefore positive advertising

$$a = \frac{C_M + 2D_M G}{2c} \tag{18}$$

for all G, since goodwill, assuming it starts with a nonnegative value, remains nonnegative. The larger root in (13) is not considered further, since it implies a negative value for C_M, and therefore negative advertising levels for some goodwill values. Further, we have

$$A_M = \frac{\alpha^2}{4r\beta} + \frac{C_M^2}{4cr} = \alpha^2\frac{c\beta\left(r+\sqrt{(2\delta+r)^2-\gamma^2/c\beta}\right)^2+\gamma^2}{4cr\beta^2\left(r+\sqrt{(2\delta+r)^2-\gamma^2/c\beta}\right)^2}. \tag{19}$$

We have the following proposition regarding the value function for marketing:

Proposition 1. *For the feedback equilibrium, the value function for marketing is positive, and increasing in goodwill.*

Proposition 1 is proved by noting that A_M and C_M are strictly positive, and D_M is nonnegative.

For the operations value function, the equating of coefficients yields the relationships

$$\begin{aligned} rA_O &= \frac{E_O^2}{4g} + \frac{\alpha}{2}E_O + \frac{C_M C_O}{2c}, \\ rC_O &= \frac{E_O H_O}{2g} + \frac{\alpha}{2}H_O + \frac{\gamma}{2}E_O - \delta C_O + \frac{C_M D_O + D_M C_O}{c}, \\ rD_O &= \frac{H_O^2}{4g} + \frac{\gamma}{2}H_O - 2\delta D_O + \frac{2D_M D_O}{c}, \\ rE_O &= \frac{E_O F_O}{g} + \alpha F_O + \frac{C_M H_O}{2c}, \\ rF_O &= \frac{F_O^2}{g} - h, \\ rH_O &= \frac{F_O H_O}{g} + \gamma F_O - \delta H_O + \frac{D_M H_O}{c} \end{aligned} \tag{20}$$

from which, in particular,

$$F_O = \frac{g}{2}\left(r \pm \sqrt{r^2 + \frac{4h}{g}}\right). \tag{21}$$

The smaller root for F_O

$$F_O = \frac{g}{2}\left(r - \sqrt{r^2 + \frac{4h}{g}}\right) \tag{22}$$

is nonpositive, so that both H_O and E_O are nonpositive, and

$$u = -\frac{E_O + 2F_O B + H_O G}{2g} \tag{23}$$

is nonnegative at least for all nonnegative values of B and G. The larger root for F_O is not considered further, since that implies nonnegative values for not only F_O but also H_O and E_O, so that production would be nonpositive for all nonnegative B and G, and also nonincreasing in both B and G, whereas the opposite should be the case. With F_O as in (22), for certain negative values of B, or positive inventory, the feedback strategy for u can be negative, which can be interpreted as withdrawing manufactured product from inventory.

H_O and E_O are derived from F_O in (22), D_M in (16), and C_M in (17). For notational convenience, define the following identities:

$$I \equiv \sqrt{(2\delta+r)^2 - \frac{\gamma^2}{c\beta}}, \quad J \equiv \sqrt{r^2 + \frac{4h}{g}}. \tag{24}$$

Then

$$H_O = -\frac{g\gamma(J-r)}{I+J} \tag{25}$$

and

$$E_O = -\frac{g\alpha(J-r)}{J+r}\left(1+\frac{\gamma^2}{c\beta(I+r)(I+J)}\right). \tag{26}$$

D_O is derived from H_O and D_M to obtain

$$D_O = -\frac{g\gamma^2(J-r)(2I+J+r)}{4I(I+J)^2}. \tag{27}$$

C_O is derived from H_O, E_O, D_O, D_M, and C_M, so that

$$C_O = -\frac{g\alpha\gamma(J-r)}{(I+r)(I+J)}\left(\frac{I+J+2r}{J+r}+\frac{\gamma^2}{c\beta(I+J)}\left(\frac{1}{I+r}+\frac{1}{J+r}+\frac{J+r}{2I(I+r)}\right)\right). \tag{28}$$

Finally, A_O is obtained from E_O, C_O, and C_M to yield

$$A_O = -\frac{g\alpha^2(J-r)}{4r(J+r)^2}\left(\begin{array}{l} 3r+J+\frac{2\gamma^2\left((I+r)(J+3r)+(J+r)^2\right)}{c\beta(I+r)^2(I+J)} \\ \quad +\frac{\gamma^4\left(I(I+r)(J+3r)+2I(J+r)^2+(J+r)^3\right)}{c^2\beta^2 I(I+r)^3(I+J)^2}\end{array}\right). \tag{29}$$

Note that A_O, C_O, D_O, E_O, F_O, H_O are all nonpositive. We have the following proposition regarding the value function for operations:

Proposition 2. *For the feedback equilibrium, the value function for operations is*

(a) *nonincreasing in goodwill for the following region:*

$$B+\frac{\gamma(2I+J+r)}{2I(I+J)}G \geqslant -\frac{\alpha}{I+r}\left(\frac{I+J+2r}{J+r}+\frac{\gamma^2}{c\beta(I+J)}\left(\frac{1}{I+r}+\frac{1}{J+r}+\frac{J+r}{2I(I+r)}\right)\right)$$

(b) *nonincreasing in backlog for the following region:*

$$B+\frac{\gamma}{I+J}G \geqslant -\frac{\alpha}{J+r}\left(1+\frac{\gamma^2}{c\beta(I+r)(I+J)}\right).$$

The proposition is proved by deriving

$$\partial V_O/\partial G = C_O + 2D_O G + H_O B$$

and

$$\partial V_O/\partial B = E_O + 2F_O B + H_O G$$

and solving for the inequalities $\partial_O/\partial G \leqslant 0$ and $\partial V_O/\partial B \leqslant 0$.

In particular, for nonnegative backlog levels, operation's value function is nonpositive, and nonincreasing in both goodwill and backlog.

From the first-order conditions (9), we have the following feedback Nash equilibrium strategies for price, advertising, and production:

$$\begin{aligned} p(G,B) &= \frac{\alpha+\gamma G}{2\beta}, \\ a(G,B) &= \frac{1}{2}\left(\frac{\alpha\gamma}{c\beta(I+r)}+(2\delta+r-I)G\right), \\ u(G,B) &= \frac{J-r}{2}\left(\frac{\alpha}{J+r}\left(1+\frac{\gamma^2}{c\beta(I+r)(I+J)}\right)+\frac{\gamma}{I+J}G+B\right). \end{aligned} \tag{30}$$

The feedback strategies have price and advertising dependent on and nondecreasing in goodwill, and production dependent on and nondecreasing in both state variables, goodwill and backlog.

5. Feedback Stackelberg equilibrium

The feedback equilibrium developed in Section 4 assumes a Nash game, i.e. that each player, marketing and operations, must develop its strategy without knowing the strategy of the other player; marketing and operations do not communicate with each other before developing their strategies. An alternative information structure for the differential game is to have marketing act as a leader in a Stackelberg game. This would be the case in an organization in which marketing communicates its strategy to operations before operations prepares its strategy.

A counterpart to the feedback Nash equilibrium is the feedback Stackelberg equilibrium. It can be shown for the current problem that the feedback Stackelberg equilibrium coincides with the feedback Nash equilibrium. The equivalence of the two equilibria is due to the lack of coupling control variable terms in the objective functions (Basar and Olsder, 1995), i.e. neither p nor a is coupled with u in operations' objective function, and u is not coupled with p or a in marketing's objective function.

6. Time paths

Inserting $p(G,B)$, $a(G,B)$, $u(G,B)$ from (30) into the state Eq. (7), we have

$$\begin{aligned} \dot G &= \frac{1}{2}\left(\frac{\alpha\gamma}{c\beta(I+r)}-(I-r)G\right), \quad G(0)=G_0, \\ \dot B &= \frac{1}{2}\left(\frac{\alpha}{J+r}\left(2r-\frac{\gamma^2(J-r)}{c\beta(I+r)(I+J)}\right) + \frac{\gamma(I+r)}{I+J}G-(J-r)B\right), \quad B(0)=B_0. \end{aligned} \tag{31}$$

The system (31) can be solved. The homogeneous system corresponding to (31) is

$$\begin{aligned} \dot G &= \frac{r-I}{2}G, \\ \dot B &= \frac{\gamma(I+r)}{2(I+J)}G+\frac{r-J}{2}B, \end{aligned} \tag{32}$$

which has characteristic roots

$$s^1 = \frac{r-I}{2}, \quad s^2 = \frac{r-J}{2} \tag{33}$$

both of which are nonpositive—r being $<I$ due to the assumed constraint (14)—and characteristic vectors

$$\mathbf{x}^1 = \begin{pmatrix}(J^2-I^2)/\gamma(I+r) \\ 1\end{pmatrix}, \quad \mathbf{x}^2 = \begin{pmatrix}0 \\ 1\end{pmatrix} \tag{34}$$

so that the general solution to (32) is

$$\begin{aligned} G(t) &= c_1\frac{J^2-I^2}{\gamma(I+r)}e^{\frac{r-I}{2}t}, \\ B(t) &= c_1 e^{\frac{r-I}{2}t}+c_2 e^{\frac{r-J}{2}t}, \end{aligned} \tag{35}$$

where c_1 and c_2 are to be determined from initial conditions. A particular solution to the system (31) is

$$\begin{aligned} G_\infty &= \frac{\alpha\gamma}{4c\beta\delta(r+\delta)-\gamma^2}, \\ B_\infty &= \frac{gr\alpha}{2h}\left(1+\frac{\gamma^2}{I+J}\left(\frac{1}{c\beta(I+r)}+\frac{J+r}{4c\beta\delta(r+\delta)-\gamma^2}\right)\right) \end{aligned} \tag{36}$$

so that the general solution to (31) is

$$\begin{aligned} G(t) &= c_1\frac{J^2-I^2}{\gamma(I+r)}e^{\frac{r-I}{2}t}+G_\infty, \\ B(t) &= c_1 e^{\frac{r-I}{2}t}+c_2 e^{\frac{r-J}{2}t}+B_\infty. \end{aligned} \tag{37}$$

The coefficients c_1 and c_2 are determined from the initial conditions to yield

$$\begin{aligned} c_1 &= \frac{\gamma(I+r)}{J^2-I^2}(G_0-G_\infty), \\ c_2 &= B_0-B_\infty-\frac{\gamma(I+r)}{J^2-I^2}(G_0-G_\infty) \end{aligned} \tag{38}$$

and

$$G(t) = (G_0 - G_\infty)e^{\frac{r-I}{2}t} + G_\infty,$$
$$B(t) = \frac{\gamma(I+r)}{J^2 - I^2}(G_0 - G_\infty)e^{\frac{r-I}{2}t} + \left(B_0 - B_\infty - \frac{\gamma(I+r)}{J^2-I^2}(G_0 - G_\infty)\right)e^{\frac{r-J}{2}t} + B_\infty. \tag{39}$$

From (39) for $G(t)$, we have

$$\dot{G} = \frac{r-I}{2}(G_0 - G_\infty)e^{\frac{r-I}{2}t} \tag{40}$$

so that the time path for G is monotonic, increasing if $G_0 < G_\infty$ and decreasing if $G_0 > G_\infty$. Also, from the feedback strategies for price and advertising in (30),

$$\dot{p} = \frac{\gamma}{2\beta}\dot{G},$$
$$\dot{a} = \frac{2\delta + r - I}{2}\dot{G} \tag{41}$$

so that we have the following:

Proposition 3. *The time paths for both price and advertising are monotonic.*

Both price and advertising are monotonically nondecreasing (nonincreasing) across time if initial goodwill is less (greater) than steady-state goodwill.

The path for production under the feedback strategy is not, in general, monotonic. From (30) and (39), we have

$$u(t) = \frac{J-r}{2}\left(\frac{\alpha}{J+r} + \frac{\alpha\gamma^2}{c\beta(I+r)(J+r)(I+J)} + \frac{\gamma G_\infty}{I+J} + B_\infty\right) + \frac{2h\gamma}{g(J^2-I^2)}(G_0 - G_\infty)e^{\frac{r-I}{2}t} + \frac{J-r}{2}\left(B_0 - B_\infty - \frac{\gamma(I+r)}{J^2-I^2}(G_0 - G_\infty)\right)e^{\frac{r-J}{2}t} \tag{42}$$

and

$$\dot{u} = -\frac{h\gamma(I-r)}{g(J^2-I^2)}(G_0 - G_\infty)e^{\frac{r-I}{2}t} - \frac{(J-r)^2}{4}\left(B_0 - B_\infty - \frac{\gamma(I+r)}{J^2-I^2}(G_0 - G_\infty)\right)e^{\frac{r-J}{2}t}. \tag{43}$$

By considering various inequalities involving I, J, G_0, and B_0, we arrive at the following proposition:

Proposition 4. *The time path for production is either monotonic or changes direction once.*

The proof of Proposition 4 is in Appendix A. The path for production may be monotonically increasing, monotonically decreasing, increasing and then decreasing, or decreasing and then increasing, depending on the values of the parameters and initial values of the state variables.

7. Special cases

7.1. Initial goodwill equal to steady-state goodwill

We now study certain specific situations that illustrate how the dynamics of the marketing-operations interface work to generate dynamic production levels. First assume that $G_0 = G_\infty$, which works to eliminate goodwill dynamics and allows a focus on how backlog affects production. In the following subsection, we add back the goodwill dynamics to assess its influence.

With $G_0 = G_\infty$, we have

$$B(t) = (B_0 - B_\infty)e^{\frac{r-J}{2}t} + B_\infty \tag{44}$$

and

$$\dot{B} = -\frac{J-r}{2}(B_0 - B_\infty)e^{\frac{r-J}{2}t} \geqslant 0 \leftrightarrow B_0 \leqslant B_\infty \tag{45}$$

so that backlog evolves monotonically. Further, we have

$$u(t) = \frac{J-r}{2}\left(\frac{\alpha}{J+r} + \frac{\alpha\gamma^2}{c\beta(I+r)(J+r)(I+J)} + \frac{\gamma G_\infty}{I+J} + B_\infty + (B_0 - B_\infty)e^{\frac{r-J}{2}t}\right)$$
$$= \frac{J-r}{2}\left(\frac{\alpha}{J+r} + \frac{\alpha\gamma^2}{c\beta(I+r)(J+r)(I+J)} + \frac{\gamma G_\infty}{I+J} + B(t)\right) \tag{46}$$

and

$$\dot{u} = \frac{(J-r)}{2}\dot{B} \tag{47}$$

so that production also follows a monotonic time path.

In the model, production can be negative if backlog is negative, i.e. there is a positive inventory. The interpretation of negative production is that the firm withdraws product from inventory, and is able to recover the production cost for the withdrawn product from inventory, e.g. by selling the product at cost. We have the following:

Proposition 5. *In the absence of goodwill dynamics,*

$$u(t) \geqslant 0 \leftrightarrow B(t) \geqslant -\frac{\alpha}{J+r} - \frac{\alpha\gamma^2}{c\beta(I+r)(J+r)(I+J)} - \frac{\gamma G_\infty}{I+J}.$$

The proof is straightforward from (46).

If backlog begins with a value at least as large as the negative amount on the right-hand-side of the relationship in the proposition, production begins with a nonnegative value and remains nonnegative thereafter.

Another scenario of interest is when backlog cost is high, that is when it is very expensive to delay delivery of the product to customers. We have the following result:

Proposition 6. *In the limit as backlog cost approaches infinity, initial production becomes an instantaneous impulse to drive backlog to zero, and production thereafter is constant and equal to demand.*

The proof of Proposition 6 is in Appendix A.

A high level of backlog cost discourages having a backlog, and the production strategy in the limit is to eliminate the backlog completely, as soon as possible. This is the strategy as well for positive initial inventory (negative initial backlog), to eliminate the inventory as quickly as possible.

7.2. Initial goodwill not equal to steady-state goodwill

Numerical investigation with specific parameter values provides an illustration of the effects on dynamic production strategy created by the dynamic adjustment of beginning goodwill to its steady-state level through advertising. With goodwill dynamics involved, the equilibrium path for production is nonmonotonic for some parameter values. Fig. 1 shows the production time paths for four different sets of parameter values. The scenarios shown are sufficient to show the possible general shapes for the time paths for production, decrease followed by increase, monotonic decrease, monotonic increase, and increase followed by decrease.

8. Conclusions

The present research expands the literature on the marketing-operations interface. It interprets this interface as a differential

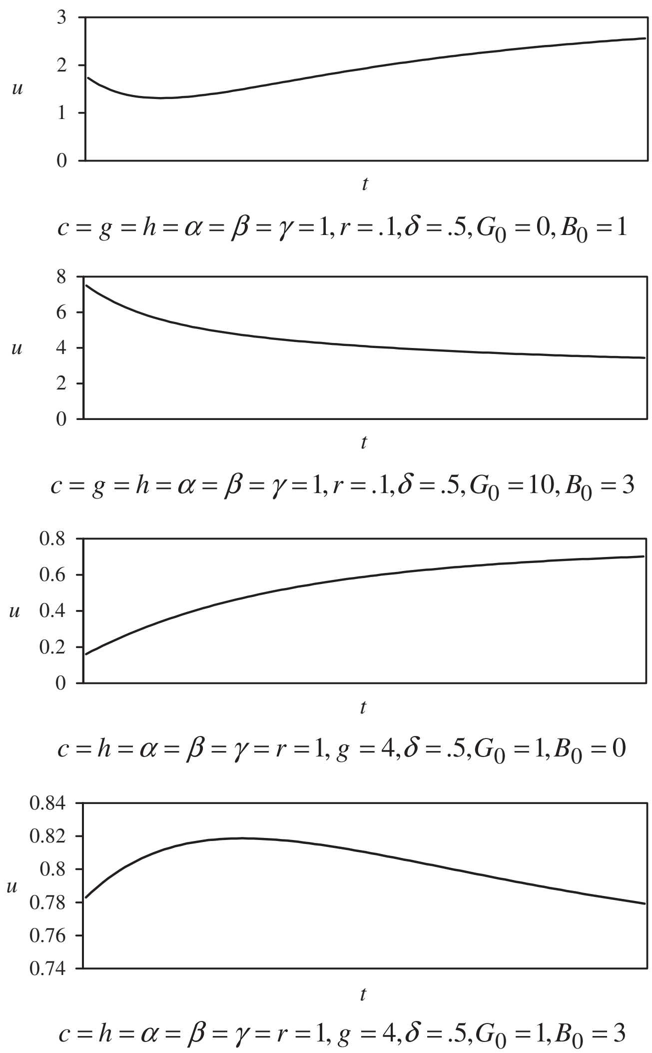

Fig. 1. Production paths.

game, and analysis of feedback Nash equilibrium strategies provides generalizable conclusions regarding dynamic marketing and operations strategies. The differential game perspective provides a useful framework for analyzing an important and too-little studied problem area, the strategic interaction involving functional areas in a decentralized organization.

The feedback Nash equilibrium analyzed allows strategic implications to be drawn regarding the state-dependent behavior of marketing's control variables, price and advertising, and operations' control variable, production. The feedback relationships developed determine the paths that price, advertising, and production follow across time. It is found that price and advertising follow time paths that are monotonic, and furthermore move in the same direction. The time path for production is either monotonic or changes direction once, and may move in the same or opposite direction as price and advertising.

We can compare the findings in the present research to those in existing studies, in particular those that provide generalizable results. In the optimal control models of Feichtinger and Hartl (1985) and Jørgensen et al. (1999), price and production are found to move in the same direction, in Jørgensen et al. (1999) if inventory is strictly positive. Also, advertising is not considered in these two studies. An implication, then, of a noncooperative game setting is that price and production do not necessarily move together. The results in the present paper can also be compared to those in three supply chain studies, by Jørgensen (1986), El Ouardighi et al. (2008), and De Giovanni (2010), that adopt a differential game approach. Jørgensen (1986) considers pricing and production, and finds that retail price is either monotonically increasing or first increasing and then decreasing, while the manufacturer's production policy is bang–bang with singular control. El Ouardighi et al. (2008) add advertising and quality to price and production and determine that advertising and price move in opposite directions, advertising increasing and price decreasing over time, in contrast to the findings in the present paper, while the time path for production may be non-monotonic. Both Jørgensen (1986) and El Ouardighi et al. (2008) analyze open-loop, and not feedback, equilibria. De Giovanni (2010) considers price, advertising, and quality, but does not include production as a control variable. De Giovanni (2010) develops feedback Nash and Stackelberg equilibria, and finds that both advertising and price are increasing in goodwill, in accordance with the findings in the present study.

This paper is the first look at the interface between functional areas within a firm as a noncooperative dynamic game. The interpretation of functional area interaction as a noncooperative game suggests other research that can be pursued. Analysis can be extended to involve other functional areas: research and development, finance, distribution, customer service, human resources, etc. Additional control variables will need to be considered depending on the interface studied. Further, coordinating mechanisms should be investigated, as ways to achieve optimal results for the firm as a whole. The coordinating mechanism would need to be appropriate to the interface considered. For example, transfer pricing could be considered as a coordinating mechanism for the marketing – operations interface. There is much research that can be done to enhance our understanding of the decisions made and strategies developed by business functions in decentralized firms.

Appendix A

Proof of Proposition 4. Say first that $J > I$. If

$$G_0 \leqslant G_\infty \quad \text{and } B_0 \leqslant B_\infty - \frac{\gamma(I+r)}{J^2-I^2}(G_\infty - G_0),$$

then $\dot{u} \geqslant 0 \forall t$. If

$$G_0 \leqslant G_\infty \quad \text{and } B_0 \geqslant B_\infty - \frac{\gamma(I+r)}{J^2-I^2}(G_\infty - G_0),$$

then

$$\dot{u} \geqslant 0 \leftrightarrow t \geqslant t^* = \frac{2\ln\left(\frac{g(J^2-I^2)(J-r)^2\left(B_0-B_\infty+\frac{\gamma(I+r)}{J^2-I^2}(G_\infty-G_0)\right)}{4h\gamma(I-r)(G_\infty-G_0)}\right)}{J-I}.$$

Furthermore, $t^* > 0$ iff

$$B_0 > B_\infty + \left(\frac{4h\gamma(I-r)}{g(J^2-I^2)(J-r)^2} - \frac{\gamma(I+r)}{J^2-I^2}\right)(G_\infty - G_0).$$

If

$$G_0 \geqslant G_\infty \quad \text{and } B_0 \geqslant B_\infty + \frac{\gamma(I+r)}{J^2-I^2}(G_0 - G_\infty),$$

then $\dot{u} \leqslant 0 \forall t$. If

$$G_0 \geqslant G_\infty \quad \text{and } B_0 \leqslant B_\infty + \frac{\gamma(I+r)}{J^2-I^2}(G_0-G_\infty),$$

then

$$\dot{u} \leqslant 0 \ \leftrightarrow \ t \geqslant t^{**} = \frac{2\ln\left(\frac{g(J^2-I^2)(J-r)^2\left(B_\infty-B_0+\frac{\gamma(I+r)}{J^2-I^2}(G_0-G_\infty)\right)}{4h\gamma(I-r)(G_0-G_\infty)}\right)}{J-I}$$

and $t^{**}>0$ iff

$$B_0 < B_\infty + \left(\frac{\gamma(I+r)}{J^2-I^2} - \frac{4h\gamma(I-r)}{g(J^2-I^2)(J-r)^2}\right)(G_0-G_\infty).$$

Now assume $I>J$. Then

$$\dot{u} = \frac{h\gamma(I-r)}{g(I^2-J^2)}(G_0-G_\infty)e^{\frac{r-I}{2}t} + \frac{(J-r)^2}{4}\left(B_\infty-B_0-\frac{\gamma(I+r)}{I^2-J^2}(G_0-G_\infty)\right)e^{\frac{r-J}{2}t}.$$

If

$$G_0 \geqslant G_\infty \quad \text{and } B_0 \leqslant B_\infty - \frac{\gamma(I+r)}{I^2-J^2}(G_0-G_\infty),$$

then $\dot{u} \geqslant 0 \forall t$. If

$$G_0 \geqslant G_\infty \quad \text{and } B_0 \geqslant B_\infty - \frac{\gamma(I+r)}{I^2-J^2}(G_0-G_\infty),$$

then

$$\dot{u} \geqslant 0 \ \leftrightarrow \ t \leqslant t^{\#} = \frac{2\ln\left(\frac{4h\gamma(I-r)(G_0-G_\infty)}{g(I^2-J^2)(J-r)^2\left(B_0-B_\infty+\frac{\gamma(I+r)}{I^2-J^2}(G_0-G_\infty)\right)}\right)}{I-J}.$$

Further, $t^{\#}>0$ iff

$$B_0 < B_\infty + \left(\frac{4h\gamma(I-r)}{g(I^2-J^2)(J-r)^2} - \frac{\gamma(I+r)}{I^2-J^2}\right)(G_0-G_\infty).$$

If

$$G_0 \leqslant G_\infty \quad \text{and } B_0 \geqslant B_\infty + \frac{\gamma(I+r)}{I^2-J^2}(G_\infty-G_0),$$

then $\dot{u} \leqslant 0 \forall t$. If

$$G_0 \leqslant G_\infty \quad \text{and } B_0 \leqslant B_\infty + \frac{\gamma(I+r)}{I^2-J^2}(G_\infty-G_0),$$

then

$$\dot{u} \leqslant 0 \ \leftrightarrow \ t \leqslant t^{\#\#} = \frac{2\ln\left(\frac{4h\gamma(I-r)(G_\infty-G_0)}{g(I^2-J^2)(J-r)^2\left(B_\infty-B_0+\frac{\gamma(I+r)}{I^2-J^2}(G_\infty-G_0)\right)}\right)}{I-J}$$

and $t^{\#\#}>0$ iff

$$B_0 > B_\infty + \left(\frac{\gamma(I+r)}{I^2-J^2} - \frac{4h\gamma(I-r)}{g(I^2-J^2)(J-r)^2}\right)(G_\infty-G_0). \qquad \square$$

Proof of Proposition 6. We have that

$$\begin{aligned}
\lim_{h\to\infty} B_\infty &= \lim_{h\to\infty}\left(\frac{gr\alpha}{2h} + \frac{gr\alpha\gamma^2}{2ch\beta(I+r)(I+J)} + \frac{gr\alpha\gamma^2(J+r)}{2h(I+J)(4c\beta\delta(r+\delta)-\gamma^2)}\right)\\
&= 0+0+\lim_{h\to\infty}\frac{gr\alpha\gamma^2(J+r)}{2h(I+J)(4c\beta\delta(r+\delta)-\gamma^2)}\\
&= \frac{gr\alpha\gamma^2}{4c\beta\delta(r+\delta)-\gamma^2}\lim_{h\to\infty}\frac{J+r}{2h(I+J)}\\
&= \frac{gr\alpha\gamma^2}{4c\beta\delta(r+\delta)-\gamma^2}\lim_{h\to\infty}\frac{1}{gJ(I+J)+2h} = 0
\end{aligned}$$

by applying l'Hôpital's rule. Then, for $t>0$,

$$\begin{aligned}
\lim_{h\to\infty} B(t) &= \lim_{h\to\infty}\left((B_0-B_\infty)e^{\frac{r-J}{2}t}+B_\infty\right)\\
&= \lim_{h\to\infty}\left(B_0e^{\frac{r-J}{2}t}+B_\infty\left(1-e^{\frac{r-J}{2}t}\right)\right) = 0+0=0.
\end{aligned}$$

Furthermore,

$$\begin{aligned}
\lim_{h\to\infty} u(0) &= \lim_{h\to\infty}\frac{J-r}{2}\left(\frac{\alpha}{J+r}+\frac{\alpha\gamma^2}{c\beta(I+r)(J+r)(I+J)}+\frac{\alpha G_\infty}{I+J}+B_0\right)\\
&= \lim_{h\to\infty}\left(\frac{\alpha(J-r)}{2(J+r)}+\frac{\alpha\gamma^2(J-r)}{2c\beta(I+r)(J+r)(I+J)}+\frac{\alpha G_\infty(J-r)}{2(I+J)}+\frac{B_0}{2}(J-r)\right)\\
&= \frac{\alpha}{2}+0+\frac{\alpha G_\infty}{2}\pm\infty = \pm\infty
\end{aligned}$$

and, for $t>0$,

$$\begin{aligned}
\lim_{h\to\infty} u(t) &= \lim_{h\to\infty}\begin{pmatrix}\frac{\alpha(J-r)}{2(J+r)}+\frac{\alpha\gamma^2(J-r)}{2c\beta(I+r)(J+r)(I+J)}+\frac{\gamma G_\infty(J-r)}{2(I+J)}+\frac{(J-r)B_0}{2e^{\frac{J-r}{2}t}}\\ +\frac{J-r}{2}B_\infty\left(1-e^{\frac{r-J}{2}t}\right)\end{pmatrix}\\
&= \lim_{h\to\infty}\begin{pmatrix}\frac{\alpha}{2}+\frac{\alpha\gamma^2}{2c\beta(I+r)(I+2J+r)}+\frac{\gamma G_\infty}{2}+\frac{B_0}{te^{\frac{J-r}{2}t}}\\ +\frac{gr\alpha(J-r)}{4h}\left(1-e^{\frac{r-J}{2}t}\right)+\frac{gr\alpha\gamma^2(J-r)}{4ch\beta(I+r)(I+J)}\left(1-e^{\frac{r-J}{2}t}\right)\\ +\frac{gr\alpha\gamma^2(J^2-r^2)}{4h(I+J)\left(4c\beta\delta(r+\delta)-\gamma^2\right)}\left(1-e^{\frac{r-J}{2}t}\right)\end{pmatrix}\\
&= \frac{\alpha}{2}+0+\frac{\gamma G_\infty}{2}+0+\lim_{h\to\infty}\begin{pmatrix}\frac{gr\alpha(J-r)}{4h}+\frac{gr\alpha\gamma^2(J-r)}{4ch\beta(I+r)(I+J)}\\ +\frac{gr\alpha\gamma^2(J^2-r^2)}{4h(I+J)(4c\beta\delta(r+\delta)-\gamma^2)}\\ +\frac{gr\alpha(J-r)}{4he^{\frac{J-r}{2}t}}+\frac{gr\alpha\gamma^2(J-r)}{4ch\beta(I+r)(I+J)e^{\frac{J-r}{2}t}}\\ +\frac{gr\alpha\gamma^2(J^2-r^2)}{4h(I+J)(4c\beta\delta(r+\delta)-\gamma^2)e^{\frac{J-r}{2}t}}\end{pmatrix}\\
&= \frac{\alpha+\gamma G_\infty}{2}+\lim_{h\to\infty}\begin{pmatrix}\frac{r\alpha}{2J}+\frac{r\alpha\gamma^2}{2c\beta(I+r)(IJ+J^2+2h/g)}\\ +\frac{r\alpha\gamma^2}{(I+J)(4c\beta\delta(r+\delta)-\gamma^2)}\\ +\frac{r\alpha}{2(J+ht/g)e^{\frac{J-r}{2}t}}+\frac{r\alpha\gamma^2}{2c\beta(I+r)\left(IJ+J^2+\frac{ht}{g}(I+J+2)\right)e^{\frac{J-r}{2}t}}\\ +\frac{r\alpha\gamma^2}{(I+J)\left(4c\beta\delta(r+\delta)-\gamma^2\right)e^{\frac{J-r}{2}t}}\end{pmatrix}\\
&= \frac{\alpha+\gamma G_\infty}{2}+0=\frac{\alpha+\gamma G_\infty}{2}
\end{aligned}$$

through repeated application of l'Hôpital's rule. □

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