SLIDS Homework 6

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- Solution proposals are individual.
- Each solution must be reproducible by your peer. Code should be added to the appendix.
- The solution to each subproblem yield 0 to 2 points. Students are expected to attempt each problem.

1

TODO: Plot DAG

Consider a decision process $p^\pi(\mathbf{x}, a, y)$ with unknown states of nature:

$$\phi = \{p(\mathbf{x}), p(y|a, \mathbf{x})\}$$

We want to study a policy class Π_w using trial data, where each policy is indexed by a scalar threshold $w \in [0, 1]$:

$$p^\pi(a|\mathbf{x}) = \begin{cases} \mathbb{1}\{x_1 x_2 \geq w\} & a = 0 \\ \mathbb{1}\{x_1 x_2 < w\} & a = 1 \end{cases} \tag{1}$$

Define the risk of a policy to be:

$$L(\pi, \phi) = \mathbb{E}^\pi[\ell(\mathbf{x}, a, y)] = \mathbb{E}^\pi[y]$$

and evaluate all policies π_w by plotting

$$w \quad \text{versus} \quad L(\pi_w; \phi)$$

where you can approximate the risk using $m_0 \geq 10^4$ samples.

Use the following unknown states of nature:

$$p(\mathbf{x}) : \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \sim \mathcal{U}(0, 1)^2 \tag{2}$$

and

$$p(y|a, \mathbf{x}) : y|a, \mathbf{x} \sim \begin{cases} \mathcal{N}(1 - x_1 x_2, 0.1) & a = 0 \\ \mathcal{N}(x_1 x_2, 0.1) & a = 1 \end{cases} \tag{3}$$

Comment on the results.

2

Let us now instead consider using randomized trial data to study policy class Π_w. The sampling process is denoted

$$\widetilde{p}(\mathbf{x}, a, y),$$

and the action is assigned randomly as

$$\widetilde{p}(a) = \begin{cases} 0.70 & a = 0 \\ 0.30 & a = 1 \end{cases} \tag{4}$$

a) Comment on the assumptions required for the risk of the decision process $L(\pi, \phi)$ point-identifiable using data from the sampling process.

b) Assume that identifiability holds so that you can use the distributions in the previous problem. Use a point-estimate of the risk to plot two curves of

$$w \quad \text{versus} \quad L(\pi_w; \phi)$$

using $n = 10$ and $n = 100$ samples from $\widetilde{p}(\mathbf{x}, a, y)$. Comment on the results.

c) Repeat the exercise above but now consider a trial population with covariates drawn as:

$$p(\mathbf{x}) : \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \sim \mathcal{U}(0, \frac{2}{3})^2 \tag{5}$$

Comment on the results.

3

Let us now instead consider using observational data to study policy class Π_w. The sampling process is denoted

$$\widetilde{p}(\mathbf{x}, a, y),$$

and how the actions are assigned is now *unknown*!

a) Comment on the assumptions required for the risk of the decision process $L(\pi, \phi)$ point-identifiable using data from the sampling process.

b) Assume that identifiability holds, so that

$$L(\pi_w; \phi) \equiv \widetilde{\mathbb{E}}\left[\frac{p^{\pi}(a|\mathbf{x})}{\widetilde{p}(a|\mathbf{x})} y\right]$$

But since $\widetilde{p}(a|\mathbf{x})$ is unknown, use any method of your choice to learn a model $\widehat{p}(a|\mathbf{x})$ to replace this quantity.

Repeat the evaluation before by plotting two curves of

$$w \quad \text{versus} \quad L(\pi_w; \phi)$$

using $n = 100$ and $n = 1000$ samples from $\widetilde{p}(\mathbf{x}, a, y)$.

Drawn the actions as:

$$\widetilde{p}(a|\mathbf{x}) = \begin{cases} s\left(\frac{1}{2}(x_1x_2+1)\right) & a=0 \\ 1-s\left(\frac{1}{2}(x_1x_2+1)\right) & a=1 \end{cases} \tag{6}$$

where $s(\cdot)$ denotes the logistic function.

Comment on the results and your assumptions.