

# Atomic Physics Cheating Paper

## 1. 库瑟福散射与经典力学

$$a \equiv \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{E}, \quad b = \frac{a}{2} \cot \frac{\theta}{2}$$

粒子计数公式:

$$dn' = N n t \left( \frac{Z_1 Z_2 e^2}{16\pi\varepsilon_0 E_k} \right)^2 \frac{d\Omega}{\sin^4(\frac{\theta}{2})}$$

微分散射截面 (Rutherford):

$$\frac{d\sigma}{d\Omega} = \sigma_C(\theta) = \left( \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{4E} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}}$$

实验室系截面:

$$\begin{aligned} \sigma_L(\theta_L) &= \left( \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{2E_L \sin^2 \theta_L} \right)^2 \\ &\times \frac{\left[ \cos \theta_L + \sqrt{1 - \left( \frac{m_1}{m_2} \sin \theta_L \right)^2} \right]^2}{\sqrt{1 - \left( \frac{m_1}{m_2} \sin \theta_L \right)^2}} \end{aligned}$$

散射分数  $f$ :

$$f = n_s \cdot 4\pi C \cot^2 \left( \frac{\theta_0}{2} \right), \quad C = \left( \frac{Z_1 Z_2 e^2}{16\pi\varepsilon_0 E} \right)^2$$

## 2. 热辐射与玻尔模型

普朗克公式:

$$E(\nu, T) d\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1} d\nu$$

玻尔半径:

$$r_n = \frac{4\pi\varepsilon_0 \hbar^2}{Z m_e e^2} \cdot n^2$$

能级与速度:

$$\begin{aligned} E_n &= -\frac{1}{2} Z^2 m_e c^2 \alpha^2 \frac{1}{n^2} \approx -\frac{Z^2 13.6}{n^2} \text{ eV} \\ v_n &= \frac{Z}{n} \alpha c, \quad \alpha = 1/137 \end{aligned}$$

原子核运动修正 (约化质量):

$$R_A = R_\infty \frac{1}{1 + m_e/m_A}$$

## 3. 光电效应与基本常数

光电效应方程:

$$\frac{1}{2} m v_m^2 = h\nu - \phi$$

常用常数与关系:

- $\hbar c \approx 197 \text{ eV} \cdot \text{nm}$  (联系长度和能量)
- $e^2/4\pi\varepsilon_0 \approx 1.44 \text{ eV} \cdot \text{nm}$  (电磁项)
- $m_e c^2 \approx 0.511 \text{ MeV}$  (电子静止能量)

相对论效应:

$$\begin{aligned} E &= mc^2 = \frac{m_0 c^2}{\sqrt{1 - (v/c)^2}} \\ E^2 &= (pc)^2 + (m_0 c^2)^2 \end{aligned}$$

## 4. 量子力学基础

德布罗意波长:  $\lambda = h/p$

不确定性原理:  $\Delta x \Delta p_x \geq \frac{\hbar}{2}, \Delta E \Delta t \geq \frac{\hbar}{2}$

概率密度:  $P(r, t) = |\psi(r, t)|^2 = \psi^* \psi$

薛定谔方程 (定态):

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

一维无限深方势阱:

$$E_n = \frac{n^2 \hbar^2}{8ma^2}, \quad \psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \quad (n \geq 1)$$

一维谐振子:  $E_n = (n + \frac{1}{2})\hbar\omega \quad (n \geq 0)$

量子隧穿 (透射系数):

$$T \approx e^{-2\sqrt{2m(V_0-E)} \cdot d/\hbar}$$

算符与期望值:  $\bar{f} = \int \psi^* \hat{f} \psi d\tau$

$$\hat{p} = -i\hbar \nabla, \quad \hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V, \quad \hat{L}_z = -i\hbar \frac{\partial}{\partial \phi}$$

## 5. 磁矩与角动量

布拉格衍射:  $2d \sin \theta = n\lambda$

磁矩公式:

$$\vec{\mu}_l = -\frac{e}{2m_e} \vec{L} = -\gamma \vec{L}$$

$$\vec{\mu}_s = -g_s \frac{e}{2m_e} \vec{S}$$

幅值与投影:

- $|\vec{\mu}_l| = \sqrt{l(l+1)} \mu_B$
- $|\vec{\mu}_J| = g_J \sqrt{j(j+1)} \mu_B$
- $\mu_z = -m_J g_J \mu_B$
- $S = \sqrt{s(s+1)} \hbar$

玻尔磁子:  $\mu_B = \frac{e\hbar}{2m_e} \approx 9.27 \times 10^{-24} \text{ J/T}$

磁场中的力:  $F_z = \mu_z \frac{\partial B}{\partial z}$

朗德 g 因子:  $g_l = 1, \quad g_s \approx 2$

$$g_J = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

## 6. 外磁场效应

能级分裂:

- 弱磁场:  $\Delta E = \mu_B B g_j m_j$
- 强磁场:  $\Delta E = \mu_B B (m_l + 2m_s)$

谱线分裂:

$$\Delta\tilde{\nu} = \frac{(M_{\text{上}} g_{\text{上}} - M_{\text{下}} g_{\text{下}}) \cdot \mu_B B}{hc}$$

单纯能量差与磁场:  $\Delta E = \Delta U = \frac{hc\Delta\lambda}{\lambda^2}$

$$B = \frac{hc\Delta\lambda}{2\lambda^2 \mu_B g_J} \quad (\text{塞曼分裂})$$

施特恩-格拉赫实验通用公式:

$$\begin{aligned} \Delta Z &= \frac{g_J \mu_B \left( \frac{\partial B}{\partial z} \right) dD}{2E_k} \cdot \Delta m_J \\ &= \frac{g_J \mu_B \left( \frac{\partial B}{\partial z} \right) dD}{mv^2} \cdot \Delta m_J \\ &= \frac{g_J \mu_B \left( \frac{\partial B}{\partial z} \right) dD}{3k_B T} \cdot \Delta m_J \end{aligned}$$

## 7. 精细结构与多电子原子

原子光谱项符号:

$$2S+1 L_J \quad (\text{宇称} \pi = (-1)^{\sum l_i})$$

- $2S+1$ : 自旋多重度
- $L$  (总轨道):  $0(S), 1(P), 2(D), 3(F), 4(G), 5(H)$
- $J$  (总角动量):  $|L-S| \leq J \leq L+S$

洪特规则 (确定基态顺序):

1. 最大  $S$ : 自旋多重度  $2S+1$  最大者能量最低。

2. 最大  $L$ : 在  $S$  相同时, 轨道角动量  $L$  最大者能量最低。

3. 确定  $J$  (自旋-轨道耦合):

- 电子数  $\leq$  半满:  $J = |L-S|$  (最小  $J$  能量最低)
- 电子数  $>$  半满:  $J = L+S$  (最大  $J$  能量最低)

氢原子精细结构:

$$E_{nj} = -\frac{13.6Z^2}{n^2} \left[ 1 + \frac{(Z\alpha)^2}{n^2} \left( \frac{n}{j+1/2} - \frac{3}{4} \right) \right]$$

L-S 耦合能量项:

$$\Delta E_{ls} = \frac{1}{2} A [J(J+1) - L(L+1) - S(S+1)]$$

其中  $A \propto \frac{Z^4}{n^3 l(l+1/2)(l+1)}$  (Landé interval rule)

**空穴原理:** 超过半充满壳层的电子数等效于空穴数 ( $L, S$  相同, 但  $J$  的基态规则相反)。

**泡利不相容与填充限额:**

- 主壳层  $2n^2$ ; 分壳层  $2(2l + 1)$
- 等效电子 ( $n, l$  相同) 需考虑泡利原理排除重复态。

类型	计算步骤
L-S	1. $\vec{L} = \sum \vec{l}_i$ 2. $\vec{S} = \sum \vec{s}_i$ 3. $\vec{J} = \vec{L} + \vec{S}$
j-j	1. $\vec{j}_i = \vec{l}_i + \vec{s}_i$ 2. $\vec{J} = \sum \vec{j}_i$

## 8. X 射线与康普顿散射

**短波限:**  $\lambda_{min} = hc/E = hc/eV$

**康普顿波长偏移:**  $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$

**衰减定律:**  $I = I_0 e^{-\mu x}$

**能量转移 (电子获得动能):**  $E'_k = E - E'$

**莫塞莱定律:**

$$E_0 = 13.6 \text{ eV} \cdot (Z - \sigma)^2 \cdot \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

**散射光子能量 (康普顿):**

$$E' = \frac{E}{1 + \frac{E}{m_p c^2}(1 - \cos\theta)}$$

## 9. 径向波函数附录

**积分公式:**  $\int_0^\infty x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}$

**类氢原子径向波函数  $R_{n,l}(r)$ :**

$$R_{1,0} = 2 \left( \frac{Z}{a_1} \right)^{3/2} e^{-Zr/a_1}$$

$$R_{2,0} = 2 \left( \frac{Z}{2a_1} \right)^{3/2} \left( 1 - \frac{Zr}{2a_1} \right) e^{-Zr/2a_1}$$

$$R_{2,1} = \frac{1}{\sqrt{3}} \left( \frac{Z}{2a_1} \right)^{3/2} \left( \frac{Zr}{a_1} \right) e^{-Zr/2a_1}$$

$$R_{3,0} = 2 \left( \frac{Z}{3a_1} \right)^{3/2} \left[ 1 - \frac{2Zr}{3a_1} + \frac{2}{27} \left( \frac{Zr}{a_1} \right)^2 \right] e^{-Zr/3a_1}$$

$$R_{3,1} = \frac{4\sqrt{2}}{3} \left( \frac{Z}{3a_1} \right)^{3/2} \left[ \frac{Zr}{a_1} - \frac{1}{6} \left( \frac{Zr}{a_1} \right)^2 \right] e^{-Zr/3a_1}$$

$$R_{3,2} = \frac{2\sqrt{2}}{27\sqrt{5}} \left( \frac{Z}{3a_1} \right)^{3/2} \left( \frac{Zr}{a_1} \right)^2 e^{-Zr/3a_1}$$

