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Problem 1 *Prove that* $\neg[r \vee (q \wedge (\neg r \to \neg p))] \equiv \neg r \wedge (p \vee \neg q)$ *by using a series of logical equivalences.*

Solution:

- Simplify the implication: $\neg r \to \neg p \equiv \neg(\neg r) \vee \neg p \equiv r \vee \neg p$. Thus, the expression becomes:
$$\neg[r \vee (q \wedge (r \vee \neg p))].$$
- Apply De Morgan's Law to the negation of the disjunction:
$$\neg[r \vee (q \wedge (r \vee \neg p))] = \neg r \wedge \neg(q \wedge (r \vee \neg p)).$$
- Apply De Morgan's to the negated conjunction:
$$\neg(q \wedge (r \vee \neg p)) = \neg q \vee \neg(r \vee \neg p).$$
So the expression is:
$$\neg r \wedge (\neg q \vee \neg(r \vee \neg p)).$$
- Simplify $\neg(r \vee \neg p)$:
$$\neg(r \vee \neg p) = \neg r \wedge \neg(\neg p) = \neg r \wedge p.$$
Thus:
$$\neg r \wedge (\neg q \vee (\neg r \wedge p)).$$
- Distribute $\neg q \vee (\neg r \wedge p)$:
$$\neg q \vee (\neg r \wedge p) = (\neg q \vee \neg r) \wedge (\neg q \vee p).$$
So the expression becomes:
$$\neg r \wedge (\neg q \vee \neg r) \wedge (\neg q \vee p).$$
- Apply the absorption law:
$$\neg r \wedge (\neg q \vee \neg r) = \neg r.$$
Resulting in:
$$\neg r \wedge (\neg q \vee p).$$
- Recognize that $\neg q \vee p = p \vee \neg q$ (commutative property), so:
$$\neg r \wedge (p \vee \neg q).$$
This matches the right hand side we were given at the start.

Problem 2 *Express the following propositions using quantifiers, then express the negation in English and using quantifiers:*
(a) Some people have no common sense.
(b) All Swedish movies are boring.
(c) No one can keep a secret.
(d) Someone in this class has a bad attitude.
Make sure you indicate the predicate and its domain.

Solution:

(a) **Proposition:** Some people have no common sense.

- **Predicate:** $C(x) =$ "x has common sense."
- **Domain:** All people.
- **Quantified Form:** $\exists x \neg C(x)$.
- **Negation (English):** "All people have common sense."
- **Negation (Quantifiers):** $\neg(\exists x \neg C(x)) \equiv \forall x C(x)$.

(b) **Proposition:** All Swedish movies are boring.

- **Predicate:** $B(x) =$ "x is boring."
- **Domain:** All Swedish movies.
- **Quantified Form:** $\forall x B(x)$.
- **Negation (English):** "Some Swedish movies are not boring."
- **Negation (Quantifiers):** $\neg(\forall x B(x)) \equiv \exists x \neg B(x)$.

(c) **Proposition:** No one can keep a secret.

- **Predicate:** $K(x) =$ "x can keep a secret."
- **Domain:** All people.
- **Quantified Form:** $\forall x \neg K(x)$.
- **Negation (English):** "Someone can keep a secret."
- **Negation (Quantifiers):** $\neg(\forall x \neg K(x)) \equiv \exists x K(x)$.

(d) **Proposition:** Someone in this class has a bad attitude.

- **Predicate:** $A(x) =$ "x has a bad attitude."
- **Domain:** All people in this class.
- **Quantified Form:** $\exists x A(x)$.
- **Negation (English):** "No one in this class has a bad attitude."
- **Negation (Quantifiers):** $\neg(\exists x A(x)) \equiv \forall x \neg A(x)$.

Problem 3 *Let $M(x)$ = "x is a millionaire" and $P(x)$ = "x drives a Porsche". The domain is all people. Translate to English:*
(a) $\forall x(M(x) \to P(x))$
(b) $\exists x(M(x) \to P(x))$
(c) $\forall x(M(x) \wedge P(x))$
(d) $\exists x(M(x) \vee P(x))$

Solution:

(a) $\forall x(M(x) \to P(x))$: "All millionaires drive a Porsche."

(b) $\exists x(M(x) \to P(x))$: Since $M(x) \to P(x) \equiv \neg M(x) \vee P(x)$, this translates to: "There exists a person who is either not a millionaire or drives a Porsche."

(c) $\forall x(M(x) \wedge P(x))$: "Everyone is both a millionaire and drives a Porsche."

(d) $\exists x(M(x) \vee P(x))$: "There exists a person who is either a millionaire or drives a Porsche."

Problem 4 *Prove that* $\cos x$ *and* $\sin x$ *are continuous* $\forall x \in \mathbb{R}$.

(I didn't understand how to prove this, so I watched a YouTube video that explained it and I only kinda get it so heres my attempt at the proof)

Solution:

To prove that $\sin x$ is continuous at every point $a \in \mathbb{R}$, we need to show that for every $\epsilon > 0$, there exists a $\delta > 0$ such that if $|x - a| < \delta$, then $|\sin x - \sin a| < \epsilon$.

Start with the trigonometric identity:

$$\sin x - \sin a = 2\cos\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right).$$

Taking the absolute value:

$$|\sin x - \sin a| = 2\left|\cos\left(\frac{x+a}{2}\right)\right|\left|\sin\left(\frac{x-a}{2}\right)\right|.$$

Since $|\cos\theta| \leq 1$ for any θ, this simplifies to:

$$|\sin x - \sin a| \leq 2\left|\sin\left(\frac{x-a}{2}\right)\right|.$$

We use the known inequality $|\sin\theta| \leq |\theta|$ for all real θ (this can be established geometrically: in the unit circle, the vertical distance $\sin\theta$ is less than or equal to the arc length θ for $\theta \geq 0$, and by symmetry for $\theta < 0$):

$$\left|\sin\left(\frac{x-a}{2}\right)\right| \leq \left|\frac{x-a}{2}\right| = \frac{|x-a|}{2}.$$

Substituting back:

$$|\sin x - \sin a| \leq 2 \cdot \frac{|x-a|}{2} = |x-a|.$$

To make $|\sin x - \sin a| < \epsilon$, choose $\delta = \epsilon$. Then, if $|x - a| < \delta = \epsilon$, we have $|\sin x - \sin a| \leq |x - a| < \epsilon$.

This δ works for any a, so $\sin x$ is continuous everywhere.

Similarly, for $\cos x$ at $a \in \mathbb{R}$:

$$\cos x - \cos a = -2\sin\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right).$$

Taking the absolute value:

$$|\cos x - \cos a| = 2\left|\sin\left(\frac{x+a}{2}\right)\right|\left|\sin\left(\frac{x-a}{2}\right)\right|.$$

Since $|\sin\theta| \leq 1$ for any θ:

$$|\cos x - \cos a| \leq 2 \cdot 1 \cdot \left|\sin\left(\frac{x-a}{2}\right)\right| = 2\left|\sin\left(\frac{x-a}{2}\right)\right|.$$

Again, using $|\sin\theta| \leq |\theta|$:

$$2\left|\sin\left(\frac{x-a}{2}\right)\right| \leq 2 \cdot \frac{|x-a|}{2} = |x-a|.$$

So:

$$|\cos x - \cos a| \leq |x-a|.$$

Choose $\delta = \epsilon$. If $|x-a| < \delta = \epsilon$, then $|\cos x - \cos a| < \epsilon$.

This works for any a, so $\cos x$ is continuous everywhere.