Valen Li

Q1:

a.) Original: $\exists_x P(x) \land \exists_x Q(x) \land A$

$\exists_x \exists_y (P(x) \land Q(y) \land A)$ Rewrite in prenex

b.) Original: $\neg(\forall_x P(x) \land \forall_x Q(x))$

De Morgans → $\exists_x \neg P(x) \lor \exists_x \neg Q(x)$

Rewrite in prenex $\exists_x \exists_y (\neg P(x) \lor \neg Q(y))$

c.) Original: $\exists_x P(x) \rightarrow \exists_x Q(x)$

~~De Morgans left side and logical of equivalence~~

Logical Equivalence: $\exists_x P(x) \rightarrow \exists_x Q(x) \equiv \neg \exists_x P(x) \lor \exists_x Q(x)$

De Morgans Left: $\forall_x \neg P(x) \lor \exists_x Q(x)$

Rewrite in prenex: $\forall_x \exists_y (\neg P(x) \lor Q(y))$

Q2:

Let D: Logic is difficult
Let S: Many students like logic
Let E: Mathematics is easy

Assuptions: $D \vee \neg S$ $E \rightarrow \neg D$

a) $S \rightarrow \neg E$

Equivalence Communative

$D \vee \neg S \equiv S \rightarrow D$ $(p \rightarrow q \equiv \neg p \vee q)$ $(p \vee \neg S \equiv \neg S \vee p)$

$E \rightarrow \neg D \equiv \neg(\neg D) \rightarrow \neg E \equiv D \rightarrow \neg E$ $(p \rightarrow q \equiv \neg q \rightarrow \neg p)$

$S \rightarrow D$ Using Hypothetical Syllogism
$D \rightarrow \neg E$
$\therefore S \rightarrow \neg E$ Valid

b.) $\neg E \rightarrow \neg S$

$D \vee \neg S \equiv \neg S \vee D \equiv S \rightarrow D$ Communative and Equivalence

If $\neg E$ is true and when $\neg S$ is false

$D \vee \neg S \equiv D \vee F \equiv D$ ✓
$E \rightarrow \neg D \equiv F \rightarrow \neg D$ always true ✓

Counter example

$D = T$
$S = T$ Invalid
$E = F$

c.) Prove $\neg E \vee D$

Asumptions: $E \rightarrow \neg D \equiv \neg E \vee \neg D$ Equivalence

$D \vee \neg S$

$\neg E \vee \neg D$ and $\neg E \vee D$

Counter example

$D = F$

$E = F$

d.) Prove $\neg D \vee \neg E$

$E \rightarrow \neg D \equiv \neg E \vee \neg D$: Equivalence

$\neg E \vee \neg D \equiv \neg D \vee \neg E$ Communative.

Valid.

e.) Prove $\neg S \rightarrow (\neg E \vee \neg D)$

We know $S \rightarrow D$ from (2a.) $\neg E \vee \neg D$

$S \rightarrow \neg E$ (2a)

$\neg S \rightarrow (\neg E \vee \neg D)$ We know $\neg E \vee \neg D$ is true or false

$\neg S \rightarrow T$ so $\neg S$ must be false ~~to stated~~ to show invalid e.i) $\neg S \to (\neg E \vee \neg D)$

$\neg E \vee \neg D$ must be false for it to be invalid

$\neg E = F$ and $\neg D = F$ Must be to be invalid
so $E = T$ $D = T$
BUT we know $E \to \neg D$

SO $T \to \neg T$ is false

SO its valid because only scenario $\neg S \to (\neg E \vee \neg D)$ is invalid is when it breaks assumptions

3. Part One
Assume true

$p_1 \leftrightarrow p_4$ shows $p_1 \to p_4$ $\quad p_4 \to p_1$
$p_2 \leftrightarrow p_3$ $\quad p_1 \to p_3$ $\quad p_3 \to p_1$
$p_1 \leftrightarrow p_3$ $\quad p_3 \to p_2$ $\quad p_2 \to p_3$

$p_1 \to p_3$
$p_3 \to p_2$
$\therefore p_1 \to p_2$

~~$p_1 \to p_2$~~
~~$p_2 \to$~~

$p_2 \to p_3$
$p_3 \to p_1$
$\therefore p_2 \to p_1$

$p_1 \to p_2$
$p_2 \to p_1$
$\therefore p_1 \leftrightarrow p_2$

~~$p_4 \to p_1$~~
~~$p_1 \to p_2, p_3$~~
~~$\therefore p_4 \to p_2, p_3$~~

$p_4 \to p_1$ and p_1 implies p_2 and p_3
So p_4 will imply all

Just many Hypo Syllogism

So because $p_1 \leftrightarrow p_4$ and $p_1 \leftrightarrow p_5$ and $p_3 \leftrightarrow p_2$

p_4 is equivalent to p_1, p_2, p_3

Part Two:

$p_1 \to p_4$
$p_4 \to p_2$
$p_2 \to p_5$
$p_5 \to p_3$
$p_3 \to p_1$

Using Hypo Syllogism you can easily show they are all equivalent.

$p_1 \to p_4$
$p_4 \to p_2$
$\therefore p_1 \to p_2$ etc.

following this order

You show they are all equivalent

4.

Let x be a man
y be a city
z be a state in U.S

Let $P(x, y, z)$: x has visited city y in state z

$\exists x \forall y \forall z \, P(x, y, z)$

5. Prove no 2 integers k and j satisfy
$k^2 = 4j+2$

$k^2 = 2(2j+1)$

k is even because $k^2 = 2(\text{integer})$ is even

so let $k = 2m$
so

$(2m)^2 = 4j+2$

$4m^2 = 4j+2$
$2m^2 = 2j+1$

$2m^2 - 2j = 1$

$2(m^2-2j) = 1$ m and j are integers
so let $m^2-2j = L$

$2(L) = 1$
$L = 1/2$ 1/2 is not an integer
contradicting given k and j
are integers

so no integers k and j satisfy $k^2 = 4j+2$

6.

Alice: C
John: ¬J
Carlos: D
Diana: ¬D

Test cases

Assume True

Alice did it	John	Carlos	Diana
F	F	T	F
T	F	T	T
F	F	F	T
T	T	T	F
2	1	3	2

a.) John did it because thats only case where 1 True

b.) Yes then carlos did it because thats only case one lying

7. Prove $x < x^2 \leftrightarrow x > 1$

$x < x^2$

$0 < x^2 - x$

$0 < x(x-1)$ Only true iff $x > 0$ and $(x-1) > 0$

OR

Both negative $x < 0$ $x - 1 < 0$

Given that x is positive real number

$x > 0$ and $(x-1) > 0$ must be true

$x - 1 > 0$ is $x > 1$ therefore $x < x^2 \rightarrow x > 1$ is true

Part b $x > 1 \Rightarrow x < x^2$

$x > 1$

$x^2 > x$

$x < x^2$ must be true

So $x < x^2 \leftrightarrow x > 1$

8

P : Assume n is an integer
Q : Prove that n^2 ends in digit from set $\{0,1,4,5,6,9\}$

$P \to Q$

$\neg Q \to \neg P$; If n^2 ends in digit from $\{2,3,7,8\}$
then n is not an integer

9.