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CS-225: Discrete Structures in CS
Homework Assignment 6
Problems provided on Canvas: Problem # 1, 2, 3, 4, 5

- **Problem #1**

$d_1 = 2$
$d_2 = 4d_1 + 3 = 4 \cdot 2 + 3$
$d_3 = 4d_2 + 3 = 4 \cdot (4 \cdot 2 + 3) + 3 = 4^2 \cdot 2 + 4 \cdot 3 + 3$
$d_4 = 4d_3 + 3 = 4 \cdot (4^2 \cdot 2 + 4 \cdot 3 + 3) + 3 = 4^3 \cdot 2 + 4^2 \cdot 3 + 4 \cdot 3 + 3$
$\vdots$
Guess:
$d_n = 4^{n-1} \cdot 2 + 4^{n-2} \cdot 3 + \cdots + 4^3 \cdot 3 + 4^2 \cdot 3 + 4 \cdot 3 + 3$
$= (4^{n-1} \cdot 3 + 4^{n-2} \cdot 3 + \cdots + 4^3 \cdot 3 + 4^2 \cdot 3 + 4^1 \cdot 3 + 4^0 \cdot 3) - 4^{n-1}$
$= \dfrac{3 \cdot 4^{n-1+1} - 3}{4 - 1} - 4^{n-1}$

$by\ the\ formula \sum_{k=0}^{n} ar^k\ (r \neq 0) = \dfrac{ar^{n+1} - a}{r - 1} (r \neq 1), with\ n = n - 1, a = 3\ and\ r = 4$

$= \dfrac{3 \cdot 4^n - 3}{3} - 4^{n-1}$
$= 4^n - 1 - 4^{n-1}$
$= 4 \cdot 4^{n-1} - 1 \cdot 4^{n-1} - 1$
$= 3 \cdot 4^{n-1} - 1$

- **Problem #2**

$t_0 = 0$
$t_1 = t_0 + 3 \cdot 1 + 1 = 0 + 3 \cdot 1 + 1 = 3 \cdot 1 + 1$
$t_2 = t_1 + 3 \cdot 2 + 1 = 3 \cdot 1 + 1 + 3 \cdot 2 + 1 = 3 \cdot (1 + 2) + 2$
$t_3 = t_2 + 3 \cdot 3 + 1 = 3 \cdot (1 + 2) + 2 + 3 \cdot 3 + 1 = 3 \cdot (1 + 2 + 3) + 3$
$t_4 = t_3 + 3 \cdot 4 + 1 = 3 \cdot (1 + 2 + 3) + 3 + 3 \cdot 4 + 1 = 3 \cdot (1 + 2 + 3 + 4) + 4$
$\vdots$
Guess:
$t_n = 3 \cdot (1 + 2 + 3 + \cdots + (n - 1) + n) + n$
$= 3 \cdot \dfrac{n(n+1)}{2} + n \quad by\ the\ formula \sum_{k=1}^{n} k = \dfrac{n(n+1)}{2}$
$= \dfrac{3n^2 + 3n + 2n}{2}$
$= \dfrac{3n^2 + 5n}{2}$

- **Problem #3**

I. BASE: $5 \in S$.
II. RECURSION: $If\ x \in S, then\ x + 5 \in S$.
III. RESTRICTION: Nothing is in S other than objects defined in I and II above.

- **Problem #4**

I. BASE: $\epsilon\ is\ in\ S, where\ \epsilon\ is\ the\ null\ string$.
II. RECURSION: $If\ s \in S, then\ 1s \in S\ and\ s0 \in S$.
III. RESTRICTION: Nothing is in S other than objects defined in I and II above.

- **Problem #5**

I. BASE: $a \in S, which\ means\ the\ only\ object\ in\ the\ base\ of\ S\ is\ a$.
II. RECURSION: $If\ s \in S, then\ as \in S\ and\ bs \in S$.
III. RESTRICTION: Nothing is in S other than objects defined in I and II above.