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CS-225: Discrete Structures in CS
Homework Assignment 4, Part 1
Exercise Set 6.1: Question # 12, 16
Exercise Set 6.2: Question # 4, 10, 14
Exercise Set 6.3: Question # 12, 37, 42

- **Set 6.1 – Q#12**

a. $A \cup B = \{x \in R| -3 \leq x < 2\}$
b. $A \cap B = \{x \in R| -1 < x \leq 0\}$
c. $A^c = \{x \in R| x < -3 \text{ } or \text{ } x > 0\}$
d. $A \cup C = \{x \in R| -3 \leq x \leq 0 \text{ } or \text{ } 6 < x \leq 8\}$
e. $A \cap C = \emptyset$
f. $B^c = \{x \in R| x \leq -1 \text{ } or \text{ } x \geq 2\}$
g. $A^c \cap B^c = \{x \in R| x < -3 \text{ } or \text{ } x \geq 2\}$
h. $A^c \cup B^c = \{x \in R| x \leq -1 \text{ } or \text{ } x > 0\}$
i. $(A \cap B)^c = \{x \in R| x \leq -1 \text{ } or \text{ } x > 0\}$
j. $(A \cup B)^c = \{x \in R| x < -3 \text{ } or \text{ } x \geq 2\}$

- **Set 6.1 – Q#16**

b. $A \cap (B \cup C) = \{b, c\}$
$(A \cap B) \cup C = \{b, c, e\}$
$(A \cap B) \cup (A \cap C) = \{b, c\}$
Hence $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
c. $(A - B) - C = \{a\}$
$A - (B - C) = \{a, b, c\}$
Hence $(A - B) - C \neq A - (B - C)$.

- **Set 6.2 – Q#4**

(a) $A \cup B \subseteq$ B, every element in $A \cup B$ is in B
(b) $A \cup B$
(c) $x \in B$
(d) A
(e) or
(f) B
(g) A
(h) B
(i) B

- **Set 6.2 – Q#10**

Suppose A, B and C are any sets. To show that $(A-B)\cap(C-B)=(A\cap C)-B$, we must show that $(A-B)\cap(C-B)\subseteq(A\cap C)-B$ and that $(A\cap C)-B\subseteq(A-B)\cap(C-B)$.
To show $(A-B)\cap(C-B)\subseteq(A\cap C)-B$:
Suppose that x is any element in $(A-B)\cap(C-B)$, we must show that $x\in(A\cap C)-B$. By definition of intersection, $x\in A-B$ and $x\in C-B$. Then by definition of set difference, $x\in A\ and\ x\notin$ B and $x\in C\ and\ x\notin$ B, then we have that $x\in A\ and\ x\in C\ and\ x\notin$ B. By definition of intersection, $x\in A\cap C$. Hence $x\in A\cap C$ and $x\notin$ B, and so, by definition of set difference, $x\in(A\cap C)-B$. So $(A-B)\cap(C-B)\subseteq(A\cap C)-B$.
To show $(A\cap C)-B\subseteq(A-B)\cap(C-B)$:
Suppose that y is any element in $(A\cap C)-B$, we must show that $y\in(A-B)\cap(C-B)$. By definition of set difference, $y\in A\cap C$ and $y\notin$ B. Then by definition of intersection, $y\in A\ and\ y\in C$. Because $y\in A\ and\ y\notin$ B, so $y\in A-B$. Because $y\in C\ and\ y\notin$ B, so $y\in C-B$. Then we have that $y\in A-B$ and $y\in C-B$. By definition of intersection, $y\in(A-B)\cap(C-B)$. So $(A\cap C)-B\subseteq(A-B)\cap(C-B)$.
So as was to be shown, $(A-B)\cap(C-B)=(A\cap C)-B$.

- **Set 6.2 – Q#14**

Suppose A, B and C are any sets and $A\subseteq$ B. Let $x\in A\cup C$. To show that $A\cup C\subseteq$ B ∪ C, we must show that $x\in B\cup C$. By definition of union, $x\in A\ or\ x\in C$.
Case 1 $(x\in A)$:
Since $A\subseteq$ B and $x\in A$, then $x\in B$ by definition of subset. Hence $x\in B$ or $x\in C$, and so, by the definition of union, $x\in B\cup C$.
Case 1 $(x\in C)$:
Because $x\in C$, we have that $x\in B\cup C$ by definition of union.
Thus, in both cases, $x\in B\cup C$, by definition of subset, $A\cup C\subseteq$ B ∪ C.

- **Set 6.3 – Q#12**

Suppose A, B and C are any sets. To show that $A\cap(B-C)=(A\cap B)-(A\cap C)$, we must show that $A\cap(B-C)\subseteq(A\cap B)-(A\cap C)$, and that $(A\cap B)-(A\cap C)\subseteq A\cap(B-C)$.
To show $A\cap(B-C)\subseteq(A\cap B)-(A\cap C)$:
Suppose that x is any element in $A\cap(B-C)$, we must show that $x\in(A\cap B)-(A\cap C)$. By definition of intersection, $x\in A\ and\ x\in B-C$. Then by definition of set difference, $x\in B\ and\ x\notin$ C. Then we have that $x\in A\ and\ x\in B$, by definition of intersection, $x\in A\cap B$. And we also have that $x\notin$ C, by definition of intersection, $x\notin$ A ∩ C. Hence, $x\in A\cap B\ and\ x\notin$ A ∩ C, by definition of set difference, $x\in(A\cap B)-(A\cap C)$. So $A\cap(B-C)\subseteq(A\cap B)-(A\cap C)$.
To show $(A\cap B)-(A\cap C)\subseteq A\cap(B-C)$:
Suppose that y is any element in $(A\cap B)-(A\cap C)$, we must show that $y\in A\cap(B-C)$. By definition of set difference, $y\in A\cap B\ and\ y\notin$ A ∩ C. Then by definition of intersection, $y\in A\ and\ y\in B\ for\ y\in A\cap B$, $y\notin$ A or $y\notin$ C for $y\notin$ A ∩ C. Then we have that $y\in A\ and\ y\in B\ and\ y\notin$ C. By definition of set difference, $y\in B-C$ because

$y \in B$ *and* $y \notin C$. Also $y \in A$, by definition of intersection, $y \in A \cap (B - C)$. So $(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$.
So as was to be shown, $A \cap (B - C) = (A \cap B) - (A \cap C)$.

- **Set 6.3 – Q#37**

$(B^c \cup (B^c - A))^c$
$= (B^c \cup (B^c \cap A^c))^c$ by Set Difference Law
$= (B^c)^c$ by Absorption Law
$= B$ by Double Complement Law

- **Set 6.3 – Q#42**

$(A - (A \cap B)) \cap (B - (A \cap B))$
$= (A \cap (A \cap B)^c) \cap (B \cap (A \cap B)^c)$ by Set Difference Law
$= (A \cap (A^c \cup B^c)) \cap (B \cap (A^c \cup B^c))$ by De Morgan's Law
$= ((A \cap A^c) \cup (A \cap B^c)) \cap ((B \cap A^c) \cup (B \cap B^c))$ by Distributive Law
$= (\emptyset \cup (A \cap B^c)) \cap ((B \cap A^c) \cup \emptyset)$ by Complement Law
$= (A \cap B^c) \cap (B \cap A^c)$ by Identity Law
$= (A \cap A^c) \cap (B \cap B^c)$ by Associative Law
$= \emptyset \cap \emptyset$ by Complement Law
$= \emptyset$ by Universal Bound Law