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CS-225: Discrete Structures in CS
Homework Assignment 9, Part 1
Exercise Set 10.1: Question # 9, 22, 37.f, 44.(a, b, c)

- **Set 10.1 – Q#9**

(i) $e_1, e_2, and\ e_7\ are\ incident\ on\ v_1.$
(ii) $v_1\ and\ v_2\ are\ adjacent\ to\ v_3.$
(iii) $e_2\ and\ e_7\ are\ adjacent\ to\ e_1.$
(iv) $e_1\ and\ e_3\ are\ loops.$
(v) $e_4\ and\ e_5\ are\ parallel\ edges.$
(vi) $v_4\ is\ isolated\ vertice.$
(vii) $degree\ of\ v_3 = 2.$
(viii) $total\ degree = \deg(v_1) + \deg(v_2) + \deg(v_3) + \deg(v_4) + \deg(v_5)$
$= 4 + 6 + 2 + 0 + 2 = 14.$

- **Set 10.1 – Q#22**

Suppose there were a simple graph with 5 vertices of degrees 2, 3, 3, 3, and 5. Then the vertex of degree 5 would have to be connected by edges to 5 distinct vertices other than itself because of the assumption that the graph is simple and hence has no loops or parallel edges. This contradicts the assumption that the graph has 5 vertices in total. Hence there is no graph with 5 vertices of degrees 2, 3, 3, 3, and 5.

- **Set 10.1 – Q#37.f**

Suppose this graph is bipartite. Then the vertex set can be partitioned into two mutually disjoint subsets such that vertices in each subset are connected by edges only to vertices in the other subset and not to vertices in the same subset. Now v_1 is in one subset of the partition, say V_1. Since v_1 is connected by edges to v_2 and v_5, both v_2 and v_5 must in the other subset, V_2. And v_2 is connected by edges to v_1 and v_3, v_5 is connected by edges to v_1 and v_4, so v_3 and v_4 must in the other subset of the partition V_1, together with v_1. But v_3 and v_4 are connected by an edge to each other. This contradicts the fact that no vertices in V_1 are connected by edges to other vertices in V_1. Hence the supposition is false, and so the graph is not bipartite.

- **Set 10.1 – Q#44**

a. No.

Let a simple graph has n(n$\geqslant$2) vertices, by definition of simple graph, a simple graph is a graph that does not have any loops or parallel edges, so every vertex of the graph is at most incident on n-1 edges. By definition of degree of a vertex, equivalently, every vertex of the graph has a degree no more than n-1. Hence, in a simple graph, every vertex has degree that is less than the number of vertices in the graph.

b. No

Suppose there were a simple graph that has 4 vertices each of different degrees. By result of #a, we know that in a simple graph, every vertex has degree that is less than the number of vertices in the graph. And the 4 vertices each has a different degree, so the supposition is that there were a simple graph that has 4 vertices of degrees 0, 1, 2, and 3. Therefore, there must be a vertex of degree 3, which means that this vertex is connected by edges to every other 3 vertex. So the degree of every other 3 vertex can not be 0, which contradicts the supposition that there must be a vertex of degree 0. Hence the supposition is false, so there cannot be a simple graph that has four vertices each of different degrees.

c. No

Suppose there were a simple graph that has $n(n \geqslant 2)$ vertices each of different degrees. And from result of #a, we know that in a simple graph, every vertex has degree that is less than the number of vertices in the graph. Thus the supposition is that there were a simple graph with $n(n \geqslant 2)$ vertices of degrees no more than n-1. Since the vertices each has a different degree, the degree of the n vertices must be 0, 1, 2, …, n-1. Therefore, there must be a vertex of degree n-1, which means that this vertex is connected by edges to every other n-1 vertex. So the degree of every other n-1 vertex cannot be 0, which contradicts the supposition that there must be a vertex of degree 0. Hence the supposition is false, so there cannot be a simple graph that has n vertices all of different degrees.