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CS-225: Discrete Structures in CS
Homework Assignment 4, Part 2
Exercise Set 5.1: Question # 21, 60
Extra Question: # 1, 2, 3

- **Set 5.1 – Q#21**

$$\sum_{m=0}^{3}\frac{1}{2^m}=\frac{1\times(\frac{1}{2})^{3+1}-1}{\frac{1}{2}-1}=-2\left(\frac{1}{2^4}-1\right)=\frac{15}{8}$$

$$by\ the\ formula\ \sum_{k=0}^{n}ar^k\ (r\neq 0)=\frac{ar^{n+1}-a}{r-1}(r\neq 1), with\ n=3, a=1\ and\ r=\frac{1}{2}$$

- **Set 5.1 – Q#60**

$$2\cdot\sum_{k=1}^{n}(3k^2+4)+5\cdot\sum_{k=1}^{n}(2k^2-1)$$

$$=\sum_{k=1}^{n}2(3k^2+4)+5\,(2k^2-1)\qquad by\ the\ distributive\ property$$

$$=\sum_{k=1}^{n}(\,16k^2+3)$$

$$=16\cdot\sum_{k=1}^{n}k^2+3n\qquad by\ the\ distributive\ property$$

$$=16\cdot\frac{n(n+1)(2n+1)}{6}+3n\qquad by\ the\ formula\ \sum_{k=1}^{n}k^2=\frac{n(n+1)(2n+1)}{6}$$

$$=\frac{8n(n+1)(2n+1)}{3}+3n$$

- **Extra Question #1**

$$\sum_{i=2}^{5}(4i-3)$$

$$=\sum_{i=1}^{5}(4i-3)-\sum_{i=1}^{1}(4i-3)$$

$shifting\ the\ lower\ limit\ of\ summation\ with\ i = 1\ and\ removing\ i = 1\ term$

$$= 4 \cdot \sum_{i=1}^{5} i - 3 \cdot 5 - (4 \cdot \sum_{i=1}^{1} i - 3 \cdot 1) \qquad by\ the\ distributive\ property$$

$$= 4 \cdot \frac{5(5+1)}{2} - 15 - 4 \cdot \frac{1(1+1)}{2} + 3 = 44$$

$$by\ the\ formula \sum_{k=1}^{n} k = \frac{n(n+1)}{n}, with\ n = 5\ and\ n = 1$$

- **Extra Question #2**

$$\sum_{j=0}^{4} (j^2 - (-1)^j)$$

$$= \sum_{j=0}^{4} j^2 - \sum_{j=0}^{4} (-1)^j \qquad by\ the\ distributive\ property$$

$$= 0^2 + \sum_{j=1}^{4} j^2 - \sum_{j=0}^{4} (-1)^j$$

$shifting\ the\ lower\ limit\ of\ summation\ with\ j = 1\ and\ adding\ j = 0\ term$

$$= \frac{4(4+1)(2 \times 4 + 1)}{6} - \frac{1 \times (-1)^{4+1} - 1}{-1 - 1} = 30 - 1 = 29$$

$$by\ formulas \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, with\ n = 4$$

$$and \sum_{k=0}^{n} ar^k\ (r \neq 0) = \frac{ar^{n+1} - a}{r - 1} (r \neq 1), with\ n = 4, a = 1\ and\ r = -1$$

- **Extra Question #3**

$$\sum_{j=3}^{7} (2^j + 4^j)$$

$$= \sum_{j=0}^{7} (2^j + 4^j) - \sum_{j=0}^{2} (2^j + 4^j)$$

$shifting\ the\ lower\ limit\ of\ summation\ with\ j = 0\ and\ removing\ j = 0, 1, 2\ terms$

$$= \sum_{j=0}^{7} 2^j + \sum_{j=0}^{7} 4^j - \sum_{j=0}^{2} 2^j - \sum_{j=0}^{2} 4^j \qquad by\ the\ distributive\ property$$

$$= \frac{1 \times 2^{7+1} - 1}{2-1} + \frac{1 \times 4^{7+1} - 1}{4-1} - \frac{1 \times 2^{2+1} - 1}{2-1} - \frac{1 \times 4^{2+1} - 1}{4-1}$$

$$by\ the\ formula\ \sum_{k=0}^{n} ar^k\ (r \neq 0) = \frac{ar^{n+1} - a}{r-1} (r \neq 1)$$

$$= 2^8 - 2^3 + \frac{4^8 - 4^3}{3} = 22072$$