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CS-225: Discrete Structures in CS
Homework Assignment 7, Part 1
Exercise Set 9.2: Question # 11.c, 14.c, 14.e, 17.a-d
Exercise Set 9.3: Question # 5.a, 24.a, 24.c, 33.e, 33.f

- **Set 9.2 – Q#11.c**

There is a 1 in the left-most position and the right-most position, then fill in the remaining 6 positions. Since there are 2 choices for each of the 6 position, by the multiplication rule, there are $2^6 = 64$ ways to fill in these 6 positions. So there are 64 bit strings of length 8 begin and end with 1.

- **Set 9.2 – Q#14**

c.

The 4 beginning letters of the license plates is TGIF, then fill in the remaining 3 digits. Since there are 10 choices for each of the remaining 3 digits, by the multiplication rule, there are $10^3 = 1000$ ways to fill in these 3 digits. So there are 1000 license plates could begin with TGIF.

e.

The 2 left-most position of the license plates is AB, then fill in the remaining 2 letters and 3 digits. Since all letters and digits are distinct, so there are 24 and 23 choices for each of the remaining 2 letters, and 10, 9, and 8 choices for each of the remaining 3 digits. Hence, by the multiplication rule, there are $24 \times 23 \times 10 \times 9 \times 8 = 397440$ ways to fill in these 5 remaining positions. So there are 397440 license plates could begin with AB and have all letters and digits distinct.

- **Set 9.2 – Q#17**

a.

The left-most digit of a four-digit integer can't be 0, so there are 9 choices for the left-most digit. And there are 10 choices for each of the remaining 3 digits. Hence, by the multiplication rule, there are $9 \times 10 \times 10 \times 10 = 9000$ integers from 1000 through 9999.

b.

The left-most digit of a four-digit integer can't be 0, so there are 9 choices for the left-most digit. The four-digit integers are odd, and odd integers end in 1, 3, 5, 7, 9, so there are 5 choices for the right-most digit. And there are 10 choices for each of the remaining 2 digits. Hence, by the multiplication rule, there are $9 \times 10 \times 10 \times 5 = 4500$ odd integers from 1000 through 9999.

c.

The left-most digit of a four-digit integer can't be 0, so there are 9 choices for the left-most digit. Since the four-digit integers have distinct digits, so there are 9, 8 and 7 choices for each of the remaining 3 digits. Hence, by the multiplication rule, there are $9 \times 9 \times 8 \times 7 =$ 4536 integers from 1000 through 9999 have distinct digits.

d.

The four-digit integers are odd, and odd integers end in 1, 3, 5, 7, 9, so there are 5 choices for the right-most digit. Since the four-digit integers have distinct digits and the left-most digit of a four-digit integer can't be 0, so there are 8, 8 and 7 choices for each of the remaining 3 digits. Hence, by the multiplication rule, there are $8 \times 8 \times 7 \times 5 =$ 2240 integers from 1000 through 9999 have distinct digits.

- **Set 9.3 – Q#5.a**

The left-most digit of a five-digit integer can't be 0, so there are 9 choices for the left-most digit. Since the five-digit integers are divisible by 5, so they end in 0 and 5, so there are 2 choices for the right-most digit. And there are 10 choices for each of the remaining 3 digits. Hence, by the multiplication rule, there are $9 \times 10 \times 10 \times 10 \times 2 =$ 18000 five-digit integers are divisible by 5.

- **Set 9.3 – Q#24**

a.

Let A = the set of integers that are multiples of 2 and B = the set of integers that are multiples of 9. Then $A \cap B = the\ set\ of\ integers\ that\ are\ multiples\ of$ 18. And n(A) = 500, since $1000 = 2 \times 500$. And n(B) = 111, since $1000 = 9 \times 111 + 1$. And n$(A \cap B) =$ 55, since $1000 = 18 \times 55 + 10$. Hence, by the inclusion/exclusion rule, $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 500 + 111 - 55 = 556$. So there are 556 integers from 1 through 1000 are multiples of 2 or multiples of 9.

c.

From the result of Q#a we know that there are 556 integers from 1 through 1000 are multiples of 2 or multiples of 9. And there are totally 1000 integers from 1 through 1000. So there are 1000-556=444 integers from 1 through 1000 are neither multiples of 2 nor multiples of 9.

- **Set 9.3 – Q#33**

Let H be the set of students who checked #1, C the set of students who checked #2, and D the set of students who checked #3.

e.

The number of students checked #2 and #3 but not #1 is $N(C \cap D) - N(H \cap C \cap D) = 3 - 2 =$ 1.

f.

The number of students checked #2 but neither of the other two is $N(C) - N(C \cap H) - N(C \cap D) + N(H \cap C \cap D) = 26 - 8 - 3 + 2 =$ 17.