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CS-225: Discrete Structures in CS
Homework Assignment 7, Part 2
Exercise Set 9.4: Question # 6, 7, 16, 27

- **Set 9.4 – Q#6**

a.
Yes. There are only 6 possible remainders that can be obtained when an integer is divided by 6: 0, 1, 2, 3, 4, 5 and 6. Thus, by the pigeonhole principle, if 7 integers are each divided by 6, then at least two of them must have the same remainder.
b.
No. For instance, {1, 2, 3, 4, 5, 6, 7} is a set of 7 integers no two of which have the same remainder when divided by 8.

- **Set 9.4 – Q#7**

Yes. Partition the set S into the following 5 disjoint subsets: {3, 12}, {4, 11}, {5, 10}, {6, 9} and {7, 8}. Each of the integers in S occurs in exactly one of the five subsets and the sum of the integers in each subset is 15. Thus if 6 integers from S are chosen, then by the pigeonhole principle, two must be from the same subset. It follows that the sum of these two integers is 15.

- **Set 9.4 – Q#16**

There are 100 integers from 1 through 100 inclusive. Of these, 20 integers are divisible by 5, since $100 \div 5 = 20$. Hence 80 integers from 1 through 100 are not divisible by 5. So 81 integers from 1 through 100 must be picked in order to be sure of getting one that is divisible by 5.

- **Set 9.4 – Q#27**

Yes. This follows from the generalized pigeonhole principle with 2000 pigeons, 365 pigeonhole, and k=4. Because $2000 \div 365 \approx 5.5 > 4$, then at least one pigeonhole contains 5 or more pigeons. In this question, which means in a group of 2000 people, must at least 5 have the same birthday.