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CS-225: Discrete Structures in CS
Homework Assignment 8, Part 1
Exercise Set 9.2: Question # 32.c, 33, 39.b, 39.d
Exercise Set 9.5: Question # 7.b, 14, 20

- **Set 9.2 – Q#32.c**

GOR may be regarded as a single symbol, so there are effectively 7symbols to be permuted. Hence, the number of arrangements is $7! = 5040$.

- **Set 9.2 – Q#33**

a.

The number of ways that they can be seated together in a row is the number of 6-permutations of a set of 6 elements. So the answer is $P(6,6) = 6! = 720$.

b.

In this case, the doctor seated on the aisle seat, then the remaining 5 people can be seated on the remaining 5 seats. And there are 2 aisle seats can be seated by the doctor. So the number of ways that the people can be seated together in a row with the doctor in an aisle seat is the number of 5-permutations of a set of 5 elements multiply 2. Hence, the answer is $P(5,5) \cdot 2 = 5! \cdot 2 = 240$.

c.

In this case, each couple wants to sit together with the husband on the left, so each couple may be regarded as a single object. Then the number of ways that they can be seated together in a row is the number of 3-permutations of a set of 3 elements. Hence, the answer is $P(3,3) = 3! = 6$.

- **Set 9.2 – Q#39**

b.

The number of ways 6 of the letters of the word ALGORITHM can be selected and written in a row is the number of 6-permutation of a set of 9 elements. So the answer is:

$$P(9,6) = \frac{9!}{(9-6)!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!} = 60480$$

d.

In this case, the first two letters must be OR. So the number of ways 6 of the letters of the word ALGORITHM can be selected and written in a row is the number of 4-permutation of a set of 7 elements. So the answer is:

$$P(7,4) = \frac{7!}{(7-4)!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!} = 840$$

- **Set 9.5 – Q#7.b**

(i)

$$\begin{bmatrix} number\ of\ subsets\ of\ 4 \\ women\ chosen\ from\ 7 \end{bmatrix} \cdot \begin{bmatrix} number\ of\ subsets\ of\ 3 \\ men\ chosen\ from\ 6 \end{bmatrix} = \binom{7}{4} \cdot \binom{6}{3}$$

$$= \frac{7!}{4!\,(7-4)!} \cdot \frac{6!}{3!\,(6-3)!}$$

$$= \frac{7!}{4! \cdot 3!} \cdot \frac{6!}{3! \cdot 3!} = 35 \cdot 20 = 700$$

(ii)

$$\begin{bmatrix} number\ of\ groups\ of\ 7 \\ with\ at\ least\ 1\ man \end{bmatrix} = \begin{bmatrix} total\ number\ of \\ groups\ of\ 7 \end{bmatrix} - \begin{bmatrix} number\ of\ all- \\ women\ groups \end{bmatrix}$$

$$= \binom{13}{7} - \binom{7}{7} = \frac{13!}{7!\,(13-7)!} - \frac{7!}{7!\,(7-7)!}$$

$$= 1716 - 1 = 1715$$

(iii)

$$\begin{bmatrix} number\ of\ groups\ of\ 7 \\ with\ at\ most\ 3\ women \end{bmatrix}$$

$$= \begin{bmatrix} number\ of\ groups \\ of\ 7\ with\ 1\ woman \end{bmatrix} + \begin{bmatrix} number\ of\ groups \\ of\ 7\ with\ 2\ women \end{bmatrix} + \begin{bmatrix} number\ of\ groups \\ of\ 7\ with\ 3\ women \end{bmatrix}$$

$$= \begin{bmatrix} number\ of\ subsets\ of\ 1 \\ woman\ chosen\ from\ 7 \end{bmatrix} \cdot \begin{bmatrix} number\ of\ subsets\ of\ 6 \\ men\ chosen\ from\ 6 \end{bmatrix} + \begin{bmatrix} number\ of\ subsets\ of\ 2 \\ women\ chosen\ from\ 7 \end{bmatrix}$$

$$\cdot \begin{bmatrix} number\ of\ subsets\ of\ 5 \\ men\ chosen\ from\ 6 \end{bmatrix} + \begin{bmatrix} number\ of\ subsets\ of\ 3 \\ women\ chosen\ from\ 7 \end{bmatrix} \cdot \begin{bmatrix} number\ of\ subsets\ of\ 4 \\ men\ chosen\ from\ 6 \end{bmatrix}$$

$$= \binom{7}{1} \cdot \binom{6}{6} + \binom{7}{2} \cdot \binom{6}{5} + \binom{7}{3} \cdot \binom{6}{4}$$

$$= \frac{7!}{1!\,(7-1)!} \cdot \frac{6!}{6!\,(6-6)!} + \frac{7!}{2!\,(7-2)!} \cdot \frac{6!}{5!\,(6-5)!} + \frac{7!}{3!\,(7-3)!} \cdot \frac{6!}{4!\,(6-4)!}$$

$$= 7 \cdot 1 + 21 \cdot 6 + 35 \cdot 15 = 658$$

- **Set 9.5 – Q#14**

a.

$$\begin{bmatrix} number\ of\ 16-bit\ strings \\ contain\ exactly\ 7\ 1's \end{bmatrix} = \binom{16}{7} = \frac{16!}{7!\,(16-7)!} = \frac{16!}{7! \cdot 9!} = 11440$$

b.

$$\begin{bmatrix} number\ of\ 16-bit\ strings \\ contain\ at\ least\ 13\ 1's \end{bmatrix}$$

$= \begin{bmatrix} number\ of\ 16-bit \\ strings\ contain\ 13\ 1's \end{bmatrix} + \begin{bmatrix} number\ of\ 16-bit \\ strings\ contain\ 14\ 1's \end{bmatrix} + \begin{bmatrix} number\ of\ 16-bit \\ strings\ contain\ 15\ 1's \end{bmatrix} +$

$\begin{bmatrix} number\ of\ 16-bit \\ strings\ contain\ 16\ 1's \end{bmatrix}$

$= \binom{16}{13} + \binom{16}{14} + \binom{16}{15} + \binom{16}{16}$

$= \frac{16!}{13!\,(16-13)!} + \frac{16!}{14!\,(16-14)!} + \frac{16!}{15!\,(16-15)!} + \frac{16!}{16!\,(16-16)!}$

$= 560 + 120 + 16 + 1 = 697$

c.

$\begin{bmatrix} number\ of\ 16-bit\ strings \\ contain\ at\ least\ 1\ 1's \end{bmatrix} = \begin{bmatrix} total\ number\ of \\ 16-bit\ strings \end{bmatrix} - \begin{bmatrix} number\ of\ all-0's \\ 16-bit\ strings \end{bmatrix} = 2^{16} - 1$

d.

$\begin{bmatrix} number\ of\ 16-bit\ strings \\ contain\ at\ most\ 1\ 1's \end{bmatrix} = \begin{bmatrix} number\ of\ all-0's \\ 16-bit\ strings \end{bmatrix} + \begin{bmatrix} number\ of\ 16-bit \\ strings\ contain\ 1\ 1 \end{bmatrix}$

$= \binom{16}{0} + \binom{16}{1}$

$= \frac{16!}{0!\,(16-0)!} + \frac{16!}{1!\,(16-1)!} = 1 + 16 = 17$

- **Set 9.5 – Q#20**

The word MILLIMICRON contains 2 M's, 3 I's, 2 L's, 1 C, 1 R, 1 O and 1 N, totally 11 letters.

a.

$\begin{bmatrix} number\ of\ distinguishable\ ways\ can\ the \\ letters\ of\ the\ word\ be\ arranged\ in\ order \end{bmatrix}$

$= \binom{11}{2} \cdot \binom{9}{3} \cdot \binom{6}{2} \cdot \binom{4}{1} \cdot \binom{3}{1} \cdot \binom{2}{1} \cdot \binom{1}{1}$

$= \frac{11!}{2!\,(11-2)!} \cdot \frac{9!}{3!\,(9-3)!} \cdot \frac{6!}{2!\,(6-2)!} \cdot \frac{4!}{1!\,(4-1)!} \cdot \frac{3!}{1!\,(3-1)!} \cdot \frac{2!}{1!\,(2-1)!} \cdot \frac{1!}{1!\,(1-1)!}$

$= 55 \cdot 84 \cdot 15 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 1663200$

b.

Fix the first position with M and last position with N, then there are 9 remaining positions to be filled by 3 I's, 2 L's, 1 C, 1 R, 1 O and 1 M.

$\begin{bmatrix} number\ of\ distinguishable\ orderings\ of\ the \\ letters\ of\ the\ word\ begin\ with\ M\ and\ end\ with\ N \end{bmatrix}$

$= \binom{9}{3} \cdot \binom{6}{2} \cdot \binom{4}{1} \cdot \binom{3}{1} \cdot \binom{2}{1} \cdot \binom{1}{1}$

$= \frac{9!}{3!\,(9-3)!} \cdot \frac{6!}{2!\,(6-2)!} \cdot \frac{4!}{1!\,(4-1)!} \cdot \frac{3!}{1!\,(3-1)!} \cdot \frac{2!}{1!\,(2-1)!} \cdot \frac{1!}{1!\,(1-1)!}$

$= 84 \cdot 15 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 30240$

c.

CR and ON each may be regarded as a single object, then there are 9 positions to be filled by 2 M’s, 3 I’s, 2 L’s, 1 CR, and 1 ON.

$$\begin{bmatrix} number\ of\ distinguishable\ orderings\ of\ the\ letters\ of\ the\ word\ contain\ the\ letters \\ CR\ next\ to\ each\ other\ in\ order\ and\ also\ the\ letters\ ON\ next\ to\ each\ other\ in\ order \end{bmatrix}$$

$$= \binom{9}{2} \cdot \binom{7}{3} \cdot \binom{4}{2} \cdot \binom{2}{1} \cdot \binom{1}{1}$$

$$= \frac{9!}{2!\,(9-2)!} \cdot \frac{7!}{3!\,(7-3)!} \cdot \frac{4!}{2!\,(4-2)!} \cdot \frac{2!}{1!\,(2-1)!} \cdot \frac{1!}{1!\,(1-1)!}$$

$$= 36 \cdot 35 \cdot 6 \cdot 2 \cdot 1 = 15120$$