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CS-225: Discrete Structures in CS
Homework Assignment 8, Part 2
Exercise Set 9.6: Question # 4, 12, 18

- **Set 9.6 – Q#4**

a.

$$\binom{30+8-1}{30} = \binom{37}{30} = \frac{37!}{30! \cdot (37-30)!} = 10295472$$

b.

If the inventory must include at least 4 A7b batteries, then 26 additional batteries are distributed among the 8 different types. So the number of distributions is:

$$\binom{26+8-1}{26} = \binom{33}{26} = \frac{33!}{26! \cdot (33-26)!} = 4272048$$

c.

$$\begin{bmatrix} number\ of\ ways\ a\ total \\ inventory\ of\ 30\ batteries \\ can\ be\ distributed\ among \\ the\ 8\ different\ types\ with \\ at\ most\ 3\ A7b\ batteries \end{bmatrix} = \begin{bmatrix} number\ of\ all\ ways \\ a\ total\ inventory \\ of\ 30\ batteries\ can \\ be\ distributed \\ among\ the\ 8 \\ different\ types \end{bmatrix} - \begin{bmatrix} number\ of\ ways\ a\ total \\ inventory\ of\ 30\ batteries \\ can\ be\ distributed\ among \\ the\ 8\ different\ types\ with \\ at\ least\ 4\ A7b\ batteries \end{bmatrix}$$

$$= 10295472 - 4272048 = 6023424$$

- **Set 9.6 – Q#12**

Think of the number 30 as divided into 30 individual units and the variable y_1, y_2, y_3, y_4 as 4 categories into which these units are placed. The number of units in category y_i indicates the value of y_i in a solution of the equation. And the number of ways to select 30 objects from the 4 categories is:

$$\binom{30+4-1}{30} = \binom{33}{30} = \frac{33!}{30!\,(33-30)!} = 5456$$

So there are 5456 nonnegative integer solutions to the equation.

- **Set 9.6 – Q#18**

a.

$$\binom{30+4-1}{30} = \binom{33}{30} = \frac{33!}{30!\,(33-30)!} = 5456$$

b.

If the pile contains only 15 quarters, then the collection can include at most 15 quarters.

$$\text{And}\quad \begin{bmatrix} number\ of\ collections \\ of\ 30\ coins\ with\ at \\ most\ 15\ quarters \end{bmatrix} = \begin{bmatrix} total\ number \\ of\ collections \\ of\ 30\ coins \end{bmatrix} - \begin{bmatrix} number\ of\ collections \\ of\ 30\ coins\ with\ at \\ least\ 16\ quarters \end{bmatrix}.$$

And if the collection includes at least 16 quarters, then 14 additional coins are collected among the 4 different types of coins. So the number of collections of 30 coins with at least 16 quarters is:

$$\binom{14+4-1}{14} = \binom{17}{14} = \frac{17!}{14!\,(17-14)!} = 680$$

And from the result of #a, total number of collections of 30 coins is 5456.

Hence, if the pile contains only 15 quarters, the number of collections of 30 coins is:

$$5456 - 680 = 4776$$

c.

If the pile contains only 20 dimes, then the collection can include at most 20 dimes. And

$$\begin{bmatrix} number\ of\ collections \\ of\ 30\ coins\ with\ at \\ most\ 20\ dimes \end{bmatrix} = \begin{bmatrix} total\ number \\ of\ collections \\ of\ 30\ coins \end{bmatrix} - \begin{bmatrix} number\ of\ collections \\ of\ 30\ coins\ with\ at \\ least\ 21\ dimes \end{bmatrix}.$$

And if the collection includes at least 21 dimes, then 9 additional coins are collected among the 4 different types of coins. So the number of collections of 30 coins with at least 21 dimes is:

$$\binom{9+4-1}{9} = \binom{12}{9} = \frac{12!}{9!\,(12-9)!} = 220$$

And from the result of #a, total number of collections of 30 coins is 5456.

Hence, if the pile contains only 20 dimes, the number of collections of 30 coins is:

$$5456 - 220 = 5236$$

d.

If the pile contains only 15 quarters and only 20 dimes, then the collection can include at most 15 quarters and at most 20 dimes. Then let A be the number of collections of 30 coins with at most 15 quarters and at most 20 dimes. Then,

$$A = \begin{bmatrix} total\ number \\ of\ collections \\ of\ 30\ coins \end{bmatrix} - \begin{bmatrix} number\ of\ collections\ of\ 30 \\ coins\ with\ at\ least\ 16 \\ quarters\ or\ at\ least\ 21\ dimes \end{bmatrix}.$$

Let B be the number of collections of 30 coins with at least 16 quarters or at least 21 dimes.

$$\text{Then}\ \ B = \begin{bmatrix} number\ of \\ collections\ of\ 30 \\ coins\ with\ at \\ least\ 16\ quarters \end{bmatrix} + \begin{bmatrix} number\ of \\ collections\ of\ 30 \\ coins\ with\ at \\ least\ 21\ dimes \end{bmatrix} - \begin{bmatrix} number\ of\ collections \\ of\ 30\ coins\ with\ at \\ least\ 16\ quarters\ and \\ at\ least\ 21\ dimes \end{bmatrix}.$$

As the set of the number of collections of 30 coins with at least 16 quarters and at least 21 dimes is an impossible set. Since this would be a total of more than 30 coins, there are no sets of coins meeting this condition. So,

$$\begin{bmatrix} number\ of\ collections\ of\ 30\ coins\ with\ at \\ least\ 16\ quarters\ and\ at\ least\ 21\ dimes \end{bmatrix} = 0$$

And from #a, #b and #c,

$[total\ number\ of\ collections\ of\ 30\ coins] = 5446,$

$[number\ of\ collections\ of\ 30\ coins\ with\ at\ least\ 16\ quarters] = 680,$

$[number\ of\ collections\ of\ 30\ coins\ with\ at\ least\ 21\ dimes] = 220.$

Hence, $A = \begin{bmatrix} total\ number \\ of\ collections \\ of\ 30\ coins \end{bmatrix} - B$

$$= \begin{bmatrix} total\ number \\ of\ collections \\ of\ 30\ coins \end{bmatrix} - \left(\begin{bmatrix} number\ of\ collections \\ of\ 30\ coins\ with\ at \\ least\ 16\ quarters \end{bmatrix} + \begin{bmatrix} number\ of\ collections \\ of\ 30\ coins\ with\ at \\ least\ 21\ dimes \end{bmatrix} \right)$$

$$= 5456 - (680 + 220) = 4556$$