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CS-225: Discrete Structures in CS
Homework Assignment 3, Part 2
Exercise Set 6.1: Question # 3, 7, 13, 18, 33, 34

- Set 6.1 – Q#3

a) No. R ⊈ T
Because there are elements in R that are not in T. For example, the number 2 is in R but 2 is not in T since 2 is not divisible by 6.
b) Yes. T ⊆ R
Because every number divisible by 6 is divisible by 2. Thus every element in T is in R, so T is an subset of R.
c) Yes. T ⊆ S
Because every number divisible by 6 is divisible by 3. Thus every element in T is in S, so T is an subset of S.

Note: The criteria on Canvas said "Showing that one set is a subset of another set", but textbook's requirement is to give Yes/No answers and explain why. The answer above followed the requirement on textbook and also showed T ⊆ R, T ⊆ S.

- Set 6.1 – Q#7

a. A ⊆ B is false
Because there are elements of A that are not in B. For example, 10 is in A because 10=6*1+4. But 10 is not in B because if it were, then 10=18b-2 for some integer b, which would imply that b=2/3, and this contradicts the fact that b is an integer. So, A ⊈ B.

b. B ⊆ A is true
Suppose x is any element of A, y is any element of B, then x=6a+4, y=18b-2 for some integer a and b. If B ⊆ A, then every element of B is in A. Then for any integer b and some integer a, x=y. By substitution,

$$6a + 4 = 18b - 2$$
$$6a = 18b - 6$$
$$a = 3b - 1$$

Thus, for any integer b, a=3b-1 is always integer (because products and difference of integers are integers). By substitution,

$$x = 6a + 4 = 6(3b - 1) + 4 = 18b - 6 + 4 = 18b - 2 = y$$

Which means, when a=3b-1, x=y, thus y satisfies the condition for being in A. Hence, every element of B is in A. So, B ⊆ A.

c. B=C is true

Suppose y is any element of B, z is any element of C, then y=18b-2, z=18c+16 for some integer b and c. If B=C, then B ⊆ C and C ⊆ B.
If B ⊆ C, then every element of B is in C. Then for any integer b and some integer c, y=z. By substitution,

$$18b-2=18c+16$$
$$18b-18=18c$$
$$c=b-1$$

Thus, for any integer b, c=b-1 is always integer (because difference of integers is integer). By substitution,

$$z=18c+16=18(b-1)+16=18b-18+16=18b-2=y$$

Which means, when c=b-1, y=z, thus y satisfies the condition for being in C. Hence, every element of B is in C. So, B ⊆ C.
If C ⊆ B, then every element of C is in B. Then for any integer c and some integer b, y=z. By substitution,

$$18b-2=18c+16$$
$$18b=18c+18$$
$$b=c+1$$

Thus, for any integer c, b=c+1 is always integer (because sum of integers is integer). By substitution,

$$y=18b-2=18(c+1)-2=18c+18-2=18c+16=z$$

Which means, when b=c+1, y=z, thus z satisfies the condition for being in B. Hence, every element of C is in B. So, C ⊆ B.
Consequently, B ⊆ C and C ⊆ B, so B=C proved.

Note: The requirement of this question on textbook is different from the criteria on Canvas. The answer above is for the requirement on textbook since I lost points for not following textbook requirement last time.
If I need to follow the criteria on Canvas, please see the answer below:

1) B ⊆ A
Suppose x is any element of A, y is any element of B, then x=6a+4, y=18b-2 for some integers a and b. If B ⊆ A, then every element of B is in A. Then for any integer b and some integer a, x=y. By substitution,

$$6\mathrm{a}+4=18\mathrm{b}-2$$
$$6\mathrm{a}=18\mathrm{b}-6$$
$$\mathrm{a}=3\mathrm{b}-1$$

Thus, for any integer b, a=3b-1 is always integer (because products and difference of integers are integers). By substitution,

$$\mathrm{x}=6\mathrm{a}+4=6(3\mathrm{b}-1)+4=18\mathrm{b}-6+4=18\mathrm{b}-2=\mathrm{y}$$

Which means, when a=3b-1, x=y, thus y satisfies the condition for being in A. Hence, every element of B is in A. So, B ⊆ A.

2) B ⊆ C
Suppose y is any element of B, z is any element of C, then y=18b-2, z=18c+6 for some

integers b and c. If B ⊆ C, then every element of B is in C. Then for any integer b and some integer c, y=z. By substitution,

$$18b - 2 = 18c + 16$$
$$18b - 18 = 18c$$
$$c = b - 1$$

Thus, for any integer b, c=b-1 is always integer (because difference of integers is integer). By substitution,

$$z = 18c + 16 = 18(b - 1) + 16 = 18b - 18 + 16 = 18b - 2 = y$$

Which means, when c=b-1, y=z, thus y satisfies the condition for being in C. Hence, every element of B is in C. So, B ⊆ C.

3) C ⊆ B

Suppose y is any element of B, z is any element of C, then y=18b-2, z=18c+6 for some integers b and c. If C ⊆ B, then every element of C is in B. Then for any integer c and some integer b, y=z. By substitution,

$$18b - 2 = 18c + 16$$
$$18b = 18c + 18$$
$$b = c + 1$$

Thus, for any integer c, b=c+1 is always integer (because sum of integers is integer). By substitution,

$$y = 18b - 2 = 18(c + 1) - 2 = 18c + 18 - 2 = 18c + 16 = z$$

Which means, when b=c+1, y=z, thus z satisfies the condition for being in B. Hence, every element of C is in B. So, C ⊆ B.

4) C ⊆ A

Suppose x is any element of A, z is any element of C, then x=6a+4, z=18c+16 for some integers a and c. If C ⊆ A, then every element of C is in A. Then for any integer c and some integer a, x=z. By substitution,

$$6a + 4 = 18c + 16$$
$$6a = 18c + 12$$
$$a = 3c + 2$$

Thus, for any integer c, a=3c+2 is always integer (because products and difference of integers are integers). By substitution,

$$x = 6a + 4 = 6(3c + 2) + 4 = 18b + 12 + 4 = 18b + 16 = z$$

Which means, when a=3c+2, x=z, thus z satisfies the condition for being in A. Hence, every element of C is in A. So, C ⊆ A.

- Set 6.1 – Q#13

a. True

All positive integers are rational numbers, so $Z^+ \subseteq Q$ is true.

b. False

Many negative real numbers are not rational numbers. For example, $-\sqrt{2} \in R\ but\ -$

$\sqrt{2} \notin \mathbb{Q}$. So R$^-$ ⊆ Q is false.

c. False

Many rational numbers are not integers. For example, $\frac{1}{2} \in Q\ but\ \frac{1}{2} \notin \mathbb{Z}$. So Q ⊆ Z is false.

d. False
$0 \in Z\ but\ 0 \notin Z^- \cup Z^+$, so $Z^- \cup Z^+ = Z$ is false.

e. True
All positive integers are not negative integers, all negative integers are not positive integers. So There doesn't exit a number can be both negative integer and positive integer. So, $Z^- \cap Z^+ = \emptyset$ is true.

f. True
All rational numbers are real numbers, so Q ⊆ R. So $Q \cap R = Q$ is true.

g. True
All integers are rational numbers, so Z ⊆ Q. So $Q \cup Z = Q$ is true.

h. True
All positive integers are real numbers, so Z$^+$ ⊆ R. So $Z^+ \cap R = Z^+$ is true.

i. False
All integers are rational numbers, so Z ⊆ Q. So $Z \cup Q = Q$, thus $Z \cup Q = Z$ is false.

- Set 6.1 – Q#18

a. No. The number 0 is not in $\emptyset$ because $\emptyset$ has no element.
b. No. The left-hand set is the empty set, it does not have any element. The right-hand set is a set with one element, namely $\emptyset$.
c. Yes. $\{\emptyset\}$ is a set with one element, namely $\emptyset$, so $\emptyset$ is an element of $\{\emptyset\}$.
d. No. $\emptyset$ is not an element of $\emptyset$ because $\emptyset$ has no element.

- Set 6.1 – Q#33

a. $P(\emptyset) = \{\emptyset\}$
b. $P(P(\emptyset)) = P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$
c. $P\left(P(P(\emptyset))\right) = P(\{\emptyset, \{\emptyset\}\}) = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

- Set 6.1 – Q#34

a. $A_1 \times (A_2 \times A_3) = \{1,2,3\} \times \{(u,m),(u,n),(v,m),(v,n)\}$
$= \{(1,(u,m)),(1,(u,n)),(1,(v,m)),(1,(v,n)),$
$(2,(u,m)),(2,(u,n)),(2,(v,m)),(2,(v,n)),$
$(3,(u,m)),(3,(u,n)),(3,(v,m)),(3,(v,n))\}$

b. $(A_1 \times A_2) \times A_3 = \{(1,u),(1,v),(2,u),(2,v),(3,u),(3,v)\} \times \{m,n\}$
$= \{((1,u),m),((1,u),n),((1,v),m),((1,v),n),$
$((2,u),m),((2,u),n),((2,v),m),((2,v),n),$
$((3,u),m),((3,u),n),((3,v),m),((3,v),n)\}$

c. $A_1 \times A_2 \times A_3 = \{(1,u,m),(1,u,n),(1,v,m),(1,v,n),$
$(2,u,m),(2,u,n),(2,v,m),(2,v,n),$
$(3,u,m),(3,u,n),(3,v,m),(3,v,n)\}$