Xiaoying Li
CS-225: Discrete Structures in CS
Homework Assignment 2, Part 2
Exercise Set 4.1: Question # 32, 58
Set 4.2: Question # 20, 25
Set 4.6: Question # 28

- Set 4.1 – Q#32

Suppose a is any odd integer and b is any even integer. By definition of odd and even, a=2x+1 and b=2y for some integers x and y. Then

$$2a + 3b = 2(2x + 1) + 3 \times 2y = 4x + 2 + 6y = 2(2x + 3y + 1)$$

And 2x+3y+1 is an integer because it's a sum of integers 2x, 2y and 1. Thus 2a+3b is twice an integer, so 2a+3b is even.
As was to be shown, if a is any odd integer and b is any even integer, then, 2a+3b is even.

- Set 4.1 – Q#58

Suppose n and n+1 are any two consecutive integers. Then the difference of the squares of any two consecutive integers is

$$(n + 1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1$$

And n is an integer, thus 2n+1 is twice an integer plus 1, so 2n+1 is odd, so $(n + 1)^2 - n^2$ is odd.
As was to be shown, the difference of the squares of any two consecutive integers is odd.

- Set 4.2 – Q#20

Suppose r and s are any two rational numbers and r < s, then r+$\frac{s-r}{2}$ is a number between r and s. By definition of rational, $r = \frac{a}{b}$, $s = \frac{c}{d}$ for some integers a, b, c, d (b and d are not 0). Then

$$r + \frac{s - r}{2} = \frac{a}{b} + \frac{c}{2d} - \frac{a}{2b} = \frac{2ad - bc - ad}{2bd} = \frac{ab - bc}{2bd}$$

And 2bd is an integer because it's a product of two integers, ab-bc is an integer because it's a difference of products of integers. And b≠0 and d≠0, so bd≠0. So $\frac{ab-bc}{2ab}$ is a rational number because it's a quotient of two integer with a nonzero denominator, thus $r + \frac{s-r}{2}$ is a rational number between r and s.
As was to be shown, there is another rational number between any two rational

numbers r and s.

- Set 4.2 – Q#25

Suppose r is any rational number. By definition of rational, $r = \frac{a}{b}$ for some integers a, b (b≠0). Then

$$3r^2 - 2r + 4 = \frac{3a^2}{b^2} - \frac{2a}{b} + 4 = \frac{3a^2 - 2ab + 4b^2}{b^2}$$

And $a^2, b^2$ are integers because they are square of integers, $3a^2, -2ab, 4b^2$ are integers because they are product of integers, and $3a^2 - 2ab + 4b^2$ is an integer because it's a sum of integers $3a^2, -2ab\ and\ 4b^2$ . And b≠0, so $b^2$ ≠0. So $\frac{3a^2-2ab+4b^2}{b^2}$ is a rational number because it's a quotient of two integer with a nonzero denominator, thus $3r^2 - 2r + 4$ is a rational number.
As was to be shown, if r is any rational number, then $3r^2 - 2r + 4$ is rational.

- Set 4.6 – Q#28

The contrapositive statement of the given statement is "For all integers m and n, if m is not even and n is not even, then mn is not even." Equivalently, "For all integers m and n, if m and n are both odd, then mn is odd."
Then suppose m and n are any two odd integers. By definition of odd, m=2a+1, m=2b+1 for some integers a,b. Then

$$mn = (2a + 1)(2b + 1) = 4ab + 2a + 2b + 1 = 2(2ab + a + b) + 1$$

And 2ab+a+b is an integer because it's a sum of integers 2ab ( is an integer because it's a product of integers 2, a and b), a and b. Let d=2ab+a+b, then d is an integer. Thus mn=2d+1, by definition of odd, mn is odd. So the statement "For all integers m and n, if m and n are both odd, then mn is odd" is true.
The contrapositive statement of the given statement is true, so the given statement is true. As was to be shown, for all integers m and n, if mn is even then m is even or n is even.