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CS-225: Discrete Structures in CS
Homework Assignment 3, Part 1
Exercise Set 4.6: Question # 12, 28 Set 4.7: Question # 8, 16.c

- Set 4.6 – Q#12

Suppose not. That is, suppose a and b are rational numbers, b≠0, and r is an irrational number, and a+br is rational.

By definition of rational, $a = \frac{u}{x}, b = \frac{v}{y}, a + br = \frac{w}{z}$ for some integers u, v, w, x, y, z, with x≠0, y≠0, z≠0. Also v≠0 because b≠0. By substitution,

$$a + br = \frac{u}{x} + \frac{v}{y}r = \frac{w}{z}$$

$$\frac{v}{y}r = \frac{w}{z} - \frac{u}{x}$$

$$r = \frac{wx - uz}{xz} \times \frac{y}{v} = \frac{wxy - uyz}{vxz}$$

wxy, uyz, vxz are integers (being products of integers), wxy-uyz is an integer (being difference of integers), and vxz≠0 (because v≠0, x≠0, y≠0). Thus, by definition of rational, $r = \frac{wxy-uyz}{vxz}$ is rational being a quotient of integers with a non-zero denominator, which contradicts the supposition that r is an irrational number.
Hence the supposition is false and the given statement "If a and b are rational numbers, b≠0, and r is an irrational number, then a+br is irrational" is true.

- Set 4.6 – Q#28

Suppose not. That is, suppose for all integers m and n, mn is even, m is odd and n is odd.
Then suppose m and n are any two odd integers. By definition of odd, m=2a+1, m=2b+1 for some integers a, b. By substitution,

$$mn = (2a + 1)(2b + 1) = 4ab + 2a + 2b + 1 = 2(2ab + a + b) + 1$$

And 2ab+a+b is an integer because it's a sum of integers 2ab (being an integer because it's a product of integers 2, a and b), a and b. Let d=2ab+a+b, then d is an integer. Thus mn=2d+1, by definition of odd, mn is odd being twice of an integer plus one, which contradicts the supposition that mn is even.
Hence the supposition is false and the given statement "for all integers m and n, if mn is even then m is even or n is even" is true.

- Set 4.7 – Q#8

Counterexample: Let $x = 1 + \sqrt{2}, y = \sqrt{2}$, then x and y are irrational, but $x - y = 1 +$

$\sqrt{2} - \sqrt{2} = 1$, which is rational.
So the statement “the difference of any two irrational number is irrational” is false.

- Set 4.7 – Q#16.c

Suppose not. That is, suppose $\sqrt{3}$ is rational. By definition of rational, $\sqrt{3} = \frac{a}{b}$ for some integers a and b with b≠0. We can assume that a and b have no common factor without loss of generality. Because if a and b have common factors, divide both a and b by their greatest common factor to obtain integers a’ and b’ with the property that a’ and b’ have no common factor and $\sqrt{3} = \frac{a\prime}{b\prime}$, then redefine a=a’, b=b’.

Squaring both sides of $\sqrt{3} = \frac{a}{b}$ gives $3 = \frac{a^2}{b^2}$, then multiply both sides by b^2 gives $3b^2 = a^2$. Thus a^2 is divisible by 3, and so, a is also divisible by 3 because 3 is a prime number. By definition of divisibility, then, a=3k for some integer k. And so $a^2 = 9k^2 = 3b^2$, dividing both sides of $9k^2 = 3b^2$ by 3 gives $b^2 = 3k^2$. Thus b^2 is divisible by 3, and so, b is also divisible by 3 because 3 is a prime number.
Consequently, both a and b are divisible by 3, which contradicts the assumption that a and b have no common factor. Thus the supposition is false, and so $\sqrt{3}$ is irrational.