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CS 325 -Winter 2020
Homework #6

- **Problem 1**

a.

The distance of the shortest path from G to C is 16.

The copies of the LP code and output are shown below:

(the objective function and constraints are shown in the LP code in the left picture)

```
max c
st
        g=0
        a-f<=5
        a-h<=4
        b-a<=8
        b-f<=7
        b-h<=9
        c-b<=4
        c-f<=3
        d-c<=3
        d-e<=9
        d-g<=2
        e-b<=10
        e-d<=25
        e-f<=2
        f-a<=10
        f-d<=18
        g-e<=7
        h-g<=3
```

```
LP OPTIMUM FOUND AT STEP      6

       OBJECTIVE FUNCTION VALUE

       1)      16.00000

 VARIABLE        VALUE          REDUCED COST
        C        16.000000          0.000000
        G         0.000000          0.000000
        A         4.000000          0.000000
        F        13.000000          0.000000
        H         3.000000          0.000000
        B        12.000000          0.000000
        D         0.000000          0.000000
        E         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          1.000000
       3)        14.000000          0.000000
       4)         3.000000          0.000000
       5)         0.000000          0.000000
       6)         8.000000          0.000000
       7)         0.000000          1.000000
       8)         0.000000          1.000000
       9)         0.000000          0.000000
      10)        19.000000          0.000000
      11)         9.000000          0.000000
      12)         2.000000          0.000000
      13)        22.000000          0.000000
      14)        25.000000          0.000000
      15)        15.000000          0.000000
      16)         1.000000          0.000000
      17)         5.000000          0.000000
      18)         7.000000          0.000000
      19)         0.000000          1.000000

NO. ITERATIONS=       6
```

b.

The distances of the shortest paths from G to all other vertices:

- A: 7
- B: 12
- C: 16
- D: 2
- E: 19
- F: 17
- H: 3

The copies of the LP code and output are shown below:

(the objective function and constraints are shown in the LP code in the left picture)

```
max a+b+c+d+e+f+h
st
        g=0
        a-f<=5
        a-h<=4
        b-a<=8
        b-f<=7
        b-h<=9
        c-b<=4
        c-f<=3
        d-c<=3
        d-e<=9
        d-g<=2
        e-b<=10
        e-d<=25
        e-f<=2
        f-a<=10
        f-d<=18
        g-e<=7
        h-g<=3
```

```
LP OPTIMUM FOUND AT STEP      9

       OBJECTIVE FUNCTION VALUE

       1)      76.00000

 VARIABLE        VALUE          REDUCED COST
        A         7.000000          0.000000
        B        12.000000          0.000000
        C        16.000000          0.000000
        D         2.000000          0.000000
        E        19.000000          0.000000
        F        17.000000          0.000000
        H         3.000000          0.000000
        G         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          7.000000
       3)        15.000000          0.000000
       4)         0.000000          3.000000
       5)         3.000000          0.000000
       6)        12.000000          0.000000
       7)         0.000000          2.000000
       8)         0.000000          1.000000
       9)         4.000000          0.000000
      10)        17.000000          0.000000
      11)        26.000000          0.000000
      12)         0.000000          1.000000
      13)         3.000000          0.000000
      14)         8.000000          0.000000
      15)         0.000000          1.000000
      16)         0.000000          2.000000
      17)         3.000000          0.000000
      18)        26.000000          0.000000
      19)         0.000000          6.000000

NO. ITERATIONS=       9
```

- **Problem 2**

The four types of men's ties' profit per tie can be calculated as below:

Silk Tie's Profit/ Tie = 6.7-0.75-20*0.125 = 3.45
Polyester Tie's Profit/ Tie = 3.55-0.75-6*0.08 = 2.32
Blend1 Tie's Profit/ Tie = 4.31- 0.75-6*0.05-9*0.05 = 2.81
Blend2 Tie's Profit/ Tie = 4.81-0.75-6*0.03-9*0.07 = 3.25

So, the objective function and all constraints can be formulated as below:

maximize 3.45s+2.32p+2.81b+3.25c
subject to
0.125s<=1000
0.08p+0.05b+0.03c<=2000
0.05b+0.07c<=1250
s>=6000
s<=7000
p>=10000
p<=14000
b>=13000
b<=16000
c>=6000
c<=8500

Using LINDO to determine the optimal solution, the code and output's copies are shown below:

```
max 3.45s+2.32p+2.81b+3.25c
st
        0.125s<=1000
        0.08p+0.05b+0.03c<=2000
        0.05b+0.07c<=1250
        s>=6000
        s<=7000
        p>=10000
        p<=14000
        b>=13000
        b<=16000
        c>=6000
        c<=8500
```

```
LP OPTIMUM FOUND AT STEP      4

       OBJECTIVE FUNCTION VALUE

       1)      120196.0

  VARIABLE        VALUE          REDUCED COST
         S       7000.000000          0.000000
         P      13625.000000          0.000000
         B      13100.000000          0.000000
         C       8500.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)       125.000000          0.000000
        3)         0.000000         29.000000
        4)         0.000000         27.200001
        5)      1000.000000          0.000000
        6)         0.000000          3.450000
        7)      3625.000000          0.000000
        8)       375.000000          0.000000
        9)       100.000000          0.000000
       10)      2900.000000          0.000000
       11)      2500.000000          0.000000
       12)         0.000000          0.476000

NO. ITERATIONS=       4
```

Therefore, the optimal solution of maximize profit is 120196, and the optimal numbers of ties of each type to maximize profit are 7000 silk ties, 13625 polyester ties, 13100 blend1 ties and 8500 blend2 ties.

- **Problem 3**

a.

Formulation of the problem as an integer program with an objective function and constraints is shown below:

a, b, c and d are numbers of coins of denominations 1, 5, 10 and 25.

minimize a+b+c+d

subject to

a+5b+10c+25d=202

a>=0

b>=0

c>=0

d>=0

a, b, c, d are integers

Using LINDO to determine the optimal solution, the code and output's copies are shown below:

```
min a+b+c+d
st
        a+5b+10c+25d=202
        a>=0
        b>=0
        c>=0
        d>=0
end
        GIN a
        GIN b
        GIN c
        GIN d
```

```
        OBJECTIVE FUNCTION VALUE

        1)      10.00000

 VARIABLE        VALUE          REDUCED COST
        A         2.000000          1.000000
        B         0.000000          1.000000
        C         0.000000          1.000000
        D         8.000000          1.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.000000
       3)         2.000000          0.000000
       4)         0.000000          0.000000
       5)         0.000000          0.000000
       6)         8.000000          0.000000

NO. ITERATIONS=      32
BRANCHES=    6 DETERM.=  1.000E    0
```

Therefore, the optimal solution of the minimum number of coins used is 10, including 2 coins of denomination 1, 0 coin of denomination 5 and 10, and 8 coins of denomination 25.

b.

Formulation of the problem as an integer program with an objective function and constraints is shown below:

a, b, c, d and e are numbers of coins of denominations 1, 3, 7, 12 and 27.

minimize a+b+c+d+e

subject to

a+3b+7c+12d+27e=293

a>=0

b>=0

c>=0

d>=0

e>=0

a, b, c, d, e are integers

Using LINDO to determine the optimal solution, the code and output's copies are shown below:

```
min a+b+c+d+e
st
        a+3b+7c+12d+27e=293
        a>=0
        b>=0
        c>=0
        d>=0
        e>=0
end
        GIN a
        GIN b
        GIN c
        GIN d
        GIN e
```

```
        OBJECTIVE FUNCTION VALUE

        1)      14.00000

  VARIABLE        VALUE          REDUCED COST
         A         0.000000          1.000000
         B         0.000000          1.000000
         C         2.000000          1.000000
         D         3.000000          1.000000
         E         9.000000          1.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          0.000000
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)         2.000000          0.000000
        6)         3.000000          0.000000
        7)         9.000000          0.000000

NO. ITERATIONS=      98
BRANCHES=   34 DETERM.=  1.000E    0
```

Therefore, the optimal solution of the minimum number of coins used is 14, including 0 coin of denomination 1 and 3, 2 coins of denomination 7, 3 coins of denomination 12, and 9 coins of denomination 27.

- **Problem 4**

a.

maximize $2x_1 - 6x_3$

$x_4 = 7 - x_1 - x_2 + x_3$

$x_5 = 3x_1 - x_2 - 8$

$x_6 = -x_1 + 2x_2 + 2x_3$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

b.

basic variables: x_4, x_5, x_6

non-basic variables: x_1, x_2, x_3