Assignment 1

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Xiaoying Li

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Problem 1

a)

$$P(X_2 > 3) = e^{-\lambda_2 t} = e^{-1\cdot 3} = e^{-3}$$

b)

$$\begin{aligned} P(\min(X_1, X_2) > 5) &= P(X_1 > 5, X_2 > 5) \\ &= P(X_1 > 5) \cdot P(X_2 > 5) \\ &= e^{-\lambda_1 \cdot t} \cdot e^{-\lambda_2 \cdot t} \\ &= e^{-4\cdot 5} e^{-1\cdot 5} \\ &= e^{-25} \end{aligned}$$

c)

$$P(X_2 < X_1) = P(\min(X_1, X_2) = X_2) = \frac{\lambda_2}{\lambda_1 + \lambda_2} = \frac{1}{1+4} = \frac{1}{5}$$

Problem 2

a)

According to the problem description, the arrival rate for packets is

$$\lambda = \frac{60}{3} = 20\, packets/second$$

which means that the time in between two packets sent out is exponentially distributed with rate λ=20 packets/second. Then,

$$P(x < 4) = 1 - e^{-\lambda t} = 1 - e^{-20\cdot 4} = 1 - e^{-80}$$

Therefore, the probability that a packet will be sent out in less than 4 seconds is $1 - e^{-80}$.

b)

A packet was sent out at time=0, the probability that at time t = 4 no further packets were sent out is

$$P(x > 4) = e^{-\lambda t} = e^{-20\cdot 4} = e^{-80}$$

c)

When 120 packets are sent out every 3 seconds on average, the arrival rate for packets is

$$\lambda = \frac{120}{3} = 40\, packets/second$$

which means that the time in between two packets sent out is exponentially distributed with rate λ=40 packets/second. Then,

$$P(x < 4) = 1 - e^{-\lambda t} = 1 - e^{-40\cdot 4} = 1 - e^{-160} > 1 - e^{-80}$$

which is obviously larger than the probability of point (a).

Problem 3

Let the time between the generation of two consecutive packets at each client be X_1 and X_2.

a)

The probability that the next packet will come from node 2 is

$$P(X_2 < X_1) = P(\min(X_1, X_2) = X_2) = \frac{\lambda_2}{\lambda_1 + \lambda_2} = \frac{2}{1+2} = \frac{2}{3}$$

b)

The probability that the router will not receive any packet in the next 2 second is

$$\begin{aligned} P(\min(X_1, X_2) > 2) &= P(X_1 > 2, X_2 > 2) \\ &= P(X_1 > 2) \cdot P(X_2 > 2) \\ &= e^{-\lambda_1 \cdot t} \cdot e^{-\lambda_2 \cdot t} \\ &= e^{-1\cdot 2} e^{-2\cdot 2} \\ &= e^{-6} \end{aligned}$$

Problem 4

a)

Packet A leaves the router before Packet B arrives means A's servicing time at router ends before B's remaining arriving time to router ends. Because of memorylessness, A's serving time is Exponential(μ), B's remaining arriving time is Exponential(λ). Then, the probability that Packet A leaves the router before Packet B arrives is

$$P = P(\text{Exponential}(\mu) < \text{Exponential}(\lambda)) = \frac{\mu}{\mu + \lambda}$$

b)

The average time Packet B waits in the buffer before being served can be discussed in two circumstances, which are Packet A leaves the router before Packet B arrives and Packet B arrives router before Packet A leaves.

When Packet A leaves the router before Packet B arrives, the time Packet B waits in the buffer before being served is 0.

And from point a) we know that the probability that Packet A leaves the router before Packet B arrives is

$$P = P(\text{Exponential}(\mu) < \text{Exponential}(\lambda)) = \frac{\mu}{\mu + \lambda}$$

When Packet B arrives router before Packet A leaves, the time Packet B waits in the buffer before being served is A's remaining serving time, which is Exponential(μ), and its average value is $\frac{1}{\mu}$.

And the probability the Packet B arrives router before Packet A leaves is,

$$P = P(\text{Exponential}(\lambda) < \text{Exponential}(\mu)) = \frac{\lambda}{\mu + \lambda}$$

Therefore, the average time Packet B waits in the buffer before being served is

$$0 \cdot \frac{\mu}{\mu + \lambda} + \frac{1}{\mu} \cdot \frac{\lambda}{\mu + \lambda} = \frac{\lambda}{\mu^2 + \mu\lambda}$$

c)

The average time Packet B spends in the system can be divided up into

$$T = T_1 + R$$

where T_1 is the time until the first thing that happens, and R is the rest of the time.

The time until the first thing happens is Exponential(μ+λ), so that

$$E(T_1) = \frac{1}{\mu + \lambda}$$

When the first thing to happen is Packet A leaves the router, which occurs with probability $\frac{\mu}{\mu+\lambda}$, then at that point Packet B's remaining time in the system is the remaining time of B arrives router and the time of B being served. Then

$$E(R|First\ thing\ is\ A\ leaves) = \frac{\mu}{\mu + \lambda}(\frac{1}{\mu} + \frac{1}{\lambda})$$

When the first thing to happen is Packet B arrives the router, which occurs with probability $\frac{\lambda}{\mu+\lambda}$, then at that point Packet B's remaining time in the system is the remaining time of A being served and the time of B being served. Then, by memorylessness,

$$E(R|First\ thing\ is\ B\ arrives) = \frac{\lambda}{\mu + \lambda}\left(\frac{1}{\mu} + \frac{1}{\mu}\right) = \frac{2\lambda}{\mu^2 + \mu\lambda}$$

Therefore, the average time packet B spends in the system is

$$\frac{1}{\mu + \lambda} + \frac{2\lambda}{\mu^2 + \mu\lambda} + \frac{\mu}{\mu + \lambda}(\frac{1}{\mu} + \frac{1}{\lambda})$$