Assignment 3

CS232 Fall 2021

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Problem 1

$$\lambda = 4\, pkt/s, \mu = \ 8\, pkt/s \to \rho = \frac{\lambda}{\mu} = \frac{4}{8} = \frac{1}{2}$$

(a)

The probability that the next packet is dropped is the probability that the number of packets in the system is K=5. Thus,

$$P[Next\ Packet\ Dropped] = p_K = \frac{(1-\rho)\rho^K}{1-\rho^{K+1}} = \frac{(1-\frac{1}{2})(\frac{1}{2})^5}{1-(\frac{1}{2})^{5+1}} = \frac{1}{63}$$

(b)

$$E[N] = \frac{\rho}{1-\rho} - \frac{(K+1)\rho^{K+1}}{1-\rho^{K+1}} = \frac{\frac{1}{2}}{1-\frac{1}{2}} - \frac{(5+1)\left(\frac{1}{2}\right)^{5+1}}{1-\left(\frac{1}{2}\right)^{5+1}} = \frac{19}{21}$$

$$E[T] = \frac{E[N]}{\lambda(1-P_K)} = \frac{\frac{19}{21}}{4(1-\frac{1}{63})} = \frac{57}{248}\ second$$

The expected time a packet needs to go through the system is $\frac{57}{248}$ second.

Problem 2

$$\lambda = 4\, pkt/s, \mu = \ 5\, pkt/s \to \rho = \frac{\lambda}{\mu} = \frac{4}{5}$$

(a)

$$E[T] = \frac{1/\mu}{1-\rho} = \frac{\frac{1}{5}}{1-\frac{4}{5}} = 1\ second$$

The expected time a packet spends in the system is 1 second.

(b)

$$E[W] = \frac{(1/\mu)\rho}{(1-\rho)} = \frac{\frac{1}{5}\cdot\frac{4}{5}}{1-\frac{4}{5}} = \frac{4}{5}\ second$$

The time a packet is expected to wait in the buffer before getting serviced is $\frac{4}{5}$ second.

Problem 3

(a)

When having a dedicated server for each application, we obtain

$$\lambda = 2\ pkt/s, \mu = \ 4\ pkt/s \rightarrow \rho = \frac{\lambda}{\mu} = \frac{2}{4} = \frac{1}{2}$$

$$E[T_{seperate}] = \frac{1/\mu}{1-\rho} = \frac{\frac{1}{4}}{1-\frac{1}{2}} = \frac{1}{2}\ second$$

When having a single powerful server serving all the five applications simultaneously, we obtain $m = 5$,

$$E[T_{combined}] = \frac{1}{m}E[T_{seperate}] = \frac{1}{5}\cdot\frac{1}{2} = \frac{1}{10}\ second$$

Obviously, $E[T_{seperate}] > E[T_{combined}]$. Therefore, having a single powerful server serving all the five applications simultaneously is better.

(b)

$$n = 2, \rho = \frac{1}{2}$$

$$P\{N(t) = n\} = p_n = (1-\rho)\rho^n \rightarrow P\{N(t) = 2\} = p_2 = \left(1-\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

The probability that there are two packets in the system is $\frac{1}{8}$.

Problem 4

(a)

$$\lambda = \frac{1}{2}\ call/s, \mu = \ 2\ call/s \rightarrow a = \frac{\lambda}{\mu} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$$

The probability that the next phone call is dropped is the probability that the number of phone calls in the server is c=5. Thus,

$$P[N(t)=n]=p_n=\frac{\frac{a^n}{n!}}{\sum_{k=0}^{c}\frac{a^k}{k!}}, for\ n=0,1,\cdots,c$$

$$\rightarrow\ P[N(t)=2]=p_2=\frac{\frac{(\frac{1}{4})^2}{2!}}{\sum_{k=0}^{2}\frac{a^k}{k!}}=\frac{\frac{1}{32}}{1+\frac{(\frac{1}{4})^1}{1!}+\frac{(\frac{1}{4})^2}{2!}}=\frac{1}{41}$$

The probability that the next phone call is dropped is $\frac{1}{41}$.

(b)

$$P[second\ phone\ call\ is\ received\ before\ the\ first\ one\ terminates]$$
$$=P[T(second\ phone\ call\ is\ received)<T(first\ phone\ call\ terminates)]$$
$$=\frac{\lambda}{\lambda+\mu}=\frac{\frac{1}{2}}{\frac{1}{2}+2}=\frac{1}{5}$$

Problem 5

(a)

$$\lambda=5\ pkts/s, \mu=\ 10\ pkts/s$$
$$\rightarrow traffic\ load\ a=\frac{\lambda}{\mu}=\frac{5}{10}=\frac{1}{2}\ Erlangs$$

The probability of a packet being dropped upon arrival is the probability of the number of packets in the system is c=1. Thus,

$$P[N(t)=n]=p_n=\frac{\frac{a^n}{n!}}{\sum_{k=0}^{c}\frac{a^k}{k!}}, for\ n=0,1,\cdots,c$$

$$\rightarrow P_b=P[N(t)=1]=p_1=\frac{\frac{(\frac{1}{2})^1}{1!}}{\sum_{k=0}^{1}\frac{a^k}{k!}}=\frac{\frac{1}{2}}{1+\frac{1}{2}}=\frac{1}{3}$$

$$\rightarrow carried\ load=a(1-P_b)=\frac{1}{2}\cdot\left(1-\frac{1}{3}\right)=\frac{1}{3}\ Erlangs$$

(b)

$$\lambda = 5\ pkts/s, P_b = \frac{1}{3}, E[W] = 2.5\ seconds$$

$$\rightarrow E[N_q] = \lambda(1 - P_b)E[W] = 5 \cdot \left(1 - \frac{1}{3}\right) \cdot 2.5 = \frac{25}{3}$$

The average number of packets in the buffer is $\frac{25}{3}$.