

Supplymentary material for “Paper Title”

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1 Parameters Used for *Case Study 1*

The basic uncertainty set for the uncertainty \mathbf{v}_t in each time period t consists two subsets $\mathcal{V}_{t,1}$ and $\mathcal{V}_{t,2}$ that are defined as

$$\mathcal{V}_{t,1} := \{\mathbf{v}_t | \mathbf{D}_1 \mathbf{v}_t \leq \mathbf{d}_1\}, \quad \mathcal{V}_{t,2} := \{\mathbf{v}_t | \mathbf{D}_2 \mathbf{v}_t \leq \mathbf{d}_2\} \quad (1)$$

where

$$\mathbf{D}_1 = \mathbf{D}_2 = [1, -1]^T, \quad \mathbf{d}_1 = [2, 0]^T, \quad \mathbf{d}_2 = [0, 2]^T. \quad (2)$$

The system matrices defining the building thermal dynamics are

$$\Phi = \begin{bmatrix} 0.0167 & 0.0048 & 0.1245 & 0.409 \\ 0.0005 & 0.0002 & 0.0039 & 0.0044 \\ 0.0253 & 0.0073 & 0.3321 & 0.0617 \\ 0.0244 & 0.0070 & 0.0526 & 0.3456 \end{bmatrix}, \quad \Gamma_u = \begin{bmatrix} 0.0986 \\ 0.0029 \\ 0.0288 \\ 0.0275 \end{bmatrix}, \quad \Gamma_w = \begin{bmatrix} 0.2536 & 0.4596 \\ 0.0070 & 0.9840 \\ 0.4450 & 0.1287 \\ 0.4477 & 0.1225 \end{bmatrix}, \quad \Gamma_v = \begin{bmatrix} 0.2536 \\ 0.0070 \\ 0.4450 \\ 0.4477 \end{bmatrix}.$$

2 Parameters Used for *Case Study 2*

The uncertainty set for the demand uncertainties consists of four uncertainty sets $\mathcal{V}_k := \{\mathbf{v} \mid \mathbf{D}_k \mathbf{v} \leq \mathbf{d}_k\}$ with parameters $(\mathbf{D}_k, \mathbf{d}_k)$ set as

$$\mathbf{D}_1 = \mathbf{D}_2 = \mathbf{D}_3 = \mathbf{D}_4 = [\mathbf{I}_3, -\mathbf{I}_3]^T \quad (3)$$

$$\mathbf{d}_1 = [0.3, 0.3, 0.3, 0, 0, 0]^T, \quad \mathbf{d}_2 = [1.2, 1.2, 1.2, -1, -1, -1]^T, \quad (4)$$

$$\mathbf{d}_3 = [1, 0.3, 0.3, -0.8, 0, 0], \quad \mathbf{d}_4 = [0.3, 1, 0.3, 0, -0.7, 0]^T. \quad (5)$$

3 Parameters Used for *Case Study 3*

The mass balance coefficient matrix is

$$\kappa = \begin{bmatrix} 0.63 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0.58 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.25 & -4 & 0 & -1 & -1.5 & 0 \\ 0 & 0.1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0.24 & -1 & 0 & 0 & 0 \\ 0 & 0 & 2.3 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.93 & 0.77 & -1 \end{bmatrix} \quad (6)$$

The variable investment cost α , fixed investment cost β , operating cost γ , purchase cost φ and sale price τ are set as

The uncertainty set consists of four subsets that are defined as

$$\mathbf{D}_1 = \mathbf{D}_2 = \mathbf{D}_3 = \mathbf{D}_4 = \mathbf{I}_{10} \otimes \begin{bmatrix} \mathbf{I}_{\text{in}} & \mathbf{0} \\ -\mathbf{I}_{\text{in}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{\text{out}} \\ \mathbf{0} & -\mathbf{I}_{\text{out}} \end{bmatrix} \quad (7)$$

$$\mathbf{d}_1 = \mathbf{e}_{10} \otimes [1620, 2150, -1380, -1850, 1300, 1620, -1100, -1380]^T, \quad (8)$$

$$\mathbf{d}_2 = \mathbf{e}_{10} \otimes [580, 810, -420, -630, 1080, 1290, -920, -1110]^T, \quad (9)$$

$$\mathbf{d}_3 = \mathbf{e}_{10} \otimes [350, 850, -250, -750, 710, 790, -650, -710, 350]^T, \quad (10)$$

$$\mathbf{d}_4 = \mathbf{e}_{10} \otimes [120, 220, -80, -180, 220, 270, -180, -230, 120]^T. \quad (11)$$

4 CCG algorithm for solving Wasserstein metric-based DRO problem

For the DRO problem

$$\min_{\mathbf{x}} \mathbf{c}^T \mathbf{x} + \max_{f(\mathbf{v}) \in \mathcal{P}} \min_{\mathbf{y}} \mathbf{b}^T \mathbf{y} \quad (12a)$$

$$\text{s.t. } \mathbf{Ax} \leq \mathbf{q}, \quad (12b)$$

$$\mathbf{Tx} + \mathbf{Wy} + \mathbf{Mv} \leq \mathbf{h} \quad (12c)$$

where the ambiguity set is defined as

$$D_W = \{\mathbb{Q} \in M(\mathcal{V}) : d_W(\mathbb{Q}, \mathbb{Q}_0) \leq \varepsilon\} \quad (13)$$

$$d_W(\mathbb{Q}, \mathbb{Q}_0) = \inf_{\Pi} \left(\int_{\mathcal{V}^2} \|\mathbf{v} - \mathbf{v}^0\| \Pi(d\mathbf{v}, d\mathbf{v}^0) \middle| \begin{array}{l} \Pi \text{ is a joint distribution of} \\ \mathbf{v} \text{ and } \mathbf{v}^0 \text{ with marginals } \mathbb{Q} \text{ and } \mathbb{Q}_0 \end{array} \right) \quad (14)$$

with ε as the Wasserstein distance. According to the results in [1], the above optimization can be reformulated as

$$\min_{\mathbf{x}, \lambda, \mathbf{z}_i, s_i} \mathbf{c}^T \mathbf{x} + \lambda \varepsilon + \frac{1}{N} \sum_{i=1}^N s_i \quad (15a)$$

$$\text{s.t. } \sup_{\mathbf{v} \in \mathcal{U}} \left[\min_{\mathbf{y}} \mathbf{b}^T \mathbf{y} - \mathbf{z}_i^T \mathbf{v} \right] + \mathbf{z}_i^T \hat{\mathbf{v}}_i \leq s_i, \quad i = 1, 2, \dots, N, \quad (15b)$$

$$\mathbf{Tx} + \mathbf{Wy} + \mathbf{Mv} \leq \mathbf{h}, \quad (15c)$$

$$\mathbf{Ax} \leq \mathbf{q}, \quad (15d)$$

$$\|\mathbf{z}_i\|_* \leq \lambda, \quad i = 1, 2, \dots, N. \quad (15e)$$

where N is the number of uncertainty samples $\hat{\mathbf{v}}_i$. Then, based on the above formulation and applying a *column-and-constraint generation* approach, the DRO problem can be solved via Algorithm 1.

$$\text{MP:} \quad \min_{\mathbf{x}, \lambda, s_i, \mathbf{z}_{t,i}, \mathbf{y}_{t,i}} \quad \mathbf{c}^T \mathbf{x} + \lambda \varepsilon + \frac{1}{N} \sum_{i=1}^N s_i \quad (16a)$$

$$\text{s.t.} \quad \mathbf{b}^T \mathbf{y}_{t,i} - \mathbf{z}_i^T \mathbf{v}_{t,i}^* + \mathbf{z}_i^T \hat{\mathbf{v}}_i \leq s_i, \quad (16b)$$

$$\mathbf{T}\mathbf{x} + \mathbf{W}\mathbf{y}_{t,i} + \mathbf{M}\mathbf{v}_{t,i} \leq \mathbf{h}, \quad (16c)$$

$$\mathbf{A}\mathbf{x} \leq \mathbf{q}, \quad (16d)$$

$$i = 1, \dots, N, \quad t = 1, \dots, r. \quad (16e)$$

$$\text{SP:} \quad \mathcal{C}(\mathbf{x}^*, \mathbf{z}_i^*) := \max_{\mathbf{v}_i} \left[\min_{\mathbf{y}_i} \mathbf{b}^T \mathbf{y}_i - \mathbf{v}_i^T \mathbf{z}_i^* \right] \quad (17a)$$

$$\text{s.t.} \quad \mathbf{T}\mathbf{x}^* + \mathbf{W}\mathbf{y}_i + \mathbf{M}\mathbf{v}_i \leq \mathbf{h}. \quad (17b)$$

References

- [1] Mohajerin Esfahani, P., Kuhn, D. (2018). Data-driven distributionally robust optimization using the Wasserstein metric: Performance guarantees and tractable reformulations. *Mathematical Programming*, 171(1), 115-166.

Algorithm 1 *column-and-constraint generation* algorithm for DRO problem (??).

Input: suboptimality gap ϵ

Output: the optimal decision variable \mathbf{x}^* and objective function value $\mathbf{c}^T \mathbf{x}^* + \lambda^* \epsilon + \frac{1}{N} \sum_{i=1}^N s_i^*$

1: Set $LB = -\infty$, $UB = \infty$, $r = 0$

2: **while** $|UB - LB| > \epsilon$ **do**

3: Solve **MP** in (16) to derive solutions $\{\mathbf{x}^*, \lambda^*, s_i^*, \mathbf{z}_i^*\}$ and update $LB = \mathbf{c}^T \mathbf{x}^* + \lambda^* \epsilon + \frac{1}{N} \sum_{i=1}^N s_i^*$

4: **for** $i = 1, \dots, N$ **do**

5: Solve **SP** $\mathcal{C}(\mathbf{x}^*, \mathbf{z}_i^*)$ in (17) to derive solutions $\{\mathbf{v}_i^*, \mathbf{y}_i^*\}$

6: **end for**

7: Update UB as

$$UB = \min \left\{ UB, \mathbf{c}^T \mathbf{x}^* + \lambda^* \epsilon + \frac{1}{N} \sum_{i=1}^N (\mathbf{b}^T \mathbf{y}_i^* - \mathbf{z}_i^{*T} \mathbf{v}_i^* + \mathbf{z}_i^{*T} \hat{\mathbf{v}}_i) \right\}$$

8: Create decision variables $\{\mathbf{y}_{r,i} \mid i = 1, \dots, N\}$, set parameters $\mathbf{v}_{r,i} = \mathbf{v}_i^*$ ($i = 1, \dots, N$), and add the following constraints to **MP** in (16)

$$\begin{cases} \mathbf{b}^T \mathbf{y}_{r,i} - \mathbf{z}_i^{*T} \mathbf{v}_{r,i} + \mathbf{z}_i^{*T} \hat{\mathbf{v}}_i \leq s_i, \\ \mathbf{T} \mathbf{x} + \mathbf{W} \mathbf{y}_{r,i} + \mathbf{M} \mathbf{v}_{r,i}^* \leq \mathbf{h}. \end{cases}$$

9: $r \leftarrow r + 1$

10: **end while**

11: **Return:** \mathbf{x}^* and $\mathbf{c}^T \mathbf{x}^* + \lambda^* \epsilon + \frac{1}{N} \sum_{i=1}^N s_i^*$
