习 题 课

题1 求：$Z=A(B+\overline{C})+\overline{A}B(C+\overline{D})+A\overline{B}C+D$

对偶式：$Z_D=(A+B\overline{C})(\overline{A}+B+C\overline{D})(A+\overline{B}+C)D$

讨论：如果对Z先进行逻辑化简，其对偶式是否与原对偶式相等？

$$
\begin{aligned}
Z&=A(B+\overline{C})+\overline{A}B(C+\overline{D})+A\overline{B}C+D\\
&=AB+A\overline{C}+\overline{A}BC+\overline{A}B\overline{D}+A\overline{B}C+D\\
&=AB+A\overline{C}+\overline{A}BC+\overline{A}B+A\overline{B}C+D\\
&=B+A\overline{C}+\overline{A}BC+A\overline{B}C+D\\
&=B+A(\overline{C}+\overline{B})+D\\
&=B+A\overline{C}+A\overline{B}+D\\
&=B+A+A\overline{C}+D\\
&=A+B+D
\end{aligned}
$$

$$
\begin{aligned}
Z_D&=(A+B\overline{C})(\overline{A}+B+C\overline{D})(A+\overline{B}+C)D\\
&=(AB+AC\overline{D}+\overline{A}B\overline{C}+B\overline{C})(A+\overline{B}+C)D\\
&=(AB+AC\overline{D}+B\overline{C})(A+\overline{B}+C)D\\
&=(ABD+B\overline{C}D)(A+\overline{B}+C)\\
&=ABD+ABCD+AB\overline{C}D\\
&=ABD
\end{aligned}
$$

结论：两个相等的逻辑表达式其对偶式也是相等的。

同理：两个相等的逻辑表达式其对应的反函数也是相等的。

题2 设X是一个小于4的8421BCD码的整数，而Y=2X，试列出Y的真值表（Y也用8421BCD码表示）。

解： 设 X：$X_3X_2X_1X_0$ Y：$Y_3Y_2Y_1Y_0$

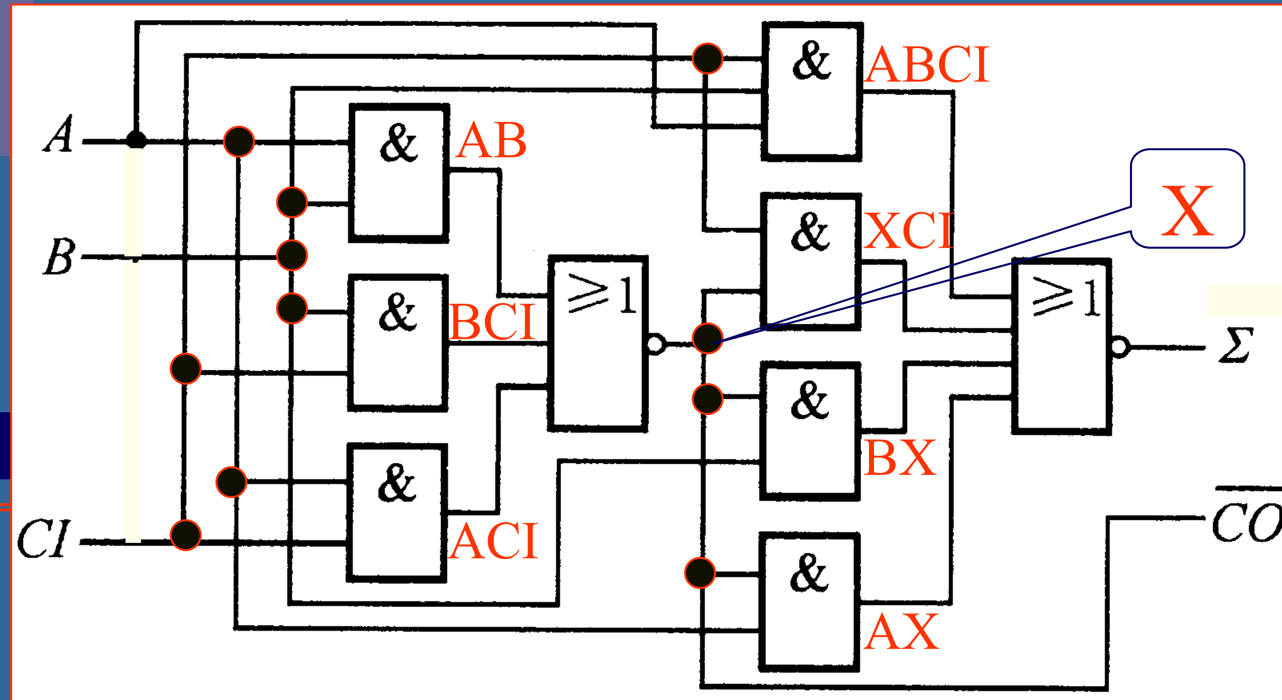
X_3	X_2	X_1	X_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	0
0	0	1	1	0	1	1	0
0	1	0	0	1	0	0	0
0	1	0	1				
0	1	1	0				
0	1	1	1				
1	0	0	0		?		
1	0	0	1				
1	0	1	0				
1	0	1	1				
1	1	0	0				
1	1	0	1				
1	1	1	0				
1	1	1	1				

X_3	X_2	X_1	X_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	0
0	0	1	1	0	1	1	0
0	1	0	0	1	0	0	0
0	1	0	1	X	X	X	X
0	1	1	0	X	X	X	X
0	1	1	1	X	X	X	X
1	0	0	0	X	X	X	X
1	0	0	1	X	X	X	X
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X

X_3	X_2	X_1	X_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	1	0
0	0	1	0	0	1	0	0
0	0	1	1	0	1	1	0
0	1	0	0	1	0	0	0
其它				X	X	X	X

题3

试分析图示电路的逻辑功能



$$\Sigma = \overline{ABCI + XCI + BX + AX}$$

$$= \overline{ABCI + X(A + B + CI)}$$

$$= \overline{ABCI} \cdot \overline{X(A + B + CI)}$$

$$= (\bar{A} + \bar{B} + \overline{CI})(\bar{X} + \bar{A}\bar{B}\overline{CI})$$

$$= (\bar{A} + \bar{B} + \overline{CI})(AB + BCI + ACI + \bar{A}\bar{B}\overline{CI})$$

$$= \bar{A}BCI + A\bar{B}CI + \bar{A}\bar{B}\overline{CI} + AB\overline{CI}$$

$$= \bar{A}(BCI + \bar{B}\overline{CI}) + A(\bar{B}CI + B\overline{CI})$$

$$= \bar{A} \cdot \overline{B \oplus CI} + A \cdot B \oplus CI$$

$$= \overline{A \oplus B \oplus CI}$$

$$\overline{CO} = X = \overline{AB + ACI + BCI}$$

功能：全加器，
输出低电平有效

题4

试用布尔代数公式化简下列各式为最简的与或式。

$$F=A\bar{B}C+\bar{A}+B+\bar{C}$$

解法一：

$$F=A\bar{B}C+\bar{A}+B+\bar{C}=(\bar{A}+A\bar{B}C)+B+\bar{C}$$

$$=\bar{A}+(\bar{B}C+B)+\bar{C}=\bar{A}+C+B+\bar{C}$$

$$=1$$

解法二：

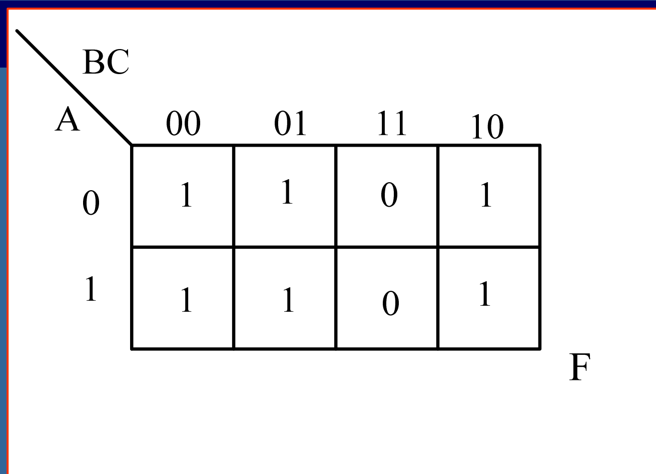
$$F=A\bar{B}C+\bar{A}+B+\bar{C}=A\bar{B}C+\overline{A\bar{B}C}=1$$

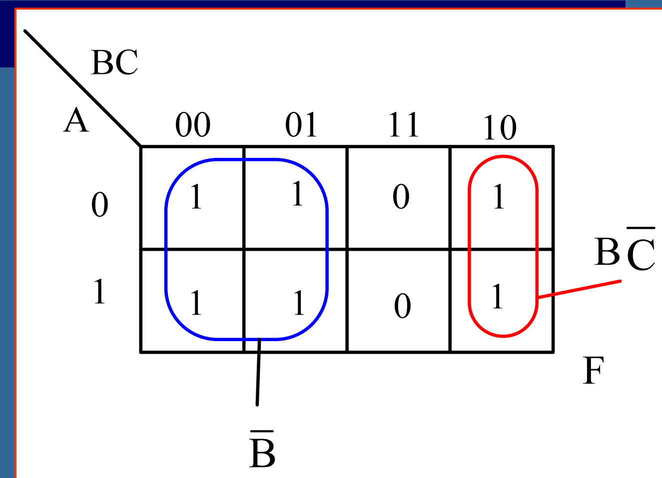
题5

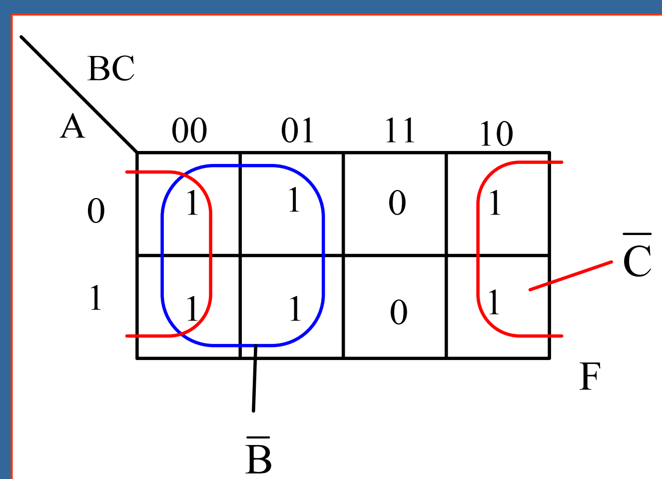
试用卡诺图化简下列各函数为最简的与或表达式。

$$F(A,B,C)=\overline{A}\,\overline{B}+\overline{C}+A\overline{B}C$$

解:

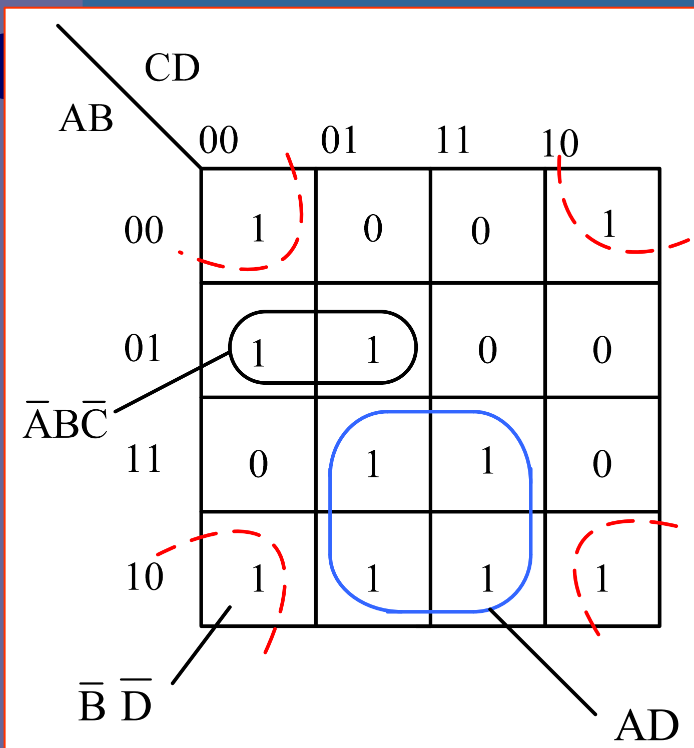




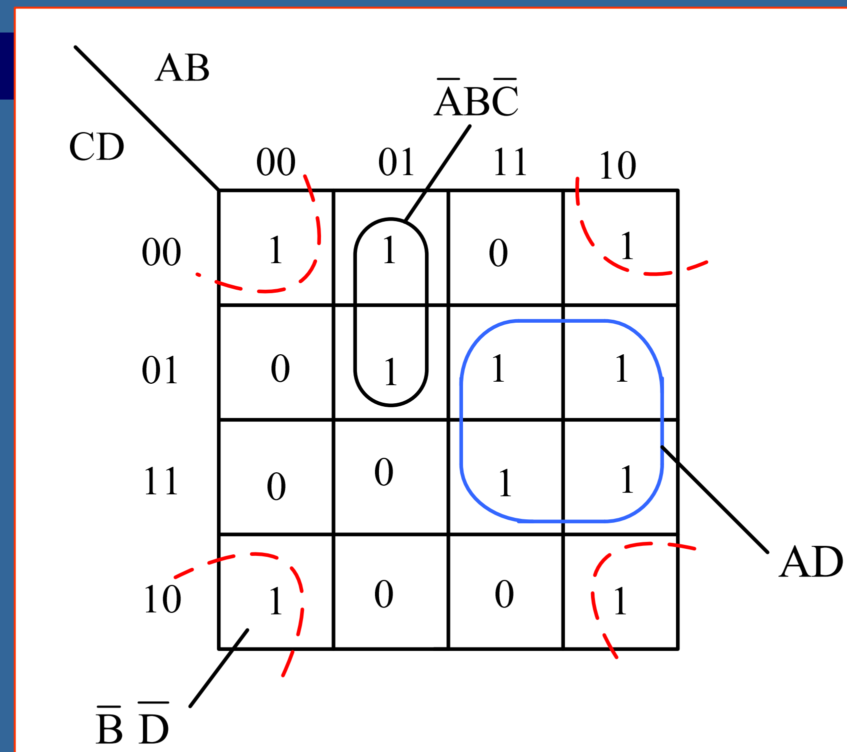


(2) $F(A,B,C,D)=\Sigma m(0,2,4,5,8,9,10,11,13,15)$

解:



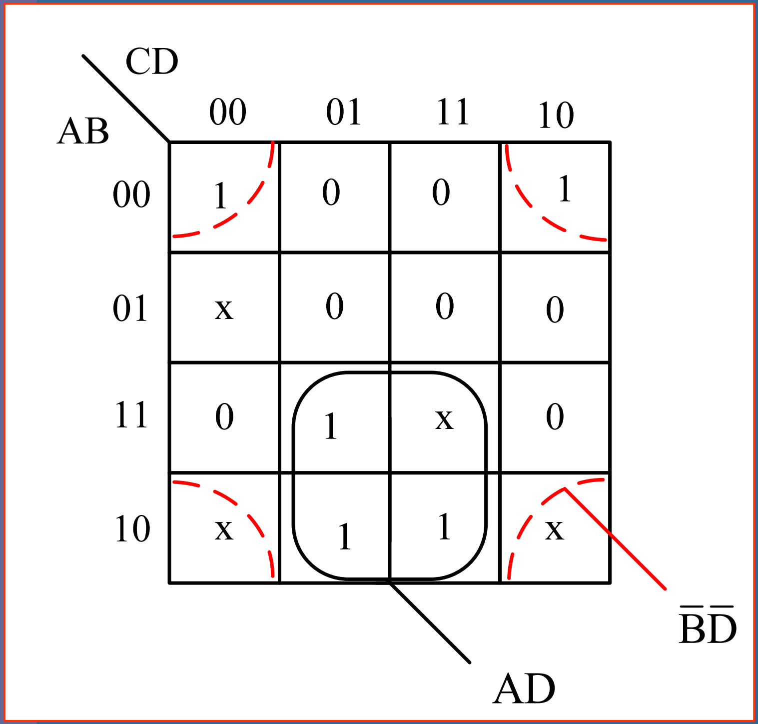
或



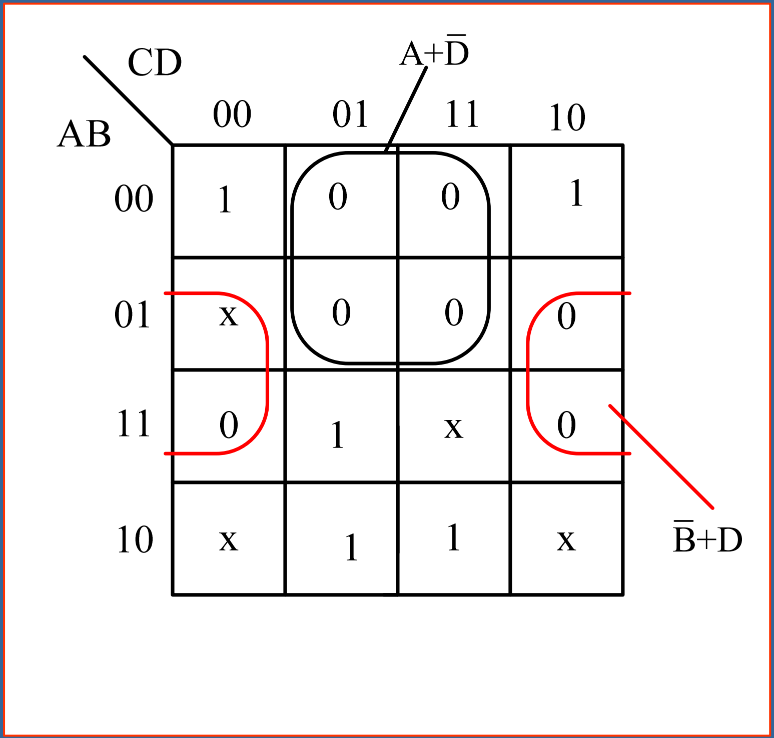
题6 试用卡诺图化简下列各函数为最简的与或表达式和或与表达式：

$$(1)\ F(A,B,C,D)=\Sigma m(0,2,9,11,13)+\Sigma d(4,8,10,15)$$

解：

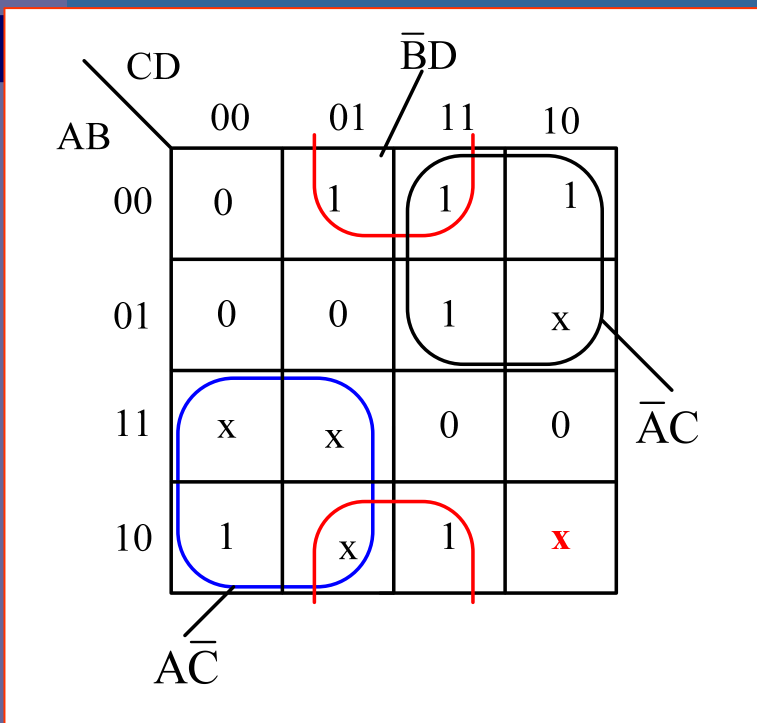


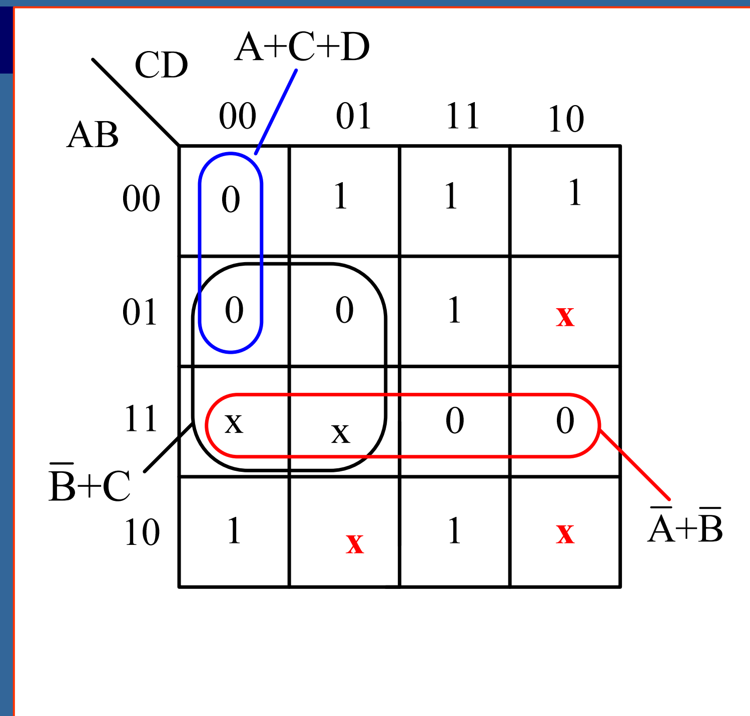
与或



或与

(2) $F(A,B,C,D)=\Pi M(0,4,5,14,15)\cdot \Pi D(6,9,10,12,13)$





题7 化简函数F为最简的与或表达式。

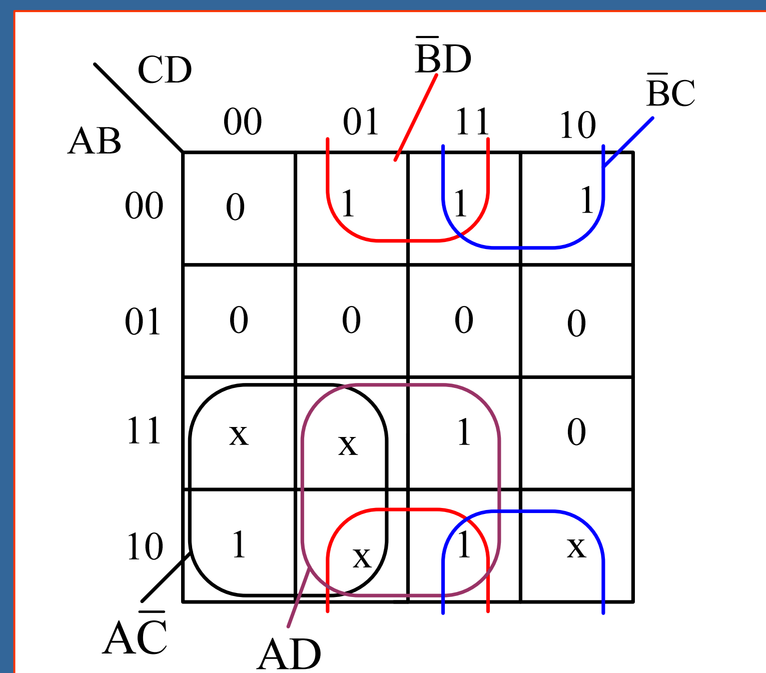
$$F(A,B,C,D)=\Sigma m(1,2,3,8,11,15)$$

且 $AB\bar{C}+A\bar{C}D+A\bar{B}C\bar{D}=0$

解： $AB\bar{C}+A\bar{C}D+A\bar{B}C\bar{D}=0$

即 $AB\bar{C}D+AB\bar{C}\bar{D}+AB\bar{C}D+A\bar{B}\bar{C}D+A\bar{B}C\bar{D}=0$

或 $\Sigma d(9,10,12,13)=0$



题8 $F(A,B,C,D)=\bar{A}\bar{B}\bar{C}+ABC+\bar{A}\bar{B}C\bar{D}$ 且 $A\oplus B=0$

解法一： $A\bar{B}+\bar{A}B=0 \Rightarrow A\bar{B}C+A\bar{B}\bar{C}+\bar{A}BC+\bar{A}B\bar{C}=0$

$$\Rightarrow A\bar{B}CD+A\bar{B}C\bar{D}+A\bar{B}\bar{C}D+A\bar{B}\bar{C}\bar{D}$$

$$+\bar{A}BCD+\bar{A}BC\bar{D}+\bar{A}B\bar{C}D+\bar{A}B\bar{C}\bar{D}$$

$$\Rightarrow \Sigma d(2,3,4,5,8,9,10,11)=0$$

AB \ CD	00	01	11	10
00	1	1	0	1
01	X	X	X	X
11	0	0	1	1
10	X	X	X	X

$$F(A,B,C,D)=\bar{A}\bar{C}+BC+C\bar{D}$$

题8 $F(A,B,C,D)=\overline{A}\,\overline{B}\,\overline{C}+ABC+\overline{A}\,\overline{B}C\overline{D}$ 且 $A\oplus B=0$

解法二： $A\oplus B=0\Rightarrow A=B$

$$F(A,B,C,D)=\overline{A}\,\overline{C}+BC+\overline{A}C\overline{D}$$

$$=\overline{A}(\overline{C}+C\overline{D})+BC=\overline{A}\,\overline{C}+BC+\overline{A}\,\overline{D}$$

为什么与解法一结果不一样？

AB \ CD	00	01	11	10
00	1	1	0	1
01	X	X	X	X
11	0	0	1	1
10	X	X	X	X

$$F(A,B,C,D)=\overline{A}\,\overline{C}+BC+\overline{A}\,\overline{D}$$

题9　设计1位减法器

解：

设：X_i、Y_i为本位的被减数和减数，Bi为由低位来的借位；D_i，B_{i+1}为本位之差和向高位的借位 。

1位全减器真值表

X_i	Y_i	B_i	B_{i+1}	D_i
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

逻辑表达式

$$B_{i+1} = \overline{X}_i Y_i + \overline{X}_i B_i + Y_i B_i$$

$$= \overline{\overline{\overline{X}_i Y_i} \cdot \overline{\overline{X}_i B_i} \cdot \overline{Y_i B_i}}$$

$$D_i = \overline{X}_i \overline{Y}_i B_i + \overline{X}_i Y_i \overline{B}_i + X_i \overline{Y}_i \overline{B}_i + X_i Y_i B_i$$

$$= \overline{\overline{\overline{X}_i \overline{Y}_i B_i} \cdot \overline{\overline{X}_i Y_i \overline{B}_i} \cdot \overline{X_i \overline{Y}_i \overline{B}_i} \cdot \overline{X_i Y_i B_i}}$$

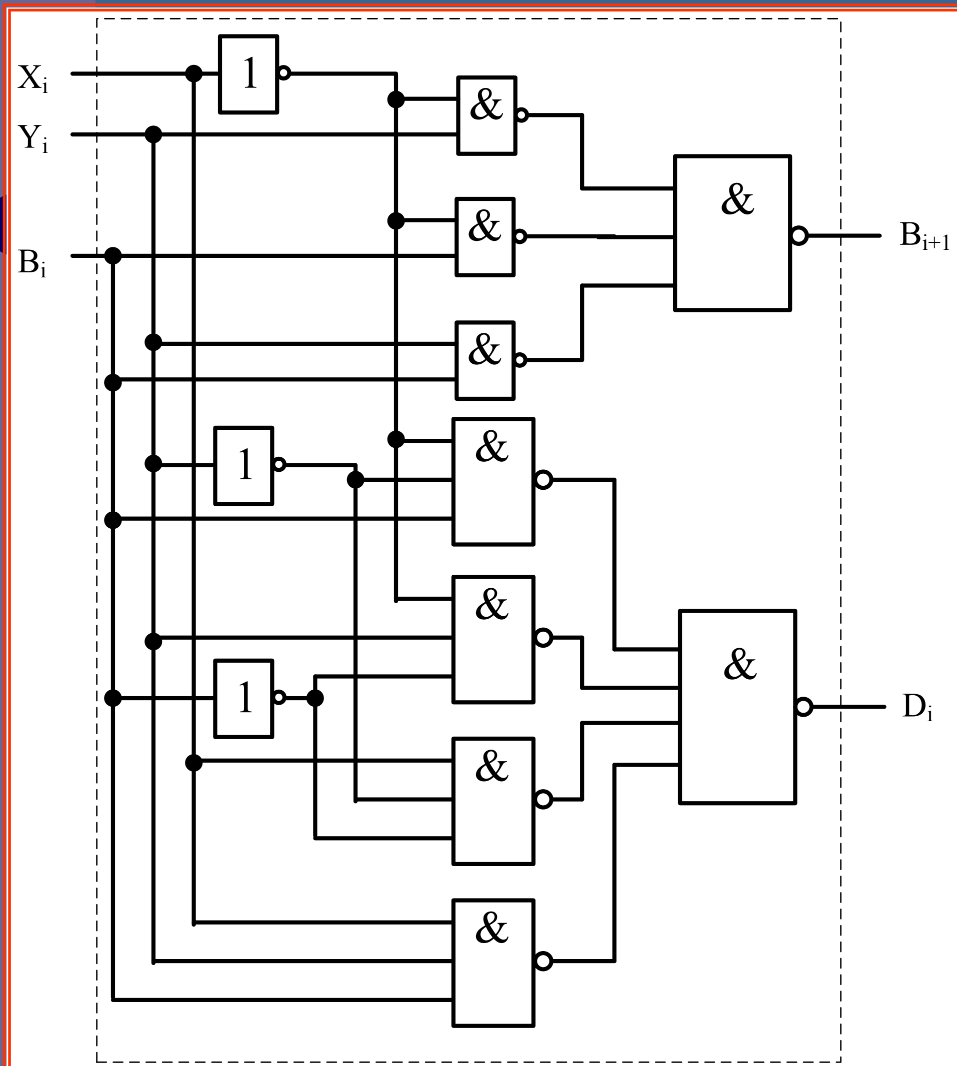
$$B_{i+1} = \bar{X}_i Y_i + \bar{X}_i B_i + Y_i B_i$$

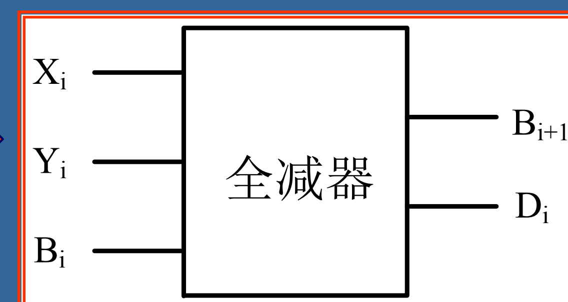
$$= \overline{\overline{\bar{X}_i Y_i} \cdot \overline{\bar{X}_i B_i} \cdot \overline{Y_i B_i}}$$

$$D_i = \bar{X}_i \bar{Y}_i B_i + \bar{X}_i Y_i \bar{B}_i + X_i \bar{Y}_i \bar{B}_i + X_i Y_i B_i$$

$$= \overline{\overline{\bar{X}_i \bar{Y}_i B_i} \cdot \overline{\bar{X}_i Y_i \bar{B}_i} \cdot \overline{X_i \bar{Y}_i \bar{B}_i} \cdot \overline{X_i Y_i B_i}}$$

逻辑图





框 图

题10 试设计一个1位全加/全减器电路。当M=0时，该电路为全加器；M=1时该电路为全减器。

解:

分析： 设M加/减控制端（M=0，加法；M=1减法），A_i被加/减数，B_i加/减数，C_i低位来的进/借位，F_i本位之和/差，C_{i+1}向高位的进/借位。

真值表

M_i	A_i	B_i	C_i	F_i	C_{i+1}
0	0	0	0	0	0
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	0	1
0	1	0	0	1	0
0	1	0	1	0	1
0	1	1	0	0	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	0	1	1	1
1	0	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	0
1	1	0	1	0	0
1	1	1	0	0	0
1	1	1	1	1	1

用卡诺图求逻辑表达式：

MA_i \ B_iC_i	00	01	11	10
00	0	0	1	0
01	0	1	1	1
11	0	0	1	0
10	0	1	1	1

C_{i+1}

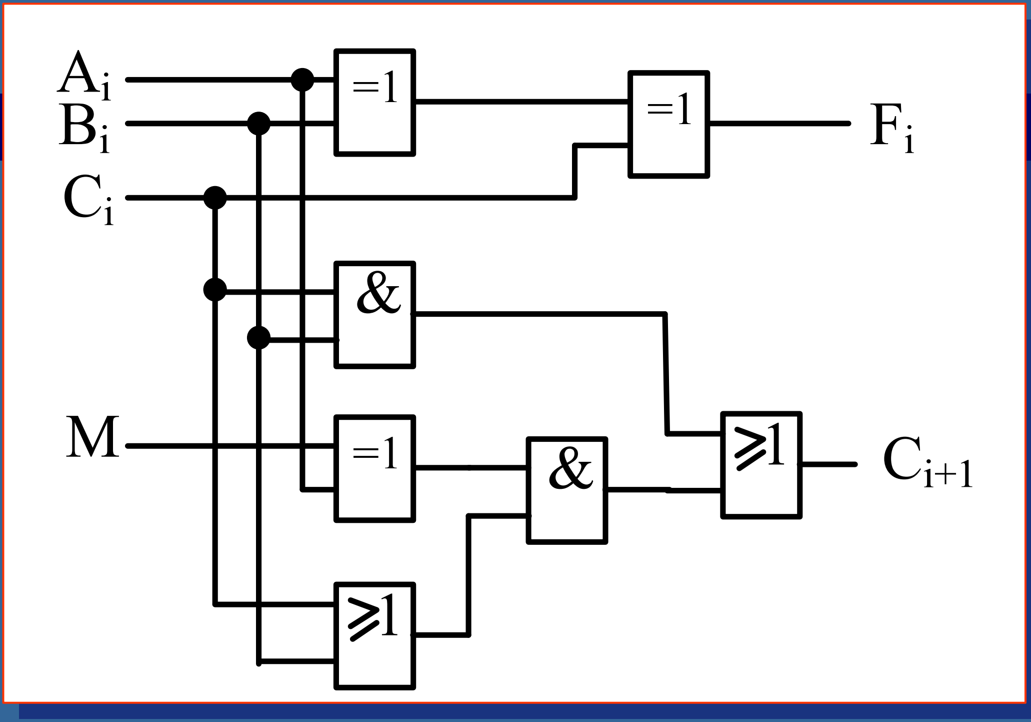
MA_i \ B_iC_i	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	1	0	1	0
10	0	1	0	1

F_i

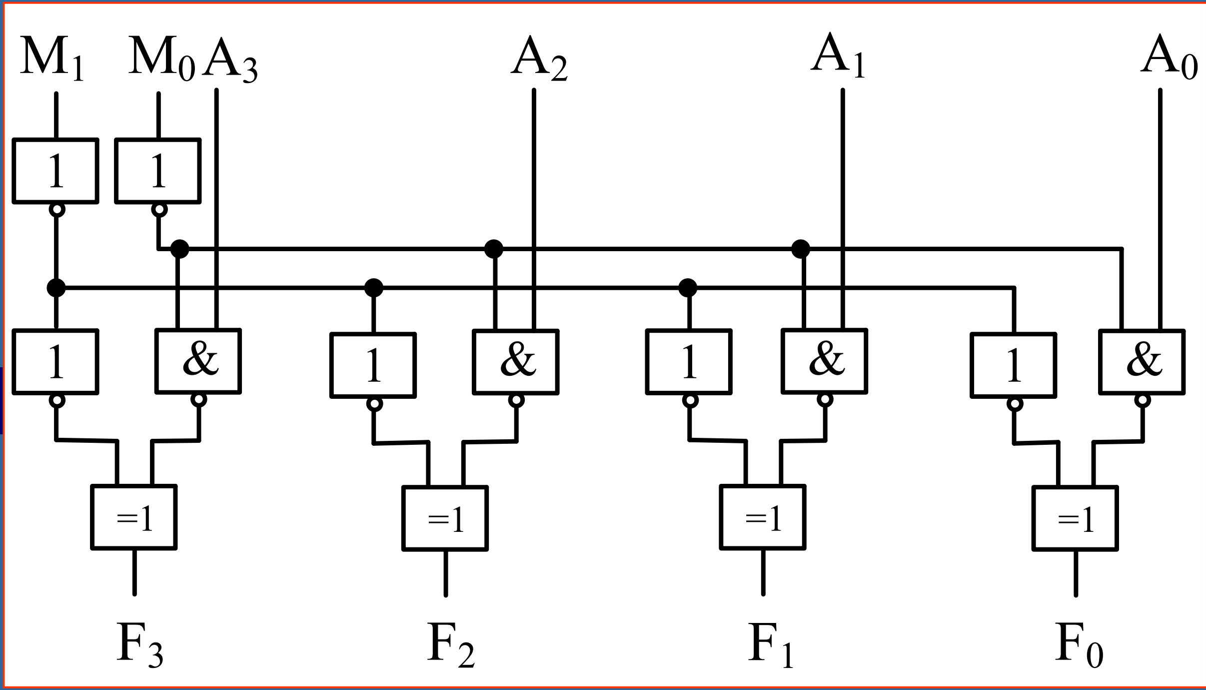
$$
\begin{aligned}
C_{i+1} &= \overline{M}A_iC_i + \overline{M}A_iB_i + M\overline{A}_iC_i + M\overline{A}_iB_i + B_iC_i \\
&= (\overline{M}A_i + M\overline{A}_i)C_i + (\overline{M}A_i + M\overline{A}_i)B_i + B_iC_i \\
&= (M \oplus A_i)(C_i + B_i) + B_iC_i
\end{aligned}
$$

$$
\begin{aligned}
F_i &= A_i\overline{B}_i\overline{C}_i + \overline{A}_i\overline{B}_iC_i + \overline{A}_iB_i\overline{C}_i + A_iB_iC_i \\
&= A_i \oplus B_i \oplus C_i
\end{aligned}
$$

画出逻辑图：



题11 分析如右图所示的逻辑电路



解:

分析：输入有6个逻辑变量，其中$A_3A_2A_1A_0$是数据输入端，M_1M_0功能控制端；输出逻辑变量是$F_3F_2F_1F_0$。

功能表

M_1	M_0	F_3	F_2	F_1	F_0
0	0	$\overline{A_3}$	$\overline{A_2}$	$\overline{A_1}$	$\overline{A_0}$
0	1	1	1	1	1
1	0	A_3	A_2	A_1	A_0
1	1	0	0	0	0

（当电路的输入变量较多时，对于电路的分析通常要围绕控制变量进行。）