

## 命题逻辑定理：

1.  $A \vdash A$ 

- 1)  $A$  (P)
- 2)  $A$  (1,  $\in$ )

2.  $A \vdash B \rightarrow A$  (肯定后件律)

- 1)  $A$  (P)
- 2)  $B$  (S)
- 3)  $A$  (2,  $\in$ )
- 4)  $B \rightarrow A$  (2, 3,  $\rightarrow+$ )

3.  $A \rightarrow B, B \rightarrow C \vdash A \rightarrow C$ 

- 1)  $A \rightarrow B$  (P)
- 2)  $B \rightarrow C$  (P)
- 3)  $A$  (S)
- 4)  $B$  (1, 3,  $\rightarrow-$ )
- 5)  $C$  (2, 4,  $\rightarrow-$ )
- 6)  $A \rightarrow C$  (3, 5,  $\rightarrow+$ )

4.  $A \rightarrow (B \rightarrow C), A \rightarrow B \vdash A \rightarrow C$ 

- 1)  $A \rightarrow (B \rightarrow C)$  (P)
- 2)  $A \rightarrow B$  (P)
- 3)  $A$  (S)
- 4)  $B \rightarrow C$  (1, 3,  $\rightarrow-$ )
- 5)  $B$  (2, 3,  $\rightarrow-$ )
- 6)  $C$  (5, 4,  $\rightarrow-$ )
- 7)  $A \rightarrow C$  (3, 6,  $\rightarrow+$ )

5.  $A, \neg A \vdash B$  (矛盾推出一切)

- 1)  $A$  (P)
- 2)  $\neg A$  (P)
- 3)  $\neg B$  (S)
- 4)  $A$  ( $\in$ )
- 5)  $\neg A$  ( $\in$ )
- 6)  $B$  (3, 4, 5,  $\neg$ )

6.  $\neg A \vdash A \rightarrow B$  (否定前件律)

- 1)  $\neg A$  (P)
- 2)  $A$  (S)
- 3)  $B$  (1, 2, TH5)
- 4)  $A \rightarrow B$  (2, 3,  $\rightarrow+$ )

7.  $A \vdash \neg A \rightarrow B$  (否定前件律)

- 1)  $A$  (P)
- 2)  $\neg A$  (S)
- 3)  $B$  (1, 2, TH5)
- 4)  $\neg A \rightarrow B$  (2, 3,  $\rightarrow+$ )

8.  $\neg\neg A \vdash A$

$\vdash$ :

- 1)  $\neg\neg A$  (P)
- 2)  $\neg A$  (S)
- 3)  $\neg\neg A, \neg A$  (1, 2,  $\epsilon$ )
- 4)  $A$  (2, 3,  $\neg$ )

$\dashv$ :

- 1)  $A$  (P)
- 2)  $\neg\neg\neg A$  (S)
- 3)  $\neg A$  (Th8 正向)
- 4)  $\neg\neg A$  (2, 3,  $\neg$ )

9.  $T, A \vdash B, \neg B \quad T \vdash \neg A$  (归谬律)

方法 1:

- 1)  $T, A$  (P)
- 2)  $B, \neg B$  (题设)
- 3)  $\neg A$  (Th5 矛盾推出一切)

方法 2:

- 1  $T$  (P)
- 2  $\neg\neg A$  (S)
- 3  $A$
- 4  $B, \neg B$  (题设)
- 5  $\neg A$  (2 3  $\neg$ )

10.  $A \rightarrow B, \neg B \vdash \neg A$

- 1)  $A \rightarrow B$  (P)
- 2)  $\neg B$  (P)
- 3)  $A$  (S)
- 4)  $B$  (1, 3,  $\rightarrow$ )
- 5)  $\neg A$  (3, 2, 4,  $\neg$  +)

11.  $A \rightarrow B \vdash \neg B \rightarrow \neg A$

- 1)  $A \rightarrow B$  (P)
- 2)  $\neg B$  (S)
- 3)  $\neg A$  (1, 2, Th10)
- 4)  $\neg B \rightarrow \neg A$  (3, 2,  $\rightarrow$  +)

12.  $\neg A \rightarrow \neg B, B \vdash A$

- 1)  $\neg A \rightarrow \neg B$  (P)
- 2)  $B$  (P)
- 3)  $\neg A$  (S)
- 4)  $\neg B$  (1, 3,  $\rightarrow$ )
- 5)  $A$  (3, 2, 4,  $\neg$ )

13.  $\neg A \rightarrow \neg B \vdash B \rightarrow A$

- 1)  $\neg A \rightarrow \neg B$  (P)

- 2)  $B$  (S)  
 3)  $\neg A$  (S)  
 4)  $\neg B$  (1.  $\rightarrow-$ )  
 5)  $B$  (2.  $\in$ )  
 6)  $A$  (3. 4. 5. 归谬律)  
 7)  $B \rightarrow A$  (2. 6.  $\rightarrow+$ )

14.  $A \rightarrow \neg B, B \vdash \neg A$

15.  $A \rightarrow \neg B \vdash B \rightarrow \neg A$

16.  $\neg A \rightarrow B, \neg B \vdash A$

- 1)  $\neg A \rightarrow B$  (P)  
 2)  $\neg B$  (P)  
 3)  $\neg A$  (S)  
 4)  $B$  (1.  $\rightarrow-$ )  
 5)  $\neg B$  (2.  $\in$ )  
 6)  $A$  (3. 4. 5.  $\neg$ )

17.  $\neg A \rightarrow B \vdash \neg B \rightarrow A$

- 1)  $\neg A \rightarrow B$  (P)  
 2)  $\neg B$  (S)  
 3)  $\neg A$  (S)  
 4)  $B$  (1.  $\rightarrow-$ )  
 5)  $\neg B$  (2.  $\in$ )  
 6)  $A$  (3. 4. 5. 归谬律)  
 7)  $\neg B \rightarrow A$  (2. 6.  $\rightarrow+$ )

18.  $\neg A \rightarrow A \vdash A$

19.  $A \rightarrow \neg A \vdash \neg A$

20.  $A \rightarrow B, A \rightarrow \neg B \vdash \neg A$

21.  $A \rightarrow B, \neg A \rightarrow B \vdash B$

22.  $\neg (A \rightarrow B) \vdash A, \neg B$

- 1)  $\neg (A \rightarrow B)$  (P)  
 2)  $\neg A$  (S)  
 3)  $A \rightarrow B$  (Th6 否定前件律)  
 4)  $A$  (1, 2, 3,  $\neg$ )  
 5)  $B$  (S)  
 6)  $A \rightarrow B$  (肯定后件律)  
 7)  $\neg B$  (1, 5, 6,  $\neg +$ )

23. 如果  $\Gamma, A \vdash C$  且  $\Gamma, B \vdash C$ , 则  $\Gamma, A \vee B \vdash C$  ( $\vee-$ )

24.  $A \wedge B \vdash B \wedge A$

$\vdash$ 

- 1)  $A \wedge B$  (P)
- 2)  $A, B$  (1,  $\wedge -$ )
- 3)  $B \wedge A$  (2,  $\wedge +$ )

 $\neg$ 

- 1  $B \wedge A$  (P)
- 2  $A, B$  (1,  $\wedge -$ )
- 3  $A \wedge B$  (2,  $\wedge +$ )

$$25. (A \wedge B) \wedge C \vdash A \wedge (B \wedge C)$$

$$26. A \wedge B \vdash \neg (A \rightarrow \neg B)$$

 $\vdash$ 

- 1)  $A \wedge B$  (P)
- 2)  $A \rightarrow \neg B$  (S)
- 3)  $A, B$  (1,  $\wedge -$ )
- 4)  $\neg B$  (2, 3,  $\rightarrow -$ )
- 5)  $\neg (A \rightarrow \neg B)$  (2, 3, 4,  $\neg +$ )

 $\neg$ 

- 1)  $\neg (A \rightarrow \neg B)$  (P)
- 2)  $\neg A$  (S)
- 3)  $A \rightarrow \neg B$  (2, Th6)
- 4)  $A$  (2, 3,  $\neg$ )
- 5)  $\neg B$  (S)
- 6)  $A \rightarrow \neg B$  (5, Th2)
- 7)  $B$  (5, 6,  $\neg$ )
- 8)  $A \wedge B$  (7, 4,  $\wedge +$ )

$$27. \neg (A \wedge B) \vdash A \rightarrow \neg B$$

 $\vdash$ 

$$28. A \rightarrow B \vdash \neg (A \wedge \neg B)$$

 $\vdash$ 

$$29. \neg (A \rightarrow B) \vdash A \wedge \neg B$$

$$30. \vdash \neg (A \wedge \neg A)$$

- 1)  $A \wedge \neg A$  (S)
- 2)  $A, \neg A$  (1,  $\wedge -$ )
- 3)  $\neg (A \wedge \neg A)$  (2, 3,  $\neg +$ )

$$31. A \vee B \vdash B \vee A$$

 $\vdash$ 

- 1)  $A$  (S)
- 2)  $B \vee A$  (1,  $\vee +$ )
- 3)  $B$  (S)
- 4)  $B \vee A$  (3,  $\vee +$ )
- 5)  $A \vee B$  (P)

- 6)  $B \vee A$  (1-4,  $\vee -$ )  
 $\vdash$   
 1)  $B$  (S)  
 2)  $B \vee A$  (3,  $\vee +$ )  
 3)  $A$  (S)  
 4)  $B \vee A$  (1,  $\vee +$ )  
 5)  $B \vee A$  (P)  
 6)  $A \vee B$  (1-4,  $\vee -$ )

$$32. (A \vee B) \vee C \vdash A \vee (B \vee C)$$

$$33. A \vee B \vdash \neg A \rightarrow B$$

$$34. A \rightarrow B \vdash \neg A \vee B$$

$$35. \vdash \neg A \vee A$$

$$36. \neg (A \wedge B) \vdash \neg A \vee \neg B$$

$$37. \neg (A \vee B) \vdash \neg A \wedge \neg B$$

- $\vdash$   
 1)  $\neg (A \vee B)$  (P)  
 2)  $A$  (S)  
 3)  $A \vee B$  (2,  $\vee +$ )  
 4)  $\neg A$  (2, 3,  $\neg +$ )  
 5)  $B$  (S)  
 6)  $A \vee B$  (5,  $\vee +$ )  
 7)  $\neg B$  (5, 6,  $\neg +$ )  
 8)  $\neg A \wedge \neg B$  (4, 7,  $\wedge +$ )  
 $\vdash$   
 1)  $\neg A \wedge \neg B$  (P)  
 2)  $\neg A, \neg B$  (1,  $\wedge -$ )  
 3)  $A \vee B$  (S)  
 4)  $\neg A \rightarrow B$  (3, th33)  
 5)  $B$  (2, 4,  $\rightarrow -$ )  
 6)  $\neg (A \vee B)$  (2, 3, 5,  $\neg +$ )

$$38. A \vee (B \wedge C) \vdash (A \vee B) \wedge (A \vee C)$$

$$39. (A \wedge B) \vee C \vdash (A \vee C) \wedge (B \vee C)$$

$$40. A \wedge (B \vee C) \vdash (A \wedge B) \vee (A \wedge C)$$

$$41. (A \vee B) \wedge C \vdash (A \wedge C) \vee (B \wedge C)$$

- $\vdash$   
 1)  $(A \vee B) \wedge C$  (P)  
 2)  $A \vee B, C$  ( $\wedge -$ )

- 3)  $\neg A \rightarrow B$  (TH33)  
 4)  $\neg (A \wedge C)$  (S)  
 5)  $\neg A \vee \neg C$  (摩根律)  
 6)  $A \rightarrow \neg C$  (TH33)  
 7)  $C$  (2.  $\in$ )  
 8)  $\neg A$  (6. 7. TH10)  
 9)  $B$  (3.  $\rightarrow$ -)  
 10)  $B \wedge C$  (7. 9.  $\wedge$ +)  
 11)  $\neg (A \wedge C) \rightarrow (B \wedge C)$  (4. 10.  $\rightarrow$ +)  
 12)  $(A \wedge C) \vee (B \wedge C)$  (11. TH33)

$\neg$

- 13)  $(A \wedge C) \vee (B \wedge C)$  (P)  
 14)  $A \wedge C$  (S)  
 15)  $A, C$  (14.  $\wedge$ -)  
 16)  $A \vee B$  (15.  $\vee$ +)  
 17)  $(A \vee B) \wedge C$  (15. 16.  $\wedge$ +)  
 18)  $B \wedge C$  (S)  
 19)  $B, C$  (18.  $\wedge$ -)  
 20)  $A \vee B$  (19.  $\vee$ +)  
 21)  $(A \vee B) \wedge C$  (18. 19.  $\wedge$ +)  
 22)  $(A \vee B) \wedge C$  ( $\vee$ -)

$$42. A \rightarrow B \wedge C \vdash (A \rightarrow B) \wedge (A \rightarrow C)$$

$$43. A \rightarrow B \vee C \vdash (A \rightarrow B) \vee (A \rightarrow C)$$

$$44. A \wedge B \rightarrow C \vdash (A \rightarrow C) \vee (B \rightarrow C)$$

$$45. A \vee B \rightarrow C \vdash (A \rightarrow C) \wedge (B \rightarrow C)$$