On Analyzing Graphs with Motif-Paths

Supplemental Material

XIAODONG LI†, REYNOLD CHENG†, KEVIN CHANG‡, CAIHUA SHAN†, CHENHAO MA†, HONGTAI CAO‡, †Department of Computer Science, University of Hong Kong, Hong Kong SAR
‡Department of Computer Science, University of Illinois at Urbana-Champaign, USA

ACM Reference Format:
Xiaodong Li†, Reynold Cheng†, Kevin Chang‡, Caihua Shan†, Chenhao Ma†, Hongtai Cao‡. 2020. On Analyzing Graphs with Motif-Paths: Supplemental Material. 1, 1 (December 2020), 5 pages. https://doi.org/10.1145/nnnnnnn.nnnnnnn

Because of the page limit in the paper, we report more details in the supplemental materials. In Section 1, we attach the proofs for lemmas (cf. Section 1.1), algorithm complexities (cf. Section 1.2). In Section 2, We report more efficiency and effectiveness evaluations, including:

- Fig. 2 MOD-Indexing time and size as graph density increases;
- Fig. 3 SMP query time on graphs of different density and materialization levels;
- Fig. 5 evaluation of generic motifs for link prediction on GAVI, EXTE and AMAZ;
- Tab. 2 link prediction performance with running time of each algorithm on each dataset;
- Tab. 3 local graph clustering performance with running time of each algorithm on each dataset;
- Fig. 4 local graph clustering performance comparison between MLGC-b and MLGC-c.;

1 PROOF

1.1 Proof of the Lemmas

Proof of Lemma 1:

Proof. For c1, there is no need to add motif-instances containing a "searched" node since all motif-instances around node marked as "searched" have been found and added into candidates for $\mathcal{P}_{s,t}$. For c2, for the motif-instances in $\mathcal{P}_{s,t}$, there are only two status of the nodes: "searched" and "discovered". We only select "discovered" node as next seed because the "searched" nodes have been used as seed before and thus using them as next seed will find duplicates. For c3, in the incremental search manner, $\mathcal{P}_{s,v}$ is found for the "undiscovered" node v when v is covered by any motif-instance for the first time. Therefore, we only add motif-instances which contain at least one node marked as "undiscovered" to push the incremental search forward. □

Proof of Lemma 2:

Author's address: Xiaodong Li†, Reynold Cheng†, Kevin Chang‡, Caihua Shan†, Chenhao Ma†, Hongtai Cao‡, †Department of Computer Science, University of Hong Kong, Hong Kong SAR
‡Department of Computer Science, University of Illinois at Urbana-Champaign, USA, {xdli,ckcheng,chshan,chma2}@cs.hku.hk;{kcchang, hongtai2}@illinois.edu.

XXXX-XXXX/2020/12-ART $15.00
https://doi.org/10.1145/nnnnnnn.nnnnnnn

Proof. Since $\mathcal{P}_{s,t}$ is the shortest sequence of motif-instances from m_s to m_t. For each motif-instance in the sequence, we pick the edge which links s, t and the nodes shared by the neighboring motif-instances, the path is the shortest one on W. Vise versa. □

Proof of Lemma 3:

Proof. Assume that $\exists(i,j) \in V \times V$, $W_{i,j} = 1$ but there is no motif-instance of $\bar{\tau}$, then there must be motif-instance m such that $m \simeq \bar{\tau}'\&(i,j) \in E_m$, where $\bar{\tau}'$ is another motif-orbit of τ with seed $s \in V_m$. By switching the seed node, $m \simeq \bar{\tau}$ with seed $s' \in V_m$, which is contradictory to the assumption. □

Here we also show that most expansive nodes satisfy $d_e \leq 2$. As pointed by [3], the degree distributions of the motifs trend to be long-tailed as the size of the motif grows, e.g., most nodes will have smaller degrees for motifs of bigger sizes. Since the motif-orbits have the same structure as the motif, and the expansive degree is at most the degree of the expansive node, thus the expansive degree also follows this rule. Also, as pointed by [1], the number of motifs increase exponentially as the motif size grows; hence most expansive degrees are rather small. We collect all expansive degrees from the motif-orbits whose size is smaller than six (i.e., the motifs used in the paper), and 65% of them are only one while 21% of them are two. As the motif orbit size grows, the numbers are more lopsided. The detailed results are shown in Fig. 1.

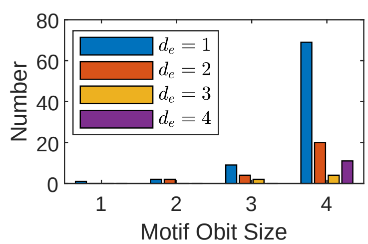


Fig. 1. The number of expansive nodes with different expansive degrees (d_e) from motif orbits of different sizes.

1.2 Time and Space Complexities

2 SUPPLEMENTAL EVALUATIONS

In this section, we evaluate the indexing cost and shortest motif-path (SMP) query time on graphs of different density levels. We follow the graph generation approach described in Section 6 of the paper and fix the node number as 300000. The edge numbers vary from 300000 to 1500000, making the average degree as 2 ($D1$), 4 ($D2$), 6($D3$), 8 ($D4$) and 10 ($D5$) respectively.

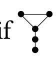
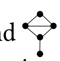
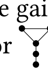
Next, we follow the setup in Section 6.1 to build the MOD-Index on each dataset with different materialization levels (i.e., $k = 0, 1, 2$ and 3). Note that $k = 0$ means that there is no motif-instances materialized in the MOD-Index. We report the indexing time (left y-axis) and space (right y-axis) in Fig. 2. Note that the motif-Index supports all motifs reported in the paper. Similar trend as Fig. 8 in the paper is also observed here. The indexing cost (both time and space) are much higher when the materialization level is set as 2 or 3. However, as we will show, $k = 1$ can already obtain good performance in the online phase with considerable time and space cost in the offline phase.

Then we randomly sample 500 node pairs in each dataset; for each pair of nodes, we run SMP with motifs from Fig. 9 in the paper. As shown in Fig. 3, the running time of different motifs varies a lot, and the biggest speedup is usually obtained when the materialization level is set as 1. For the cliques, their motif-orbits are within the

Table 1. Summary of algorithm complexities.

Alg.	Time	Space
BASE	$O(N_\tau^3)$	$O(N_\tau^2)$
MODC	$O(\sum_{s\in V}\sum_{\tau'\in B_k} D_{\tau'}^k)$	$O(I)$
MODCt	$O(\lvert V\rvert \times d_{\max}^2)$	$O(\lvert V\rvert \times \binom{d_{\max}}{2})$
MODQ	$O(D_\tau^{\phi_\tau - k})$	$O(I)$
SMP	$O(\lvert V\rvert \times D_\tau^{\phi_\tau - k} + N_\tau)$	$O(\lvert V\rvert + N_\tau + I)$
ESMP	$O(\lvert V\rvert \times D_\tau^{\phi_\tau - k} + \lvert E\rvert + N_\tau)$	$O(\lvert V\rvert + \lvert E\rvert + N_\tau + I)$
MGD	$O(\lvert V\rvert \times D_\tau^{\phi_\tau - k} + N_\tau)$	$O(\lvert V\rvert + N_\tau + I)$
MKI	$O((\lvert V\rvert \times D_\tau^{\phi_\tau - k})^{2L} + N_\tau)$	$O(\lvert V\rvert^{2L} + N_\tau + I)$
MLGC	$O(\lvert V\rvert \times \hat{k} \times D_\tau^{\phi_\tau - k} + N_\tau)$	$O(d_{\max} \times \hat{k} + \binom{\hat{k}}{\lvert V_\tau\rvert} + I)$
MBET	$O(\lvert V\rvert \times (D_\tau^{\phi_\tau - k} + \lvert V\rvert + \lvert E_W\rvert) + N_\tau)$	$O(\binom{d_{\max}^{\phi_\tau}}{\lvert V_\tau\rvert} + \lvert V\rvert^2 + I)$

Here $N_\tau = \binom{|V|}{|V_\tau|}$, $D_\tau = \binom{d_{\max}}{\tau.d_e}$ and $I = \sum_{s\in V}\sum_{\tau'\in B_k}\binom{d_{\max}^{\phi_{\tau'}}}{|V_{\tau'}|-1}$.

first level of the MOD-Index, hence the 1st materialization level is enough for them. For motif  and , their motif-orbits are on the second and the third levels of the MOD-Index, hence the best performance gain is noticed on the second materialization level or the third materialization level. This trend is more clear for , since it is more frequent in these datasets. In general, the cost is acceptable when considering the speedup of the query time, especially for the first materialization level.

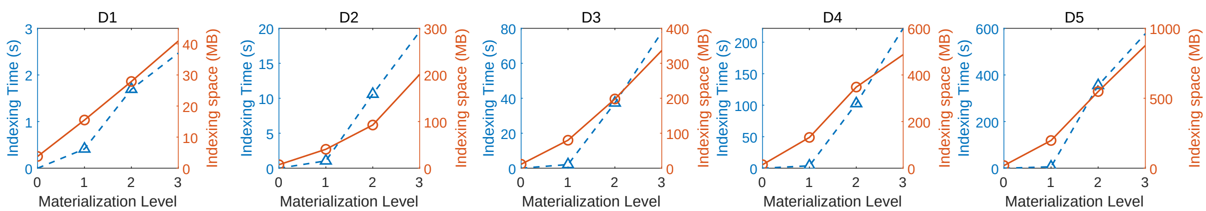


Fig. 2. MOD-Indexing time and space cost as graph density grows.

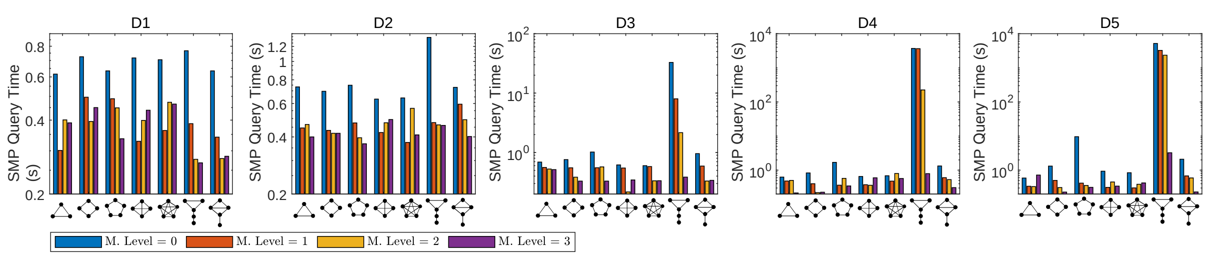


Fig. 3. Shortest motif-path query time on graphs of different density and materialization levels (M. Level).

Next, we report the detailed running time of Table 5 and Table 6 of the paper in Table 2 and Table 3 respectively. The algorithms with the defragmentation approach in [2] are also included, i.e., MKI-c, MGD-c and MLGC-c. We also report the results of MLGC-c as the value of $\hat{k}$ varies in Fig. 4, which is not included in the paper for the limited space. Finally, we report more effectiveness results of using different motifs in link prediction. As shown in Fig. 5, we also involve MCN, which is not included in the paper for the limited space. Results show that MCN obtains similar results as MGD when serious fragmentation occurs (e.g., when  is used), but its running time is much higher and outperformed by the enhanced version of motif-path.

Table 2. MKI/MGD performance with AUC and running time reported. Numbers of top-3 highest AUC are marked bold.

Method	**GAVI**		**EXTE**		**DBLP**		**AMAZ**		**YOUT**	
	AUC	Time	AUC	Time	AUC	Time	AUC	Time	AUC	Time
CN	0.72	1.2ms	0.56	1.7ms	0.77	12.9ms	0.62	13.5ms	0.54	0.2s
JC	0.7	1.1ms	0.48	1.6ms	0.55	14.4ms	0.52	12.8ms	0.44	0.2s
AA	0.75	1.2ms	0.57	1.6ms	**0.81**	12.0ms	0.65	12.6ms	0.52	0.2s
PA	0.59	1.2ms	0.76	1.7ms	0.64	13.4ms	0.63	14.9ms	0.76	0.2s
FM	0.65	1.2ms	0.65	1.5ms	0.59	14.0ms	0.64	15.4ms	0.69	0.2s
HT	0.6	1.4ms	0.7	1.7ms	0.64	13.5ms	0.59	15.2ms	0.53	70.1s
RPR	0.61	1.5ms	0.51	1.6ms	0.76	12.7ms	0.62	13.4ms	0.48	72.0s
MCN	0.67	1.0s	0.62	3.2s	0.75	38.8s	0.61	2.4s	0.65	16.6m
MLP+GB	**0.89**	7.0m	**0.83**	76.9m	**0.82**	35.9m	**0.72**	1.4m	**0.83**	70.4h
KI	0.69	36.2ms	0.6	0.5s	0.69	2.2s	0.6	97.5ms	0.63	1.1m
MKI	0.71	0.1s	0.66	1.2s	0.74	23.5s	0.63	6.1s	0.66	5.7m
MKI-c	0.62	6.4m	0.67	18.0m	-	-	-	-	-	-
MKI-b	**0.76**	0.1s	**0.87**	2.0s	**0.75**	31.2s	**0.73**	10.6s	**0.76**	7.5m
GD	0.5	2.2ms	0.5	3.1ms	0.5	23.0ms	0.5	26.7ms	0.5	1.8s
MGD	0.67	50.1ms	0.63	0.2s	0.65	2.5s	0.66	1.8s	0.55	1.4m
MGD-c	0.52	1.3m	0.51	3.2m	-	-	-	-	-	-
MGD-b	**0.75**	32.7ms	**0.84**	30.6ms	0.72	2.8s	**0.81**	0.9s	**0.87**	1.8s

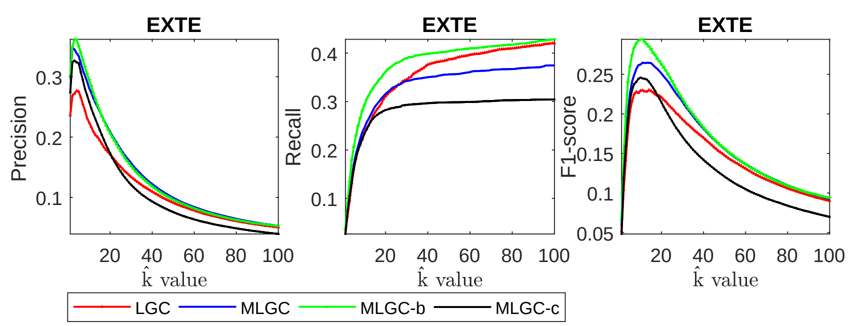


Fig. 4. Local graph clustering results on EXTE, with Precision, Recall and F1-score reported.

REFERENCES

[1] Xiaowei Chen, Yongkun Li, Pinghui Wang, and John Lui. 2016. A general framework for estimating graphlet statistics via random walk. *Proceedings of the VLDB Endowment* (2016).

Table 3. MLGC performance with F1-score and running time reported. Numbers of top-3 highest F1-score are marked bold.

Method	**GAVI**		**EXTE**		**DBLP**		**AMAZ**		**YOUT**	
	F1	Time	F1	Time	F1	Time	F1	Time	F1	Time
TECTONIC	0.39	7.0s	**0.44**	24.6s	-	-	0.37	-	-	-
MAPPR	0.39	0.2s	**0.42**	0.1s	**0.34**	5.1s	0.35	4.4s	0.15	16.8s
EdMot	0.33	21.1s	0.38	1.1m	-	-	-	-	-	-
LGC	**0.42**	9ms	0.36	12.2ms	0.33	1.3s	**0.63**	89.6ms	**0.17**	0.7s
MLGC	**0.41**	3.1ms	0.3	8.7ms	**0.35**	1.3s	**0.59**	7.8ms	**0.16**	8.7s
MLGC-c	0.39	23.1s	0.29	1.1m	-	-	-	-	-	-
MLGC-b	**0.42**	3.7ms	**0.38**	9.9ms	**0.35**	1.5s	**0.65**	8.8ms	**0.23**	13.8s

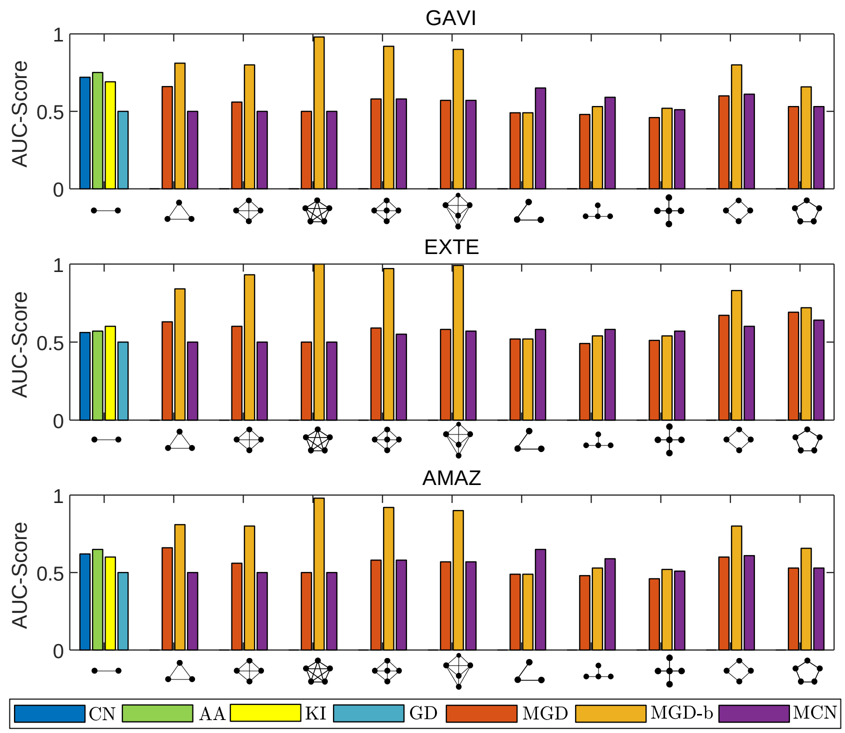


Fig. 5. Evaluation of generic motifs (edge, cliques, quasi-cliques, stars and cycles) for link prediction on GAVI, EXTE and AMAZ.

[2] Pei-Zhen Li, Ling Huang, Chang-Dong Wang, and Jian-Huang Lai. 2019. EdMot: An Edge Enhancement Approach for Motif-aware Community Detection. In *Proceedings of the 25th ACM SIGKDD*. 479–487.
[3] Sridevi Kamla Maharaj. 2018. *Graphlet Analysis Of Networks*. Ph.D. Dissertation. UC Irvine.