Information Sciences 644 (2023) 119271

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Two-stage adaptive differential evolution with dynamic dual-populations for multimodal multi-objective optimization with local Pareto solutions

Guoqing Li [a,*], Wanliang Wang [b], Caitong Yue [c], Weiwei Zhang [d], Yirui Wang [a]

[a] Faculty of Electrical Engineering and Computer Science, Ningbo University, Ningbo 315211, China
[b] College of Computer Science and Technology, Zhejiang University of Technology, Hangzhou 310023, China
[c] School of Electrical and Information Engineering, Zhengzhou University, Zhengzhou 450001, China
[d] College of Computer and Communication Engineering, Zhengzhou University of Light Industry, Zhengzhou 450001, China

ARTICLE INFO

Keywords:
Multimodal multi-objective optimization
local Pareto solutions
Dynamic dual-populations
Adaptive differential evolution
Two-stage evolution

ABSTRACT

Several distinctive Pareto Sets (PSs) with an identical Pareto Front (PF) and local PSs with acceptable quality are comprised in multimodal multi-objective optimization problems (MMOPs). Recently, many multimodal multi-objective evolutionary algorithms (MMEAs) have been proposed. However, even though most of MMEAs have the ability to discover equivalent global PSs, these methods encounter failures in developing local PSs. The main reasons are that local PSs are dominated by global PSs and are removed from the population during the evolutionary process. To tackle this matter, a two-stage adaptive differential evolution with a dynamic dual-populations strategy, termed TADE_DDS, is developed. In TADE_DDS, a dynamic population strategy is put forward to divide the population into a global population that locates equivalent global PSs and a local population that aims to locate local PSs. Subsequently, the whole procedure is completed by two evolutionary stages associated with a dynamic population strategy, and an adaptive differential evolution algorithm is adopted for both global and local populations. The first-stage evolution aims to find more favorable local PSs and the second-stage evolution concentrates on finding a variety of global PSs. Additionally, a local environmental selection and a global environmental selection are performed for developing the diversity of local PSs and improving the convergence of global PSs and local PSs, respectively. TADE_DDS and several popular MMEAs are implemented on standard test problems. Experimental results demonstrate that TADE_DDS is equipped to locate both global and local PSs, and is superior to its competing algorithms.

1. Introduction

Multiple conflicting objectives are optimized simultaneously in multi-objective optimization problems (MOPs). One objective value that is optimal may lead to poor performance for the other objectives. Therefore, a trade-off solution set is desired for MOPs and is defined as Pareto optimal solutions. The decision vectors and objective vector of solutions are termed Pareto set (PS) and Pareto Front (PF), respectively. Previously, plentiful multi-objective evolutionary algorithms (MOEAs) [1,2,3] are remarkable and have obtained promising performance when dealing with various kinds of MOPs. These MOEAs have an emphasis on finding PF with promising

* Corresponding author.
E-mail address: li241700@126.com (G. Li).

https://doi.org/10.1016/j.ins.2023.119271
Received 27 October 2022; Received in revised form 24 May 2023; Accepted 27 May 2023
Available online 8 June 2023

convergence in objective space and have achieved the superior performance for solving MOPs. However, diverse PSs have rarely been studied in MOPs since there is only one PS corresponding to PF.

Given such a scenario, there is a point on the PF preferred by the decision-maker corresponding to a decision vector on PS that is not sufficient for their demands. In this case, diverse and equivalent PSs are essential. Fortunately, researchers have already concentrated on PS's diversity and proposed multimodal multi-objective optimization problems (MMOPs) [4,5]. There are multiple equivalent PSs with identical PF and local PSs for MMOPs. Two equivalent global PSs and a local PS with an acceptable threshold δ are exhibited as an example of MMOPs in Fig. 1. MMOPs cases can be found frequently in feature selection [6,7], path planning [8], and diet design [9]. Traditional MOEAs have poor performance in dealing with MMOPs since they only find one equivalent PS, and they are hard to solve MMOPs. Recently, several multimodal MOEAs (MMEAs) have been designed in succession, and they have shown promising performance in MMOPs. For instance, ring-based MMEA [10], clustering-based MMEAs [11], decomposition-based MMEAs [12], weighted indicator-based MMEAs [13], and knee-based MMEAs [14]. These mentioned MMEAs are available for obtaining diverse PSs. Subsequently, several MMEAs [15,16,17] that balance the distribution of PS and PF are suggested, which promotes the exploitation of MMEAs. Furthermore, several special MMOPs are also involved, such as imbalanced MMOPs [18,19] and large-scale sparse MMOPs [20], and some effective MMEAs are proposed for solving such MMOPs. The main purpose is to discover global PSs and maintain a balance between PS and PF for these proposed MMEAs.

Regrettably, local PSs involved in MMOPs are overlooked for the aforementioned MMEAs. Local PSs are critical in one case where equivalent global PSs are not found, and local PSs with acceptable quality are preferable. For example, A_1 and A_2 are two equivalent Pareto solutions, and A_3 is a local Pareto solution in Fig. 1. Given this scenario that two equivalent Pareto solutions (A_1 and A_2) are not found by MMEAs, A_3 is a preferable solution. However, A_3 is difficult to exploit and maintain for MMEAs. There are several main reasons for this case. Firstly, two global Pareto solutions (A_1 and A_2) dominate the local Pareto solution A_3, and the local Pareto solution A_3 quickly converges to A_1 in the evolution procedure. Secondly, numerous local Pareto solutions similar to A_3 are developed which give rise to the population settling into a local optimal area, and many global Pareto solutions (i.e. A_1 and A_2) are not located. Finally, the local Pareto solution with acceptable quality (A_3) is hard to evaluate in terms of all solutions that are dominated by equivalent global PSs. In this case, searching for local PSs is therefore a meaningful and challenging study. To the best of our knowledge, several recently proposed MMEAs have been conceived to solve MMOPs with local PSs. A dual-clustering MMEA is developed to solve several MMOPs with local PSs [21], a modified double-niched evolutionary algorithm [22] has been proposed to search for polygon-based MMOPs with local PS, and a clearing-based MMEAs [23] is devised to locate local PSs. However, there is enormous potential for improving the performance of locating local PSs. Particularly, it is a tremendous challenge to balance the search capability and maintain a predominant distribution of global and local PSs.

As a consequence, previous MMEAs for dealing with MMOPs with local PS are a difficult task as it involves locating both global and local PSs. To deal with this matter, we develop a two-stage adaptive differential evolution with dynamic dual-populations strategy (TADE_DDS) for solving MMOPs with local Pareto solutions. TADE_DDS is not only capable of exploiting diverse PSs, but also providing a high performing distribution of unique PSs, simultaneously. Three main contributions are included as follows:

1) Dynamic population approach is involved in TADE_DDS, and the population is divided into a global population that is responsible for developing equivalent global PSs and a local population that is concerned with exploiting local PSs.
2) The evolutionary process is subdivided into two stages. The global population decreases and the local population increases to focus on developing local PSs in the first-stage evolution. Instead, the global population increases and the local population decreases for the purpose of exploiting diverse PSs and maintaining local solutions in the next stage of evolution. Adaptive differential evolution, similar to the successful history parameter adaptation, is used for the global population and local population, respectively.

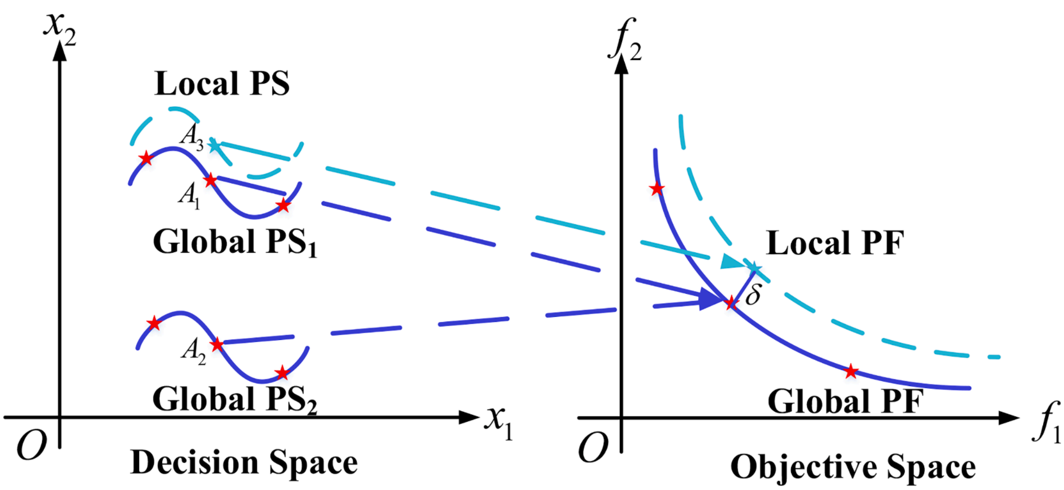


Fig. 1. A practical demonstration of MMOPs.

3) Two diverse environmental selection operators, termed local environment selection and global environment selection respectively, are associated with two stages of evolution. The local environmental selection involved in the first-stage evolution adopts a clustering-based technique to select favorable local individuals for developing local PSs. Meanwhile, the global environment selection operation is performed in the second-stage evolution for selecting various solutions with superior convergence.

The remaining sections of this paper are organized below. Related works are presented in section 2. Section 3 shows the general framework of TADE_DDS and Section 4 discusses and analyzes experimental results. Section 5 concludes this paper and analyses future work.

2. Related works and motivation

In this section, we present several relevant definitions, differential evolutionary algorithms, and research on multimodal multi-objective evolutionary algorithms. Furthermore, we also analyze the motivation of this paper.

2.1. Definition of MMOPs

Recently, MMOPs have been widely studied and a specific definition is given below [24]:

Definition 1. In MOPs, there may be multiple equivalent and distinct PSs with the same PF. The special kind of MOPs is defined as MMOPs.

Definition 2. Two distinct individuals X and $X^{'}$ are viewed as equivalent if $\|f(X) - f(X^{'})\| \leqslant \delta$, where δ is a threshold.

In Definition 2, $\delta = 0$ suggests that global solutions are involved in MMOPs and local solutions are not involved in MMOPs. Conversely, $\delta > 0$ indicates that there are both global Pareto optimal set (global PS) and local Pareto optimal set (local PS), simultaneously.

Additionally, the definition of both global PS and local PS is given below:

Definition 3. For an arbitrary solution X in G_S, if there is no solution dominating X, then the solution set G_S is called global PS.

Definition 4. For an arbitrary solution X in L_S, if there is no arbitrary solution $X^{'}$ and $\|f(X) - f(X^{'})\| \leqslant \delta$, then the solution set L_S is called local PS.

2.2. Differential evolution

Differential evolution (DE) [25] is widely employed in solving optimization problems, consisting of initialization, mutation, crossover, and selection operations. First, an individual x in the population Pop with N individuals is initialized as below:

$$x = L + \sigma \times (U - L) \tag{1}$$

where L and U are the lower and upper boundaries of decision vectors, and $\sigma \in [0, 1]$ is a random number. Subsequently, mutation, crossover, and selection are performed in the following manner until the iteration-stopping criteria are met.

1) Mutation: For each individual x , the mutation vector v based on DE/rand/1/bin and several mutation operators are generated as follows:

$$v = x_{r1} + F \times (x_{r2} - x_{r3}) \tag{2}$$

$$v = x_{r1} + F \times [(x_{r2} - x_{r3}) + (x_{r4} - x_{r5})] \tag{3}$$

$$v = x_{best} + F \times (x_{r1} - x_{r2}) \tag{4}$$

$$v = x_{best} + F \times [(x_{r1} - x_{r2}) + (x_{r3} - x_{r4})] \tag{5}$$

$$v = x + F \times (x_{r1} - x_{r2}) + F \times (x_{pbest} - x) \tag{6}$$

where x_{r1}, x_{r2}, x_{r3}, x_{r4}, and x_{r5} are random individuals that are different from each other and not equal to x, F is a mutation factor. x_{best} and x_{pbest} indicate the best individual and the best individual among several individuals, respectively.

2) Crossover: After the mutation operation, the individual x and the mutation vector v are subjected to the crossover operator to generate a trial vector u. Binary crossover is widely used and implemented as follows.

$$u = \begin{cases} v \text{ if } (r \leqslant Cr) \text{ or } j = j_{rand} \\ x \text{ otherwise} \end{cases} \tag{7}$$

where Cr is the crossover rate, $j_{rand} \in [1, d]$ is a random decision variable index, and d is the number of decision variables.

3) Selection: The superior individual between the individual x and trial vector u is maintained in the selection operator for a

minimization problem according to Eq. (8).

$$x = \begin{cases} u \text{ if } f(u) \leqslant f(x) \\ x \text{ otherwise} \end{cases} \tag{8}$$

where $f(u)$ and $f(x)$ are the objective values of the individual x and trial vector u, respectively. Additionally, the selection operation in MOPs is modified as follows since it is not available to compare multi-objective values, simultaneously.

$$x = \begin{cases} u \text{ if } u \prec x \\ x \text{ otherwise} \end{cases} \tag{9}$$

where $u \prec x$ indicates the individual x is dominated by trial vector u.

2.3. Previous studies in MMOPs

Recently, Liang [4] proposed the concept of MMOPs, and Tanabe [24] introduced a standard definition and review of MMOPs. MMOPs attracted the attention of researchers and proposed many MMEAs. DE and PSO were the most prominently used to solve MMOPs. Among them, several self-organizing based MMEAs [26,27,28], were presented in sequence. The self-organizing network was trained to obtain niche-based neighbors, and superior global PSs were obtained. Also, grid search-based and zoning search-based MMEAs [29,30] provided superior performance for solving MMOPs. Likewise, several MMEAs involved DE were relatively outstanding, including MMEAs with DE [31], clustering-based MMEAs using DE [32], and improved MMEAs using DE [33]. Such MMEAs adopted diverse niching strategies that were involved in multimodal optimization to search for global PSs.

Similarly, there were indicator-based MMEAs, decomposition-based MMEAs, and preference-based MMEAs. A niching indicator-based MMEAs [34] that perform the fitness calculation between the offspring and its closest individual and weighted indicator-based MMEAs [13] which combine diverse information of solutions in dual space are suggested. Subsequently, inspired by decomposition-based MOEAs, a similar framework [35] was designed to solve MMOPs, and decomposition-based MMEAs in dual space [36] were proposed to survive diverse solutions. Furthermore, Wang proposed preference-inspired MMEAs [37], in which the coevolutionary framework was adopted and a dual-diversity archive was introduced. In comparison to DE-based and PSO-based MMEAs, these MMEAs have more emphasis on obtaining diverse solutions in dual space when searching for equivalent global PSs.

The main purpose of the mentioned-above MMEAs was to locate equivalent global PSs and to maintain diverse solutions. In this way, several local PSs were missing. Very recently, a few MMEAs were specifically devised to handle MMOPs with local PSs. Liu [22] proposed a Pareto-based ranking front strategy to locate multiple layers and adopted the double-niched method to develop local PS and PF. Lin [21] used a dual clustering strategy in which one cluster developed local PSs and the other cluster found diverse Pareto solutions in objective space to solve MMOPs with local Pareto solutions. Additionally, a layer-based evolutionary algorithm associated with a dynamic clearing strategy was designed [23] to develop local PS. These algorithms were capable of locating local PSs and obtaining promising performance on different categories of benchmarks in comparison to several MMEAs that were devoted to solving MMOPs with only equivalent global PSs. It is worth noting that these algorithms focus on searching for equivalent global PSs and local PSs simultaneously, but they were hard to balance the ability to search for various kinds PSs during the evolutionary process. The main reason is that MMEAs that focus on searching for equivalent global PSs inevitably lead to missing local PSs and that concentrating on searching for local PSs may sacrifice the performance of equivalent global PSs. It is therefore necessary for MMEAs to balance the performance of global PSs and local PSs in solving MMOPs with local PSs, simultaneously.

2.4. Motivation

Superior MMEAs require the following three capabilities:1). Diverse PS including global and local PSs are simultaneously located by MMEAs. 2). Balancing the exploitation of diverse kinds PSs during the evolution procedure. 3). Maintaining the convergence and diversity of PSs in dual space, simultaneously. However, most of the aforementioned MMEAs are only able to discover global solutions, yet they overlook local PS since global PSs dominate local PSs and local PSs converge quickly to global PSs. Furthermore, numerous local PSs may also influence the performance of MMEAs because the local PSs trap in the local optimum region and find a few global PSs. Although a few MMEAs are specifically designed to tackle MMOPs with local PS, they are difficult to balance in developing and maintaining the performance of all PSs including both global and local PSs. Thereby, this paper suggests a two-stage adaptive DE with a dynamic dual-populations strategy for addressing MMOPs with local PS. In the proposed MMEAs, the global population and local population are responsible for developing and balancing the exploitation of all kinds PSs. Sustainably, global environmental selection and local environmental selection maintain the convergence and diversity of solutions. The detailed general framework of the proposed algorithm and its components are presented in Section 3.

3. Proposed TADE_DDS

In this section, we concentrate on describing the framework of TADE_DDS and its several components. Furthermore, the computational complexity of the proposed algorithm is also discussed.

3.1. General framework of TADE_DDS

Algorithm 1 presents the framework of TADE_DDS in detail. TADE_DDS mainly involves four components, including dynamic dual-populations strategy, adaptive differential evolution, local environmental selection strategy, and global environmental selection strategy. Firstly, the population *Pop* with *N* individuals is randomly initialized in line 1, and the current evolutionary iteration number *g* is set to 0 on line 2. Then, the evolutionary procedure is performed on lines 3–14. The whole evolutionary process is divided equally into two evolutionary stages according to the current evolutionary iteration number *g*. In the first-stage evolution on lines 4–7, the first-stage dynamic dual-population strategy divides the population *Pop* into a global population *GPop* and a local population *LPop* in line 5. Subsequently, an adaptive differential evolution strategy on line 6 is performed, and a new population *Pop* is generated by local environmental selection on line 7. If the current iteration number *g* is more than or equal to half of the total iterations number $G_{\max}$, the second-stage evolution is implemented on lines 8–12. Notably, the population division and environmental selection in the second-stage evolutionary process are different from the first-stage evolutionary process. Particularly, two different approaches toward population division are described in section 3.2. We use the second-stage dynamic dual-population strategy to divide the population into two subpopulations in line 9. Then, the adaptive differential evolution strategy is performed on line 10, and a global environmental selection strategy is used to produce the new population *Pop* on line 11. Repeat the evolutionary process until the end of the iteration. Ultimately, the population *Pop* is derived as the final Pareto optimal solution on line 15.

Algorithm 1. TADE_DDS

Input: $N, G_{\max}$
Output: *Pop*
Pop ← initialize population with *N* individuals randomly;
$g = 0$
while $g < G_{\max}$ **do**
if g is less than the half of $G_{\max}$ **do**
$\{GPop, LPop\}$ ← Divide the population using the first-stage dynamic dual-population strategy;
$Off = AdaptiveDE(GPop, LPop)$
$Pop = LocalES(Pop, Off, N)$
else if g is more than or equal to the half of $G_{\max}$ **do**
$\{GPop, LPop\}$ ← Divide the population using the second-stage dynamic dual-population strategy;
$Off = AdaptiveDE(GPop, LPop)$
$Pop = GlobalES(Pop, Off, N)$
end
$g = g + 1;$
end
return *Pop*

3.2. Dynamic dual-populations strategy

MMOPs involve various PSs, and the challenge is how to discover and balance global PSs and local PSs, simultaneously. The dynamic dual-populations strategy is therefore proposed. The population is classified into two subpopulations that are named global population *GPop* and local population *LPop*, respectively. The global population is responsible for locating equivalent global PSs, and the local population is recommended for finding local PSs. Here, we first introduce the first-stage dynamic dual-population strategy. In the first-stage evolution, the global population size N_G and the local population size N_L are calculated as follows:

$$N_G = \left\lceil \frac{(N_{min} - N) \times g}{G_{\max}} + N \right\rceil \tag{10}$$

$$N_L = N - N_G \tag{11}$$

where g and $G_{\max}$ are the current iteration number and the total iterations number, respectively, N_{min} is the minimum value of N_G, and set to $\lceil 0.1 \times N \rceil$, and is the ceiling function.

To balance the distribution of two subpopulations, a local diversity-based indicator is employed to separate the population into a global population and local populations. The local diversity value of $x (x \in Pop)$ in line 5 is defined below:

$$I_d(x) = \frac{1}{1 + \alpha \times \frac{\sum_{i=1}^{k} dis(x, z_{i,dec}(x))}{M_{dec}} + \beta \frac{\sum_{i=1}^{k} dis(f(x), z_{i,obj}(f(x)))}{M_{obj}}} \tag{12}$$

$$M_{dec} = \frac{\sum_{x \in Pop}^{N} \sum_{j=1}^{k} dis(x, z_{j,dec}(x))}{N}, M_{obj} = \frac{\sum_{x \in Pop}^{N} \sum_{j=1}^{k} dis(f(x), z_{j,obj}(f(x)))}{N} \tag{13}$$

$$\alpha = \beta = e^{(1-\lambda)} \tag{14}$$

where $\sum_{i=1}^{k} dis(x, z_{i,dec}(x))$ and $\sum_{i=1}^{k} dis(f(x), z_{i,obj}(f(x)))$ indicate the distances between the individual x and its k nearest neighbors in dual space, respectively; M_{dec} and M_{obj} are the mean distances of $\sum_{i=1}^{k} dis(x, z_{i,dec}(x))$ and $\sum_{i=1}^{k} dis(f(x), z_{i,obj}(f(x)))$, respectively; $z_{i,dec}(x)$ and $z_{i,obj}(f(x))$ are the i-th nearest neighbors of the decision vector x and objective vector $f(x)$, respectively; α and β are the weighted value of M_{dec} and M_{obj}, respectively; λ is Pareto ranking layer of the individual x.

The I_d metric reasonably estimates the local diversity of all individuals, and smaller I_d values are desired. Additionally, $e^{(1-\lambda)}$ is involved in Eq. (12), which indicates that $I_d(x)$ also weakly focuses on the convergence of each individual. N_G individuals with better I_d values are defined as the global population, and the remaining N_L individuals are defined as the local population. Notably, the global population and local population are not the same as global PSs and local PSs. On the contrary, the global population is responsible for developing global PSs and the local population is designed for exploring local PSs. In this manner, the global population size declines and the local population size increases gradually in the first-stage evolution according to Eq. (10) and Eq. (11).

Next, we present the second-stage dynamic dual-population methodology involved in the second-stage evolution. In Eq. (11), the local population size increases linearly for the purpose of locating local PSs, resulting in it being difficult to find diverse PSs. To exploit global PSs and maintain local PSs, N_G in the second-stage evolution is varied and calculated by the following:

$$N_G = \lceil N \times \log_2(1 + \frac{g}{G_{\max}}) \rceil \tag{15}$$

Similarly, N_L is calculated in an identical manner as Eq. (11). It is observed that the minimum value N_G is larger than $0.5 \times N$ and is continuously increasing. To locate desired global PSs, the population *Pop* are sorted according to the Pareto ranking. The first N_G individuals are defined as the global population, and the remaining individuals are defined as the local population. If so, the population is also divided into a global population and a local population by different methodologies.

3.3. Adaptive differential evolution

Several DE algorithms have been devised to solve MMOPs and obtain remarkable performance. In view of this case, an adaptive differential evolution, named AdaptiveDE, in the proposed TADE_DDS is performed on Algorithm 2 to update the global population and local population in the same manner, independently. Firstly, each individual x in the global population and $x^{'}$ in the local population performs the mutation by Eq. (6) to generate the mutation vectors v and $v^{'}$ on line 3, respectively. Next, the crossover operator is performed by Eq. (7) to produce the trial vector u and $u^{'}$ on line 4, respectively. Then, the offspring individuals o and $o^{'}$ in the selection operator are generated by Eq. (9) on line 5. Subsequently, F and Cr involved in the mutation operator crossover operator are updated dynamically on line 8. The adaptive parameter methodology is the key to adaptive differential evolution, and we adopt a successful-history-based parameter adaptation strategy [38,39] to update the parameters F and Cr.

Algorithm 2. AdaptiveDE

Input: *GPop*,*LPop*
Output:*Off*
$Off = \varnothing$;
for each individual x in *GPop* and each individual $x^{'}$ in *LPop* **do**
$\{v, v^{'}\} \leftarrow$ mutation operator
$\{u, u^{'} \backslash\} \leftarrow$ crossover operator;
$\{o, o^{'}\} \leftarrow$ selection operator;
$Off = \{Off \cup o \cup o^{'}\}$;
end
$\{F, Cr\} \leftarrow$ adaptive update the parameters F and Cr;
return *Off*

3.4. Local environmental selection

The primary goals of environmental selection are to develop diverse PSs and maintain local PSs in decision space. Environmental selection is also split into two stages accompanied by the evolutionary process. In Algorithm 3, the local environment selection in the first-stage evolution, termed *LocalES*, is to find plenty of local PS and few global PS. The global environmental selection in the second-stage evolution, named *GlobalES*, is to locate diverse PSs and maintain local PSs. Such that diverse solutions are sustained in the populations.

In Algorithm 3, the parents and offspring are partitioned into *Num* subpopulations using affinity propagation clustering (APClustering) [40] on line 2. For each subpopulation C_i, we also devise a methodology in Algorithm 4 to seek local Pareto solutions on lines 3–5, marked as *SeekLocalSolution*. Subsequently, all local solutions $LS = \{LS_1, LS_2, ..., LS_{Num}\}$ are developed by *SeekLocalSolution*, and then the diversity of *LS* is evaluated by Eq. (12) on line 6. Ultimately, the first N individuals with superior diversity value form a new population *Pop* on line 7. If so, plenty of local solutions with acceptable quality and few global equivalent PSs in Algorithm 3 are maintained in *Pop*.

In Algorithm 4, suppose that $x (x \in C_i)$ is not dominated by any individual whose distance between each individual x and other individuals in C_i is less than θ, and x is denoted as a local solution. θ is first calculated on line 2 by Eq. (6):

$$\theta = \frac{1}{\sqrt{2}\pi \times (1 - e^{(1-d)})} \times (\prod_{l=1}^{d}(x_{l,max} - x_{l,min}))^{1/d} \tag{16}$$

where $x_{l,max}$ and $x_{l,min}$ indicate the maximum and minimum values of l-th decision vector in the subpopulation C_i, respectively, d denotes the decision variables number. For each subpopulation C_i, the set of these individuals similar to the individual x is marked as local Pareto solutions LS_i, and the remaining individuals are labeled as poor solutions DS_i on lines 3–10. Fig. 2 shows an example of locating local Pareto solutions. As shown in Fig. 2, three individuals (a_1, a_3, and a_4) are dominated by a_2, but a_3 is dominated by a_2 within θ and the other two individuals (a_1 and a_4) are not dominated by a_2 within θ. Therefore, the global Pareto solution (a_2) and two local Pareto solutions (a_1 and a_4) are found. In this case, even though the Pareto optimal solutions dominate several individuals, these individuals that similar to a_1 and a_4 are still considered as the local solutions.

Algorithm 3. LocalES

```
Input: Pop, Off,N
    Output:Pop
    LS = ∅;
    {C_1, C_2, ...C_i, ..., C_Num} = APClustering(Pop, Off);
    for i = 1 to Num do
    {LS_i, DS_i} = SeekLocalSolution(C_i);
    end
    I_d← evaluate the diversity of LS by Eq. (12);
    Pop← the first N individuals with better I_d values in LS;
    return Pop
```

Algorithm 4.*SeekLocalSolution*

```
Input:C_i
    Output: LS_i,DS_i
    LS_i = DS_i = ∅;
    θ← calculate the θ value by Eq. (16);
    for each individual x ∈ C_i do
    Nb← find several individuals whose distance between x and other individuals in C_i is smaller than θ;
    if any individual in Nb not dominate x do
    LS_i = {LS_i ∪ x};
    else
    DS_i = {DS_i ∪ x};
    end
    end
    return LS_i,DS_i
```

3.5. Global environmental selection

The main aim of global environmental selection is to exploit further equivalent global PSs and maintain available local PSs. Distinct from the local environmental selection, the whole population *Pop* instead of multiple subpopulations consistently performs the local

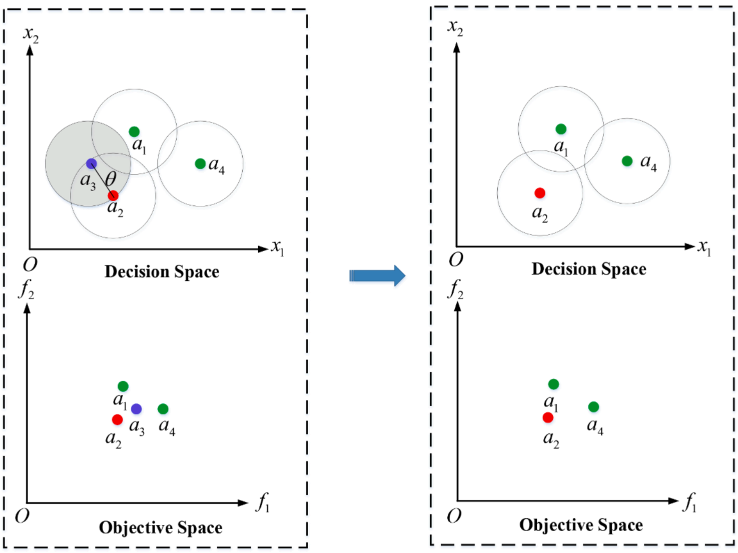


Fig. 2. An example of locating local Pareto solutions.

solution selection strategy in Algorithm 5 and finds local solution sets LS on line 1. If the size of LS is less than N, we propose a novel approach, termed as *ReselectLS*, to reselect several promising local solutions in DS on lines 2–4. If the size of LS is still more than N, the local solution selection strategy is executed again to ensure that the population size is N on lines 6–8. Ultimately, N individuals are maintained in the population *Pop* on line 9.

It integrates local convergence quality and local diversity quality in Algorithm 6 to balance the distribution of each individual in DS. Firstly, the local diversity quality is calculated similarly to Eq. (12) on lines 2–3. It is noted that α and β involved in Eq. (12) are set $\alpha = 1$ and $\beta = 1 - e^{(1-\lambda)}$, respectively. The main purpose of this is that convergence is more emphasized in global environmental selection. Then, the local convergence quality is measured on line 4 by Eq. (17):

$$I_c(x) = \frac{\sum_{x \in DS, y \in DS}^{|DS|} R(x, y)}{|DS|} \tag{17}$$

$$R(x, y) = \begin{cases} -1 \text{ if } x \prec y \& dis(x, y) < \theta \\ 1 \text{ if } y \prec x \& dis(x, y) < \theta \\ 0 \text{ otherwise} \end{cases} \tag{18}$$

where $x \prec y (y \prec x)$ indicates that x dominates y(y dominates x), θ is calculated by Eq. (16). Specifically, the I_c and I_d are normalized on line 6, and the sum of I_c and I_d is denoted as the local convergence quality I on line 7. Smaller I values are desired. Finally, sorting I value and the first N' individuals with better I value are used as the final solutions on lines 8–9. In this case, N solutions are maintained in the global environment selection operation.

To demonstrate the environment selection operator, the obtained solutions in dual space are displayed in Fig. 3. It is clear that lots of local PSs and PF and few global PSs and PF are maintained at the end of the local environment selection. In local environmental selection, poor local solutions are dominated within θ , and few superior global PSs and many local Pareto solutions within each subpopulation are maintained. Continuously, many global PSs and PF are found at the beginning of the global environment selection associated with increasing global population size. Finally, superior global PSs and PF are explored and local PS and PF are maintained at the end of the global environment selection. Thus, these solutions with favorable convergence and diversity are maintained.

Algorithm 5. GlobalES

Input: *Pop, Off,N*
Output:*Pop*
$\{LS, DS\} = SeekLocalSolution(\{Pop, Off\});$

(*continued on next page*)

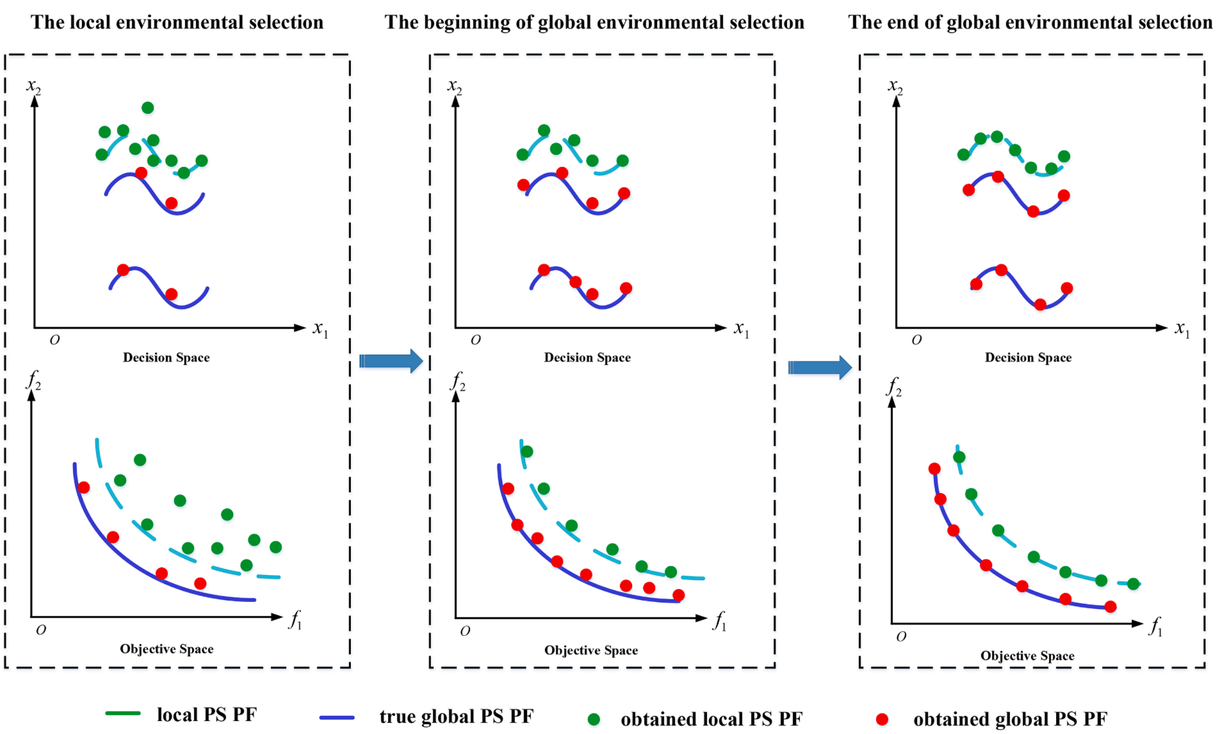


Fig. 3. An example of environmental selection processes.

Table 1
The 1/PSP value of different algorithms.

Problems	MO_Ring_ PSO_SCD	MMODE_ CSCD	MMO_ SO_QPSO	MMEAWI	MPMMEA	MMONBSA	MMOEA /DC	CEA-LES	TADE_DDS
MMF1	3.00E-02 (1.20E-03) +	2.49E-02 (9.46E-04) +	2.30E-02 (1.29E-03) +	2.75E-02 (1.28E-03) +	3.99E-02 (4.32E-03) +	2.30E-02 (7.80E-04) +	2.60E-02 (9.95E-04) +	2.51E-02 (1.07E-03) +	**2.17E-02 (6.62E-04)**
MMF2	2.63E-02 (5.20E-03) +	1.04E-02 (2.54E-03) ~	1.73E-02 (3.48E-03) +	1.73E-02 (3.36E-03) +	2.62E-02 (9.34E-03) +	1.64E-02 (2.76E-03) +	**8.61E-03 (2.29E-03)-**	1.68E-02 (2.95E-03) +	9.74E-03 (1.46E-03)
MMF4	1.60E-02 (9.41E-04) +	1.25E-02 (4.12E-04) -	**1.14E-02 (4.20E-04)-**	1.47E-02 (6.95E-04) +	4.27E-02 (1.03E-02) +	1.28E-02 (3.14E-04) ~	1.66E-02 (7.82E-04) +	1.48E-02 (5.38E-04) +	1.30E-02 (4.05E-04)
MMF5	5.53E-02 (3.41E-03) +	4.60E-02 (1.72E-03) +	4.10E-02 (2.43E-03) +	5.28E-02 (3.92E-03) +	7.30E-02 (8.06E-03) +	4.48E-02 (1.92E-03) +	4.87E-02 (2.39E-03) +	4.18E-02 (1.59E-03) +	**3.89E-02 (1.29E-03)**
MMF7	1.61E-02 (9.12E-04) +	1.33E-02 (8.57E-04) ~	**1.19E-02 (8.07E-04)-**	1.44E-02 (7.35E-04) +	2.31E-02 (4.21E-03) +	1.24E-02 (3.66E-04) -	1.61E-02 (4.56E-04) +	1.33E-02 (4.48E-04) ~	1.34E-02 (4.93E-04)
MMF8	4.02E-02 (1.80E-03) +	3.42E-02 (3.23E-03) +	3.74E-02 (3.68E-03) +	4.94E-02 (8.32E-03) +	1.95E-01 (7.34E-02) +	**2.73E-02 (2.04E-03)-**	3.47E-02 (3.84E-03) +	3.38E-02 (2.90E-03) ~	3.22E-02 (1.28E-03)
MMF10	1.74E-01 (1.94E-02) +	1.66E-01 (1.32E-02) +	1.68E-01 (1.29E-02) +	3.32E-02 (9.91E-02) +	2.14E-02 (3.26E-02) ~	1.62E-01 (2.78E-03) +	3.44E-02 (9.88E-02) +	6.89E-03 (4.99E-04) ~	**6.86E-03 (1.23E-03)**
MMF11	2.11E-01 (2.61E-02) +	2.12E-01 (2.97E-02) +	2.27E-01 (2.49E-02) +	4.54E-03 (2.86E-04) -	**4.61E-03 (4.63E-04) -**	2.18E-01 (2.79E-02) +	7.11E-03 (3.66E-04) -	8.28E-03 (5.09E-04) ~	8.00E-03 (7.36E-04)
MMF12	1.85E-01 (4.38E-02) +	2.05E-01 (4.90E-02) +	2.05E-01 (4.01E-02) +	**1.77E-03 (1.39E-04)-**	1.80E-03 (7.99E-04) -	2.16E-01 (4.24E-02) +	2.80E-03 (2.39E-04) ~	2.92E-03 (2.14E-04) +	2.76E-03 (1.67E-04)
MMF13	3.21E-01 (8.70E-02) +	3.86E-01 (1.12E-01) +	4.61E-01 (1.06E-01) +	**2.47E-02 (6.82E-04)-**	7.03E-02 (2.69E-02) +	3.84E-01 (1.16E-01) +	4.93E-02 (2.32E-03) +	4.80E-02 (2.08E-03) +	4.48E-02 (1.92E-03)
MMF14	4.60E-02 (1.17E-03) +	4.43E-02 (1.37E-03) +	3.82E-02 (5.04E-04) -	**3.70E-02 (7.27E-04)-**	5.37E-02 (3.56E-03) +	4.47E-02 (1.04E-03) +	4.59E-02 (1.13E-03) +	3.85E-02 (5.81E-04) -	4.34E-02 (8.98E-04)
MMF15	1.59E-01 (1.66E-02) +	1.44E-01 (1.51E-02) +	1.47E-01 (2.44E-02) +	**3.82E-02 (7.10E-04)-**	5.13E-02 (3.12E-03) -	1.52E-01 (1.67E-02) +	6.33E-02 (2.67E-03) +	5.05E-02 (1.35E-03) -	5.75E-02 (3.09E-03)
MMF1_e	3.98E-01 (1.33E-01) +	2.74E-01 (6.51E-02) +	2.76E-01 (4.65E-02) +	6.24E-01 (2.55E-01) +	5.79E + 00 (4.97E + 00) +	4.44E-01 (1.33E-01) +	4.08E-01 (1.71E-01) +	2.40E-01 (3.00E-02) +	**2.05E-01 (1.08E-02)**
MMF14_a	5.27E-02 (1.04E-03) +	5.37E-02 (1.53E-03) +	**4.30E-02 (4.16E-04)-**	4.91E-02 (9.72E-04) -	8.36E-02 (8.34E-03) +	5.07E-02 (1.09E-03) -	6.49E-02 (2.25E-03) +	4.70E-02 (5.85E-04) -	5.18E-02 (7.51E-04)
MMF15_a	1.67E-01 (1.31E-02) +	1.66E-01 (9.89E-03) +	1.63E-01 (1.72E-02) +	**4.76E-02 (1.44E-03)-**	7.07E-02 (7.98E-03) +	1.72E-01 (9.33E-03) +	9.21E-02 (9.64E-03) +	5.40E-02 (1.27E-03) -	6.47E-02 (2.61E-03)
MMF10_l	4.50E-01 (8.91E-01) +	1.62E-01 (4.33E-03) +	1.67E-01 (8.26E-03) +	6.17E + 00 (2.04E + 00) +	3.10E + 00 (2.68E + 00) +	1.62E-01 (2.32E-03) +	9.73E-03 (9.77E-03) +	**6.18E-03 (5.27E-04)-**	6.65E-03 (3.54E-04)
MMF11_l	3.19E-01 (3.33E-01) +	1.38E + 00 (9.53E-01) +	1.10E + 00 (6.00E-01) +	1.80E + 00 (1.34E-01) +	1.49E + 00 (2.42E-01) +	1.28E + 00 (7.54E-01) +	**4.90E-03 (7.62E-04)-**	5.59E-03 (2.37E-04) ~	5.68E-03 (3.54E-04)
MMF12_l	6.28E-01 (5.41E-01) +	1.98E + 00 (8.97E-01) +	8.16E-01 (7.20E-01) +	2.14E + 00 (1.86E-01) +	2.18E + 00 (2.05E-01) +	1.21E + 00 (9.55E-01) +	**2.14E-03 (1.27E-04)** ~	2.24E-03 (1.03E-04) ~	2.19E-03 (1.14E-04)
MMF13_l	3.34E-01 (1.04E-01) +	3.74E-01 (9.63E-02) +	4.79E-01 (9.25E-02) +	5.24E-01 (6.44E-03) +	5.15E-01 (1.06E-01) +	4.25E-01 (1.13E-01) +	1.29E-01 (8.14E-02) +	1.23E-01 (2.46E-03) +	**5.64E-02 (1.55E-03)**
MMF15_l	1.50E-01 (1.66E-02) +	1.39E-01 (1.67E-02) +	1.48E-01 (3.11E-02) +	5.76E-01 (5.09E-02) +	5.62E-01 (8.27E-02) +	1.48E-01 (2.30E-02) +	4.70E-02 (9.04E-04) -	**4.11E-02 (9.55E-04)-**	4.97E-02 (1.64E-03)
MMF15_a_l	1.58E-01 (1.11E-02) +	1.55E-01 (1.64E-02) +	1.60E-01 (1.90E-02) +	2.54E-01 (3.56E-03) +	2.63E-01 (2.00E-02) +	1.63E-01 (1.45E-02) +	8.11E-02 (1.34E-02) +	7.41E-02 (4.96E-03) +	**5.94E-02 (1.62E-03)**
MMF16_l1	1.05E-01 (7.27E-03) +	1.01E-01 (8.02E-03) +	1.03E-01 (1.14E-02) +	1.88E-01 (2.24E-03) +	1.93E-01 (2.22E-02) +	1.09E-01 (1.08E-02) +	4.85E-02 (1.95E-03) +	6.57E-02 (5.99E-03) +	**4.34E-02 (8.31E-04)**

(*continued on next page*)

Table 1 (*continued*)

Problems	MO_Ring_ PSO_SCD	MMODE_ CSCD	MMO_ SO_QPSO	MMEAWI	MPMMEA	MMONBSA	MMOEA /DC	CEA-LES	TADE_DDS
MMF16_l2	1.96E-01 (2.12E-02) +	1.81E-01 (1.96E-02) +	1.92E-01 (2.77E-02) +	7.57E-01 (6.70E-02) +	7.49E-01 (7.59E-02) +	2.01E-01 (1.77E-02) +	1.21E-01 (4.79E-03) +	1.42E-01 (2.02E-02) +	**5.03E-02 (1.33E-03)**
MMF16_l3	1.46E-01 (1.25E-02) +	1.40E-01 (1.10E-02) +	1.37E-01 (1.49E-02) +	2.77E-01 (6.20E-03) +	2.86E-01 (5.63E-03) +	1.47E-01 (1.05E-02) +	1.46E-01 (8.41E-02) +	8.52E-02 (6.78E-04) +	**5.31E-02 (2.43E-03)**
+/-/~	24/0/0	21/1/2	20/4/0	17/7/0	20/3/1	20/3/1	18/4/2	12/6/6	

(*continued*)

Algorithm 5. GlobalES

if $|LS|\langle N$ **do**
$N' = N - |LS|$;
$P_{conv} = ReselectLS(DS, N')$;
$LS = LS \cup P_{conv}$;
else if $|LS|\rangle N$ **do**
$\{LS, DS\} = SeekLocalSolution(LS)$;
end
$Pop = LS$;
return Pop

Algorithm 6. ReselectLS

Input: DS, N'
Output: P_{conv}
$I_d = I_c = \varnothing$;
for each individual $x \in DS$ **do**
$I_d(x) \leftarrow$ evaluate the diversity value of the individual x by Eq. (12);
$I_c(x) \leftarrow$ evaluate the local convergence value of the individual x by Eq. (17);
end
$\{I_d, I_c\} = normal(I_d, I_c)$;
$I = I_d + I_c$;
$I \leftarrow$ sort DS according to I values;
$P_{conv} = DS(1 : N')$;
return P_{conv}

3.6. Computational complexity

Dynamic dual-populations strategy, adaptive differential evolution, and local and global environmental selection strategy are associated with TADE_DDS. The computational complexity of the dynamic dual-populations strategy is $O(mN^2)$ since Pareto sorting is mentioned in the dynamic dual-populations strategy, where m and N denote objective number and population size, respectively. The complexity of adaptive differential evolution is $O(N)$. Furthermore, the environmental selection is composed of global and local environmental selection, and the two computational complexity are also $O(N)$. To sum up, the total complexity of TADE_DDS is $O(mN^2)$.

4. Experimental study

In this section, we set several associated experiments to validate the performance of the proposed algorithm in comparison to several competing algorithms. We also analyze the influence on the performance of TADE_DDS of dynamic double populations, environment selection, and population size.

4.1. Experimental setting

To validate the performance of TADE_DDS, a criterion MMOPs benchmark [41], including 24 test problems, is chosen as test problems. Particularly, the last 9 test cases have local PSs, and the remaining test problems merely have global equivalent PSs. Additionally, four evaluation matrices, denoted $1/PSP$, $1/HV$, *IGDX*, and *IGDF*, are available to assess the performance of all algorithms. $1/PSP$ and $1/HV$ indicate the reciprocal of Pareto Sets Proximity and Hypervolume, respectively. *IGDX* and *IGDF* show the distance between the true solution and obtained solution in both decision and objective space, respectively. Thereby, $1/PSP$ and *IGDX* reflect the performance of decision space, while HV and IGDF describe the performance of objective space. Smaller values of $1/PSP$, *IGDX*, $1/HV$, and *IGDF* are desirable.

To demonstrate the superior performance of TADE_DDS, eight popular MMEAs, including MMODE_CSCD [32], MMO_SO_QPSO [28], MMEAWI [13], MO_Ring_PSO_SCD [10], MPMMEA [20], MMONBSA [42], MMOEA/DC [21], and CEA-LES [23], are set to the competing algorithms of TADE_DDS. Among them, MMOEA/DC and CEA-LES are capable of solving MMOPs and locating outstanding

Table 2
The IGDX performance of different algorithms.

Problems	MO_Ring_PSO_SCD	MMODE_CSCD	MMO_SO_QPSO	MMEAWI	MPMMEA	MMONBSA	MMOEA/DC	CEA-LES	TADE_DDS
MMF1	2.98E-02 (1.20E-03) +	2.49E-02 (9.24E-04) +	2.29E-02 (1.29E-03) +	2.75E-02 (1.22E-03) +	3.98E-02 (4.18E-03) +	2.29E-02 (7.67E-04) +	2.60E-02 (9.83E-04) +	2.50E-02 (1.07E-03) +	**2.16E-02 (6.54E-04)**
MMF2	2.54E-02 (4.97E-03) +	1.04E-02 (2.54E-03) ~	1.68E-02 (3.40E-03) +	1.72E-02 (3.35E-03) +	2.51E-02 (7.81E-03) +	1.59E-02 (2.57E-03) +	**8.57E-03 (2.20E-03)-**	1.67E-02 (2.86E-03) +	9.70E-03 (1.42E-03)
MMF4	1.59E-02 (9.14E-04) +	1.25E-02 (4.15E-04) -	**1.14E-02 (4.21E-04)-**	1.47E-02 (6.90E-04) +	4.24E-02 (1.00E-02) +	1.27E-02 (3.05E-04) ~	1.66E-02 (7.79E-04) +	1.48E-02 (5.37E-04) +	1.29E-02 (4.02E-04)
MMF5	5.51E-02 (3.41E-03) +	4.58E-02 (1.71E-03) +	4.09E-02 (2.43E-03) +	5.27E-02 (3.89E-03) +	7.29E-02 (8.07E-03) +	4.47E-02 (1.94E-03) +	4.87E-02 (2.39E-03) +	4.18E-02 (1.59E-03) +	**3.88E-02 (1.28E-03)**
MMF7	1.61E-02 (9.02E-04) +	1.32E-02 (8.49E-04) ~	**1.19E-02 (7.96E-04)-**	1.44E-02 (7.33E-04) +	2.27E-02 (3.87E-03) +	1.24E-02 (3.59E-04) -	1.60E-02 (4.50E-04) +	1.33E-02 (4.47E-04) ~	1.34E-02 (4.86E-04)
MMF8	3.99E-02 (1.69E-03) +	3.37E-02 (3.01E-03) ~	3.72E-02 (3.68E-03) +	4.94E-02 (8.30E-03) +	1.93E-01 (7.32E-02) +	**2.72E-02 (1.99E-03)-**	3.46E-02 (3.73E-03) +	3.38E-02 (2.90E-03) ~	3.22E-02 (1.27E-03)
MMF10	1.67E-01 (1.16E-02) +	1.65E-01 (1.39E-02) +	1.67E-01 (1.32E-02) +	3.31E-02 (9.87E-02) +	2.14E-02 (3.26E-02) ~	1.62E-01 (2.84E-03) +	3.44E-02 (9.88E-02) +	6.89E-03 (4.99E-04) ~	**6.85E-03 (1.23E-03)**
MMF11	2.11E-01 (2.59E-02) +	2.12E-01 (2.97E-02) +	2.27E-01 (2.48E-02) +	**4.53E-03 (2.81E-04)-**	4.61E-03 (4.63E-04) -	2.17E-01 (2.79E-02) +	7.10E-03 (3.65E-04) -	8.28E-03 (5.09E-04) ~	7.98E-03 (7.35E-04)
MMF12	1.85E-01 (4.37E-02) +	2.05E-01 (4.90E-02) +	2.05E-01 (4.00E-02) +	**1.77E-03 (1.39E-04)-**	1.80E-03 (7.99E-04) -	2.16E-01 (4.24E-02) +	2.80E-03 (2.39E-04) ~	2.92E-03 (2.14E-04) +	2.76E-03 (1.67E-04)
MMF13	2.30E-01 (1.50E-02) +	2.35E-01 (2.09E-02) +	2.52E-01 (1.22E-02) +	**2.47E-02 (6.88E-04)-**	7.03E-02 (2.68E-02) +	2.35E-01 (1.84E-02) +	4.92E-02 (2.32E-03) +	4.80E-02 (2.09E-03) +	4.47E-02 (1.91E-03)
MMF14	4.60E-02 (1.17E-03) +	4.43E-02 (1.37E-03) +	3.82E-02 (5.04E-04) -	**3.70E-02 (7.27E-04)-**	5.37E-02 (3.56E-03) +	4.47E-02 (1.04E-03) +	4.59E-02 (1.12E-03) +	3.85E-02 (5.81E-04) -	4.34E-02 (8.98E-04)
MMF15	1.59E-01 (1.66E-02) +	1.44E-01 (1.51E-02) +	1.47E-01 (2.44E-02) +	**3.82E-02 (7.10E-04)-**	5.13E-02 (3.12E-03) -	1.52E-01 (1.66E-02) +	6.32E-02 (2.67E-03) +	5.05E-02 (1.35E-03) -	5.75E-02 (3.09E-03)
MMF1_e	3.56E-01 (9.66E-02) +	2.54E-01 (4.57E-02) +	2.65E-01 (3.92E-02) +	5.01E-01 (1.42E-01) +	2.08E + 00 (7.94E-01) +	4.00E-01 (9.90E-02) +	3.66E-01 (1.29E-01) +	2.36E-01 (2.61E-02) +	**2.00E-01 (9.63E-03)**
MMF14_a	5.26E-02 (1.04E-03) +	5.37E-02 (1.52E-03) +	**4.30E-02 (4.15E-04)-**	4.91E-02 (9.72E-04) -	8.36E-02 (8.34E-03) +	5.07E-02 (1.07E-03) -	6.49E-02 (2.26E-03) +	4.70E-02 (5.85E-04) -	5.18E-02 (7.51E-04)
MMF15_a	1.64E-01 (1.12E-02) +	1.66E-01 (9.96E-03) +	1.60E-01 (1.55E-02) +	**4.76E-02 (1.44E-03)-**	7.07E-02 (7.98E-03) +	1.71E-01 (8.96E-03) +	9.21E-02 (9.63E-03) +	5.40E-02 (1.27E-03) -	6.47E-02 (2.62E-03)
MMF10_l	1.70E-01 (1.28E-02) +	1.60E-01 (1.11E-03) +	1.64E-01 (4.10E-03) +	1.97E-01 (1.12E-02) +	1.34E-01 (8.52E-02) +	1.61E-01 (2.38E-03) +	9.73E-03 (9.77E-03) +	**6.18E-03 (5.27E-04)-**	6.64E-03 (3.50E-04)
MMF11_l	2.02E-01 (1.72E-02) +	2.25E-01 (2.99E-02) +	2.37E-01 (1.92E-02) +	2.49E-01 (1.92E-04) +	2.48E-01 (1.15E-03) +	2.33E-01 (2.59E-02) +	**4.90E-03 (7.62E-04)-**	5.59E-03 (2.37E-04) ~	5.67E-03 (3.54E-04)
MMF12_l	1.92E-01 (4.47E-02) +	2.27E-01 (3.78E-02) +	1.99E-01 (4.34E-02) +	2.45E-01 (2.85E-04) +	2.45E-01 (3.93E-04) +	2.07E-01 (4.52E-02) +	**2.14E-03 (1.27E-04) ~**	2.24E-03 (1.03E-04) ~	2.19E-03 (1.14E-04)
MMF13_l	2.34E-01 (1.66E-02) +	2.29E-01 (1.82E-02) +	2.48E-01 (1.06E-02) +	2.50E-01 (5.51E-04) +	2.56E-01 (3.16E-02) +	2.40E-01 (1.79E-02) +	1.03E-01 (4.72E-02) +	1.15E-01 (2.30E-03) +	**5.27E-02 (1.44E-03)**
MMF15_l	1.50E-01 (1.66E-02) +	1.39E-01 (1.67E-02) +	1.48E-01 (3.11E-02) +	2.56E-01 (8.00E-04) +	2.62E-01 (5.24E-03) +	1.48E-01 (2.30E-02) +	4.69E-02 (8.96E-04) -	**4.11E-02 (9.55E-04)-**	4.97E-02 (1.64E-03)
MMF15_a_l	1.56E-01 (1.06E-02) +	1.54E-01 (1.62E-02) +	1.56E-01 (1.36E-02) +	2.06E-01 (2.29E-03) +	2.16E-01 (7.49E-03) +	1.61E-01 (1.42E-02) +	8.11E-02 (1.34E-02) +	7.41E-02 (4.96E-03) +	**5.94E-02 (1.61E-03)**
MMF16_l1	1.05E-01 (7.27E-03) +	1.01E-01 (8.02E-03) +	1.03E-01 (1.14E-02) +	1.43E-01 (3.60E-04) +	1.51E-01 (7.75E-03) +	1.09E-01 (1.08E-02) +	4.85E-02 (1.95E-03) +	6.57E-02 (5.99E-03) +	**4.34E-02 (8.31E-04)**

(continued on next page)

Table 2 (*continued*)

Problems	MO_Ring_ PSO_SCD	MMODE_ CSCD	MMO_ SO_QPSO	MMEAWI	MPMMEA	MMONBSA	MMOEA/DC	CEA-LES	TADE_DDS
MMF16_l2	1.96E-01 (2.12E-02) +	1.81E-01 (1.94E-02) +	1.92E-01 (2.77E-02) +	3.28E-01 (1.03E-03) +	3.31E-01 (4.38E-03) +	1.98E-01 (1.72E-02) +	1.06E-01 (1.02E-03) +	1.37E-01 (1.35E-02) +	**5.03E-02 (1.33E-03)**
MMF16_l3	1.46E-01 (1.24E-02) +	1.40E-01 (1.10E-02) +	1.37E-01 (1.49E-02) +	1.98E-01 (4.09E-04) +	2.05E-01 (1.75E-03) +	1.46E-01 (1.01E-02) +	1.23E-01 (4.93E-02) +	8.52E-02 (6.78E-04) +	**5.31E-02 (2.43E-03)**
+/-/~	24/0/0	20/1/3	20/4/0	17/7/0	20/3/1	20/3/1	18/4/2	12/6/6	

local PSs in decision space. As suggested in the benchmark problems, the population size N and the total iterations number $G_{\max}$ for all MMEAs are set to $200 \times Nops$ and $50 \times Nops$, respectively, where $Nops$ is the number of global and local PSs. The performance of $1/PSP$, $1/HV$, $IGDX$, and $IGDF$ for all algorithms is shown in Tables 1-4, respectively, and the best values for each test case are bolded.

4.2. Experimental results

The section shows the performance of TADE_DDS and its competing algorithms on both decision space and objective space, and TADE_DDS needs to find local PSs and balance the distribution of global PSs and local PSs. The mean and standard deviation of four indicators for all algorithms over 21 runs are shown. Furthermore, to confirm the variances between competing algorithms and TADE_DDS in terms of four performance matrices, the symbols '+', '-', and '~' associated with the Wilcoxon rank sum test [43] suggest that TADE_DDS is superior to, inferior to, or similar to comparison algorithms, respectively.

To begin with, the decision space performance of nine algorithms is discussed. In Table 1, TADE_DDS gets the best 1/PSP value on 9 out of 24 test scenarios. It is reassuring to know that TADE_DDS obtains the best 1/PSP value of 5 out of 9 MMOPs with local PSs, MMOEA/DC achieves the best 1/PSP values on MMF11_l and MMF12_l, and CEA-LES gets the best 1/PSP on MMF10_l and MMF15_l. The remaining competing algorithms have not gained the best 1/PSP values on 9 MMOPs. Also, we can find that TADE_DDS achieves good performance according to the Wilcoxon rank sum test. Particularly, MMOEA/DC and CEA-LES are devoted to solving MMOPs with local PS, and TADE_DDS gets 18 and 12 good 1/PSP values in comparison to MMOEA/DC and CEA-LES, respectively. Therefore, TADE_DDS is superior to MMOEA/DC and CEA-LES.

As shown in Table 2, the IGDX of TADE_DDS and its compared algorithms are similar in terms of 1/PSP and IGDX indicators. It can be seen that TADE_DDS also gets the best IGDX values on 9 out of 24 test problems, including MMF1, MMF5, MMF10, MMF1_e, MMF13_l, MMF15_a_l, MMF16_l1, MMF16_l2, and MMF16_l3. TADE_DDS obtains a smaller IGDX over 17 out of 24 test problems in comparison to MO_Ring_PSO_SCD, MMODE_CSCD, MMO_SO_QPSO, MPMMEA, and MMONBSA according to the Wilcoxon rank sum test in Table 2. The best IGDX values are obtained by MMEAWI on MMF11, MMF12, MMF13, MMF14, MMF15, and MMF15_a, but it is inferior to TADE_DDS on 17 test problems. Unfortunately, MO_Ring_PSO_SCD, MMODE_CSCD, MMO_SO_QPSO, MMEAWI, MPMMEA, and MMONBSA have not obtained the best IGDX values and are obviously far worse than MMOEA/DC, CEA-LES, and TADE_DDS on 9 MMOPs with local PS. The primary drivers are that they only locate global PSs but overlook local PSs in decision space. On the contrary, the proposed TADE_DDS, CEA-LES, and MMOEA/DC take into account the local PSs along with global equivalent PSs. The first 15 test cases on the MMOPs test suite include only global PSs, and the IGDX values of TADE_DDS are smaller than MMOEA/DC and CEA-LES on 12 and 7 test cases, respectively. Additionally, TADE_DDS is also competitive to MMOEA/DC and CEA-LES on over half of 9 MMOPs with local PSs. Taking into account the performance both with global PSs and with local PSs, TADE_DDS is superior to MMOEA/DC and CEA-LES in two scenarios. From the above comparison of the 1/PSP and IGDX among TADE_DDS and all competing algorithms, we can conclude that the proposed TADE_DDS get a promising performance on decision space in comparison to its competing algorithms.

The performance of objective space is then analyzed. In Table 3, TADE_DDS has not achieved the best 1/HV value in 24 cases, and the HV value of TADE_DDS is slightly superior to MMOEA/DC and CEA-LES. Specifically, TADE_DDS, MMOEA/DC, and CEA-LES are distinctly inferior to the other six MMEAs. Intuitively, there is not a massive difference within acceptable levels between TADE_DDS and its competing algorithms as per 1/HV values. It is worth noting that several algorithms have null HV values on some test problems since the 1/HV values are negative, and we mark the HV values as empty (–). In Table 4, although TADE_DDS only achieves the smallest IGDF on MMF15_a_l, it obtains a smaller IGDF on 23, 15, 16, 15, 16, and 17 test problems in comparison to MO_Ring_PSO_SCD, MMODE_CSCD, MMO_SO_QPSO, MMEAWI, MPMMEA, and MMONBSA, respectively. Thereby, it is significantly preferable to all competing algorithms apart from MMOEC/DC and CEA-LES in terms of IGDF values. The best IGDF value of 6 and 8 out of 24 cases are gained by MMOEC/DC and CEA-LES, respectively. Particularly, it can be noticed that the IGDF and IGDX values of TADE_DDS are slightly poorer than and better than MMOEC/DC and CEA-LES on 9 scenarios with local PSs. Therefore, it is hard to balance the decision and objective space performance for all MMEAs in terms of searching for global and local PSs, simultaneously. In summary, the objective space performance is satisfactory and acceptable.

To further analyze the dual space performance of all algorithms, the Friedman average ranking values [44] of IGDX, IGDF, and four indicators on 21 times for TADE_DDS and all competing algorithms are shown in Fig. 4. Smaller average ranking values are favored. It is clear that the average ranking values of IGDX and IGDF for TADE_DDS, CEA-LES, and MMOEA/DC are remarkable in comparison to the other six algorithms, and they are perfect for solving MMOPs with local PS. As opposed to the other two algorithms, TADE_DDS obtains the best IGDX average ranking value but a poor IGDF average ranking value. Instead, MMOEA/DC gets the best IGDF average

Table 3
The 1/HV performance of different algorithms.

Problems	MO_Ring_ PSO_SCD	MMODE_ CSCD	MMO_ SO_QPSO	MMEAWI	MPMMEA	MMONBSA	MMOEA/DC	CEA-LES	TADE_DDS
MMF1	1.14E + 00 (1.97E-04) ~	1.14E + 00 (2.52E-04) -	1.14E + 00 (2.83E-04) -	1.15E + 00 (1.17E-03) ~	1.15E + 00 (1.62E-03) +	**1.14E + 00 (1.74E-04)-**	1.15E + 00 (3.39E-04) +	1.15E + 00 (3.01E-03) +	1.15E + 00 (8.88E-04)
MMF2	1.17E + 00 (2.93E-03) ~	**1.15E + 00 (6.90E-04)-**	1.16E + 00 (3.30E-03) ~	1.15E + 00 (2.10E-03) -	1.19E + 00 (1.50E-02) +	1.16E + 00 (1.87E-03) ~	1.16E + 00 (4.45E-03) ~	–(–) +	1.18E + 00 (4.46E-02)
MMF4	1.85E + 00 (1.09E-03) -	1.85E + 00 (4.78E-04) -	1.85E + 00 (9.83E-04) -	**1.85E + 00 (4.44E-04)-**	2.09E + 00 (1.68E-01) +	1.85E + 00 (1.15E-03) -	1.86E + 00 (4.28E-03) +	1.87E + 00 (2.07E-02) +	1.85E + 00 (3.31E-03)
MMF5	**1.14E + 00 (1.97E-04) ~**	1.14E + 00 (2.13E-04) -	1.14E + 00 (2.49E-04) -	1.15E + 00 (3.55E-04) ~	1.15E + 00 (1.09E-03) +	1.14E + 00 (2.17E-04) -	1.15E + 00 (1.78E-04) +	1.15E + 00 (4.45E-03) +	1.14E + 00 (6.38E-04)
MMF7	1.14E + 00 (2.63E-04) +	**1.14E + 00 (7.38E-05)-**	1.14E + 00 (1.32E-04) ~	1.15E + 00 (2.04E-04) +	1.15E + 00 (6.91E-04) +	1.14E + 00 (1.54E-04) -	1.15E + 00 (6.88E-04) +	1.15E + 00 (1.57E-03) +	1.14E + 00 (3.60E-04)
MMF8	2.38E + 00 (8.87E-03) -	**2.37E + 00 (1.54E-03)-**	2.38E + 00 (1.41E-02) -	2.37E + 00 (1.89E-03) -	–(–) +	2.37E + 00 (2.86E-03) -	2.38E + 00 (1.30E-02) -	3.82E + 00 (3.92E + 00)~	3.17E + 00 (1.78E + 00)
MMF10	7.97E-02 (5.16E-04) -	7.93E-02 (3.09E-03) -	7.88E-02 (3.65E-04) -	7.85E-02 (2.34E-03) -	8.60E-02 (2.36E-03) +	**7.78E-02 (7.52E-05)-**	8.27E-02 (1.06E-03) -	8.42E-02 (6.27E-04) +	8.31E-02 (5.12E-04)
MMF11	6.90E-02 (2.40E-05) -	6.89E-02 (1.49E-05) -	**6.89E-02 (1.15E-05)-**	6.90E-02 (4.32E-05) -	7.17E-02 (1.01E-03) +	6.89E-02 (1.85E-05) -	7.10E-02 (1.22E-04) +	7.12E-02 (2.86E-04) +	7.06E-02 (1.42E-04)
MMF12	6.39E-01 (1.68E-03) -	**6.36E-01 (5.56E-05)-**	6.37E-01 (1.16E-03) -	6.36E-01 (8.95E-05) -	6.41E-01 (6.95E-03) -	6.36E-01 (2.73E-04) -	7.62E-01 (1.09E-01) ~	7.63E-01 (1.48E-01) ~	7.47E-01 (1.01E-01)
MMF13	5.45E-02 (5.88E-05) -	5.43E-02 (2.72E-05) -	**5.43E-02 (2.11E-05)-**	5.43E-02 (1.65E-05) -	5.53E-02 (9.13E-04) -	5.43E-02 (2.52E-05) -	5.72E-02 (4.45E-04) ~	5.72E-02 (4.30E-04) ~	5.71E-02 (4.05E-04)
MMF14	3.36E-01 (2.63E-02) -	3.48E-01 (2.97E-02) ~	3.29E-01 (2.02E-02) -	**3.15E-01 (6.55E-03)-**	4.73E-01 (1.16E-01) +	3.35E-01 (1.32E-02) ~	3.50E-01 (1.05E-02) ~	3.27E-01 (6.11E-03) -	3.48E-01 (1.84E-02)
MMF15	2.41E-01 (1.11E-02) -	2.49E-01 (9.41E-03) ~	2.35E-01 (1.30E-02) -	**2.20E-01 (7.74E-03)-**	2.96E-01 (5.43E-02) +	2.34E-01 (6.82E-03) -	2.60E-01 (9.36E-03) ~	2.43E-01 (7.62E-03) -	2.53E-01 (1.19E-02)
MMF1_e	1.17E + 00 (2.18E-02) -	1.19E + 00 (3.28E-02) -	1.18E + 00 (2.73E-02) -	1.17E + 00 (1.39E-02) -	1.15E + 00 (1.77E-03) -	**1.15E + 00 (1.64E-03)-**	8.52E + 00 (2.23E + 01)-	–(–) ~	– (–)
MMF14_a	3.34E-01 (1.72E-02) -	3.55E-01 (1.92E-02) ~	3.28E-01 (2.67E-02) -	**3.12E-01 (7.22E-03)-**	3.31E-01 (7.04E-03) -	3.30E-01 (8.59E-03) -	3.45E-01 (1.87E-02) ~	3.25E-01 (9.91E-03) -	3.52E-01 (2.25E-02)
MMF15_a	2.42E-01 (1.23E-02) -	2.51E-01 (1.34E-02) ~	2.41E-01 (1.38E-02) -	**2.24E-01 (5.68E-03)-**	2.29E-01 (4.61E-03) -	2.39E-01 (9.75E-03) -	2.75E-01 (1.68E-02) +	2.38E-01 (8.83E-03) -	2.52E-01 (1.18E-02)
MMF10_l	7.89E-02 (3.72E-04) -	7.79E-02 (6.99E-04) -	7.81E-02 (1.13E-04) -	**7.77E-02 (2.90E-05)-**	8.44E-02 (3.09E-03) +	7.77E-02 (5.69E-05) -	8.19E-02 (5.22E-04) -	8.34E-02 (2.61E-04) +	8.33E-02 (5.21E-04)
MMF11_l	6.89E-02 (8.72E-06) -	**6.88E-02 (3.40E-06)-**	6.88E-02 (5.08E-06) -	6.89E-02 (1.32E-05) -	7.20E-02 (1.13E-03) +	6.88E-02 (5.86E-06) -	7.08E-02 (2.83E-04) +	7.10E-02 (1.84E-04) +	7.04E-02 (1.61E-04)
MMF12_l	6.37E-01 (6.91E-04) -	6.35E-01 (3.82E-05) -	6.36E-01 (7.07E-04) -	**6.35E-01 (3.67E-05)-**	7.61E-01 (3.13E-01) +	6.35E-01 (9.64E-05) -	6.83E-01 (4.90E-02) ~	7.43E-01 (1.02E-01) ~	7.04E-01 (7.05E-02)
MMF13_l	5.43E-02 (1.73E-05) -	**5.42E-02 (6.76E-06)-**	5.42E-02 (9.01E-06) -	5.42E-02 (8.86E-06) -	5.52E-02 (1.04E-03) -	5.42E-02 (1.24E-05) -	5.65E-02 (8.86E-04) ~	5.71E-02 (3.66E-04) +	5.68E-02 (3.19E-04)
MMF15_l	2.33E-01 (8.64E-03) -	2.44E-01 (7.88E-03) -	2.34E-01 (1.08E-02) -	**2.21E-01 (3.40E-03)-**	3.06E-01 (4.47E-02) +	2.34E-01 (7.77E-03) -	2.56E-01 (4.80E-03) +	2.45E-01 (4.32E-03) -	2.51E-01 (1.09E-02)
MMF15_a_l	2.33E-01 (9.90E-03) -	2.46E-01 (7.41E-03) -	2.27E-01 (7.32E-03) -	**2.24E-01 (3.41E-03)-**	2.32E-01 (4.01E-03) -	2.34E-01 (6.33E-03) -	2.67E-01 (1.62E-02) +	2.37E-01 (6.79E-03) -	2.52E-01 (8.43E-03)
MMF16_l1	2.37E-01 (1.18E-02) -	2.41E-01 (1.11E-02) ~	2.25E-01 (6.82E-03) -	**2.17E-01 (2.97E-03)-**	2.97E-01 (4.80E-02) +	2.29E-01 (5.29E-03) -	2.55E-01 (4.13E-03) +	2.34E-01 (9.31E-03) -	2.47E-01 (1.07E-02)

(*continued on next page*)

Table 3 (*continued*)

Problems	MO_Ring_ PSO_SCD	MMODE_ CSCD	MMO_ SO_QPSO	MMEAWI	MPMMEA	MMONBSA	MMOEA/DC	CEA-LES	TADE_DDS
MMF16_l2	2.33E-01 (7.74E-03) -	2.43E-01 (1.04E-02) -	2.27E-01 (8.57E-03) -	**2.21E-01 (3.85E-03)-**	3.04E-01 (4.41E-02) +	2.33E-01 (1.48E-02) -	2.47E-01 (4.89E-03) ~	2.33E-01 (1.28E-02) -	2.49E-01 (7.12E-03)
MMF16_l3	2.32E-01 (6.30E-03) -	2.38E-01 (7.58E-03) ~	2.23E-01 (6.98E-03) -	**2.18E-01 (2.49E-03)-**	3.22E-01 (3.32E-02) +	2.30E-01 (4.44E-03) -	2.58E-01 (1.78E-02) +	2.48E-01 (4.62E-03) +	2.40E-01 (8.15E-03)
+/-/~	1/20/3	0/18/6	0/22/2	1/21/2	17/8/0	0/22/2	11/4/9	11/8/5	

ranking value but a poor IGDX average ranking value. It is difficult to gain good IGDF and IGDX average ranking values simultaneously in terms of TADE_DDS, CEA-LES, and TADE_DDS. Particularly, the average ranking values of four indicators in Fig. 4(3) show that TADE_DDS is the most outstanding. Therefore, taking the above discussion into account, the proposed TADE_DDS is competitive. Additionally, we also analyzed the complexity of all algorithms. Assume that the population size is N, and the decision variables number and objective number are d and m, respectively. The computational complexity of MO_Ring_PSO_SCD, MMOEA/DC, and MMODE_CSCD is all $O(m+d)N^2$, and the computational complexity of MMO_SO_QPSO and MPMMEA is $O(N^2)$ and $O(mn(n+d))$, respectively. The computational complexity of MMEAWI, MMONBSA, CEA-LES, and TADE_DDS is all $O(mN^2)$. It is obvious that the complexity of TADE_DDS is superior to all competing algorithms apart from MMO_SO_QPSO. Although MMO_SO_QPSO achieves the best complexity, its performance is poor and the complexity of TADE_DDS is close to the complexity of MMO_SO_QPSO in view of the smaller m. Therefore, the computational complexity of TADE_DDS is also preferred in comparison to its competing algorithms.

To demonstrate the capability of locating local solutions, the obtained PSs and PF for TADE_DDS and MMEAWI on MMOPs with local Pareto solutions are shown in Fig. 5. As we can observe from Fig. 5 that diverse PSs and local PSs are found in TADE_DDS, as well as equivalent global PF and local PF. However, only global equivalent PSs and PF are found by MMEAWI, and local PSs are overlooked. It confirms that TADE_DDS is capable of handling MMOPs with local Pareto solutions.

4.3. *Influence of population size*

To further investigate the influence of population size, the population size of all algorithms in this section is set to $N = 50 \times Nops$, $N = 100 \times Nops$, $N = 200 \times Nops$, and $N = 250 \times Nops$, respectively, and the number of fitness evaluations *MaxFes* is fixed at $10000 \times Nops$. TADE_DDS and all competing algorithms with different population sizes are performed in several MMOPs, and the IGDX and IGDF values are plotted in Fig. 6 and Fig. 7.

We can see from Fig. 6 that TADE_DDS is superior to MMODE_CSCD, MMO_SO_QPSO, MMEAWI, MO_Ring_PSO_SCD, MPMMEA, and MMONBSA and similar to MMOEA/DC and CEA-LES on MMF10_l, MMF12_l, MMF15_l, and MMF16_l3 as the population increases. Compared with MO_Ring_PSO_SCD, MMODE_CSCD, MMO_SO_QPSO, MMEAWI, MPMMEA, and MMONBSA, TADE_DDS with different population sizes significantly acquire good IGDX and IGDF values on MMOPs with local PS. Likewise, TADE_DDS achieves preferred IGDX on MMF10_l, MMF12_l, and MMF16_l3, and the IGDX of TADE_DDS, MMOEA/DC, and CEA-LES are similar for MMF1 and MMF8. However, the IGDF of MMOEA/DC is superior to TADE_DDS and CEA-LES. Additionally, we can also notice that there is a greater tendency for the IGDX and IGDF values of TADE_DDS and its competing algorithms on MMF1 and MMF8 to decrease as the population increases, but there is no obvious distinction on MMF10_l, MMF16_l3, MMF15_l, and MMF12_l for all competing algorithms apart from MMOEA/DC and CEA-LES as the population increases. From the above analysis, we can conclude that TADE_DDS with various N values is also competitive.

4.4. *Discussion on dynamic dual-populations strategy*

In this section, the effectiveness of the dynamic dual-population strategy in TADE_DDS is discussed. A variation of TADE_DDS that only involves a dynamic population reduction strategy (Eq. (10)), and another variation of TADE_DDS that only contains a dynamic population increase strategy (Eq. (15)) are devised and are denoted as TADE_DDS_v1 and TADE_DDS_v2, respectively. TADE_DDS, TADE_DDS_v1, and TADE_DDS_v2 are performed on 9 cases, and IGDX and IGDF are presented in Fig. 8.

We can see from Fig. 8 that the IGDX and IGDF obtained by TADE_DDS are extremely superior to TADE_DDS_v1 and TADE_DDS_v2 in all cases except for MMF13_l, and the IGDX and IGDF of TADE_DDS and TADE_DDS_v2 on MMF13_l are similar. The IGDX of TADE_DDS_v2 is clearly better than TADE_DDS_v1 on MMF13_l, MMF15_l, MMF15_a_l, and MMF16_l2, and TADE_DDS_v1 and TADE_DDS_v2 perform similarly on the remaining test problems. Compared with TADE_DDS_v1, TADE_DDS_v2 get good IGDF values on all test problems apart from MMF10_l and MMF16_l3. Therefore, TADE_DDS_v1 is also inferior to TADE_DDS_v2. Because N_G is decreasing and N_L is increasing in TADE_DDS_v1 resulting in numerous local Pareto solutions are survived. On the contrary, TADE_DDS_V2 survives a larger number of global solutions with increasing N_G. Based on the above, the proposed dynamic dual-population strategy can find diverse PSs and local PS, simultaneously. Furthermore, several different phases of PS obtained by TADE_DDS are shown in Fig. 9. It is clear that numerous poor local PF and several local PF are found by TADE_DDS on the first-stage evolution for MMF10_l, and then numerous local PF and several global PF are located. Finally, superior global PF and local PF are maintained. Therefore, we can deduce that the proposed dynamic dual-populations strategy is effective in TADE_DDS.

Table 4
The IGDF performance of different algorithms.

Problems	MO_Ring_PSO_SCD	MMODE_CSCD	MMO_SO_QPSO	MMEAWI	MPMMEA	MMONBSA	MMOEA/DC	CEA-LES	TADE_DDS
MMF1	2.06E-03 (9.26E-05) +	**1.37E-03 (8.72E-05)-**	1.79E-03 (9.74E-05) ~	2.36E-03 (1.60E-04) +	2.04E-03 (1.31E-04) +	1.43E-03 (7.52E-05) -	1.72E-03 (7.18E-05) -	1.78E-03 (6.93E-05) ~	1.76E-03 (8.54E-05)
MMF2	1.24E-02 (1.54E-03) +	**4.68E-03 (3.09E-04)-**	8.64E-03 (7.81E-04) +	6.52E-03 (1.14E-03) ~	1.35E-02 (6.44E-03) +	7.85E-03 (8.80E-04) +	5.04E-03 (7.62E-04) -	8.96E-03 (1.21E-03) +	6.50E-03 (1.37E-03)
MMF4	1.78E-03 (1.37E-04) +	**1.21E-03 (6.80E-05)-**	1.69E-03 (1.15E-04) ~	1.92E-03 (1.63E-04) +	2.69E-03 (4.18E-04) +	1.36E-03 (9.09E-05) -	1.82E-03 (9.70E-05) +	1.97E-03 (1.13E-04) +	1.66E-03 (1.52E-04)
MMF5	1.98E-03 (7.96E-05) +	**1.38E-03 (3.97E-05)-**	1.77E-03 (7.92E-05) ~	2.13E-03 (1.44E-04) +	2.02E-03 (1.13E-04) +	1.44E-03 (4.97E-05) -	1.79E-03 (3.24E-05) +	1.95E-03 (6.65E-05) +	1.72E-03 (8.43E-05)
MMF7	1.88E-03 (9.51E-05) +	**1.21E-03 (3.27E-05)-**	1.61E-03 (3.81E-05) ~	2.04E-03 (7.92E-05) +	1.83E-03 (6.76E-05) +	1.37E-03 (4.13E-05) -	1.87E-03 (4.84E-05) +	1.75E-03 (6.16E-05) +	1.63E-03 (5.38E-05)
MMF8	2.78E-03 (1.51E-04) +	2.34E-03 (2.53E-04) -	2.45E-03 (1.47E-04) -	2.00E-03 (7.43E-05) -	2.56E-03 (6.23E-04) -	**1.55E-03 (6.58E-05)-**	1.78E-03 (5.40E-05) -	2.46E-03 (1.99E-04) -	2.61E-03 (1.34E-04)
MMF10	2.04E-01 (2.31E-02) +	1.62E-01 (1.27E-02) +	1.75E-01 (1.44E-02) +	3.77E-02 (7.97E-02) +	6.53E-02 (9.15E-02) ~	1.62E-01 (6.54E-03) +	4.04E-02 (7.87E-02) +	**2.46E-02 (1.48E-03) ~**	2.47E-02 (4.19E-03)
MMF11	8.69E-02 (6.58E-03) +	8.59E-02 (6.86E-03) +	8.85E-02 (7.60E-03) +	**1.54E-02 (1.43E-03)-**	1.58E-02 (1.30E-03) -	8.87E-02 (6.43E-03) +	1.96E-02 (1.06E-03) -	2.24E-02 (1.15E-03) ~	2.26E-02 (2.00E-03)
MMF12	6.61E-02 (1.34E-02) +	6.86E-02 (1.72E-02) +	6.52E-02 (1.34E-02) +	2.93E-03 (2.13E-04) -	**2.89E-03 (1.17E-03)-**	7.26E-02 (1.44E-02) +	3.91E-03 (2.16E-04) -	4.72E-03 (1.74E-04) +	4.44E-03 (3.26E-04)
MMF13	9.66E-02 (1.80E-02) +	9.18E-02 (3.06E-02) +	1.07E-01 (2.46E-02) +	2.48E-02 (2.54E-03) -	**1.80E-02 (2.40E-03)-**	1.03E-01 (2.89E-02) +	3.37E-02 (2.16E-03) -	3.61E-02 (2.20E-03) ~	3.56E-02 (2.88E-03)
MMF14	6.81E-02 (1.24E-03) +	6.22E-02 (1.43E-03) ~	6.02E-02 (1.32E-03) -	5.99E-02 (1.36E-03) -	7.33E-02 (6.07E-03) +	6.47E-02 (1.59E-03) +	5.76E-02 (1.81E-03) -	**5.57E-02 (9.78E-04)-**	6.26E-02 (1.69E-03)
MMF15	1.85E-01 (4.29E-03) +	1.80E-01 (3.82E-03) +	1.72E-01 (5.08E-03) +	8.45E-02 (1.61E-03) -	1.06E-01 (5.90E-03) -	1.80E-01 (2.44E-03) +	1.19E-01 (4.03E-03) ~	**1.06E-01 (1.57E-03)-**	1.21E-01 (7.20E-03)
MMF1_e	7.88E-03 (1.15E-03) -	8.16E-03 (1.02E-03) -	8.08E-03 (1.76E-03) -	1.94E-02 (9.24E-03) +	3.93E-03 (6.11E-04) -	3.71E-03 (5.42E-04) -	**3.18E-03 (2.32E-04)-**	5.57E-03 (1.78E-03) -	8.83E-03 (1.23E-03)
MMF14_a	6.76E-02 (1.64E-03) +	6.26E-02 (1.49E-03) -	6.08E-02 (1.13E-03) -	6.34E-02 (1.41E-03) ~	8.46E-02 (7.38E-03) +	6.42E-02 (1.52E-03) ~	5.63E-02 (1.31E-03) -	**5.53E-02 (8.41E-04)-**	6.40E-02 (1.65E-03)
MMF15_a	1.87E-01 (3.49E-03) +	1.82E-01 (3.34E-03) +	1.73E-01 (2.54E-03) +	8.96E-02 (2.16E-03) -	1.17E-01 (1.18E-02) ~	1.82E-01 (2.05E-03) +	1.28E-01 (6.16E-03) +	**9.65E-02 (1.62E-03)-**	1.19E-01 (3.85E-03)
MMF10_l	1.80E-01 (1.04E-02) +	1.56E-01 (8.57E-03) +	1.58E-01 (1.24E-02) +	1.91E-01 (9.69E-03) +	1.36E-01 (7.50E-02) +	1.51E-01 (7.37E-03) +	1.52E-02 (1.41E-02) +	**1.07E-02 (5.78E-04)-**	1.31E-02 (1.36E-03)
MMF11_l	8.42E-02 (5.91E-03) +	8.58E-02 (6.55E-03) +	8.09E-02 (7.12E-03) +	9.36E-02 (7.93E-04) +	9.15E-02 (2.51E-03) +	8.56E-02 (5.88E-03) +	**1.13E-02 (2.02E-03)-**	1.25E-02 (5.11E-04) -	1.43E-02 (8.53E-04)
MMF12_l	6.06E-02 (1.21E-02) +	7.56E-02 (1.32E-02) +	6.34E-02 (1.49E-02) +	8.29E-02 (3.17E-04) +	8.26E-02 (2.43E-04) +	7.07E-02 (1.39E-02) +	**2.04E-03 (1.24E-04)-**	2.32E-03 (7.07E-05) -	2.61E-03 (1.22E-04)
MMF13_l	9.01E-02 (2.13E-02) +	8.47E-02 (2.91E-02) +	1.04E-01 (2.22E-02) +	1.47E-01 (7.22E-03) +	1.18E-01 (2.38E-02) +	1.06E-01 (3.23E-02) +	2.19E-02 (1.47E-02) -	**2.09E-02 (2.19E-03)-**	2.31E-02 (2.18E-03)
MMF15_l	1.66E-01 (2.49E-03) +	1.61E-01 (5.43E-03) +	1.55E-01 (4.44E-03) +	1.80E-01 (1.23E-03) +	1.90E-01 (5.11E-03) +	1.64E-01 (4.04E-03) +	**8.58E-02 (1.61E-03)-**	8.87E-02 (1.40E-03) -	1.04E-01 (2.59E-03)
MMF15_a_l	1.66E-01 (2.48E-03) +	1.60E-01 (5.13E-03) +	1.58E-01 (3.21E-03) +	1.73E-01 (3.14E-03) +	1.87E-01 (5.76E-03) +	1.64E-01 (2.46E-03) +	1.19E-01 (1.56E-02) ~	1.33E-01 (5.19E-03) +	**1.13E-01 (2.09E-03)**
MMF16_l1	1.29E-01 (2.33E-03) +	1.25E-01 (1.63E-03) +	1.22E-01 (3.71E-03) +	1.39E-01 (2.10E-03) +	1.45E-01 (7.35E-03) +	1.28E-01 (2.41E-03) +	**6.62E-02 (9.80E-04)-**	1.20E-01 (3.60E-03) +	8.07E-02 (2.12E-03)

(*continued on next page*)

Table 4 (*continued*)

Problems	MO_Ring_PSO_SCD	MMODE_CSCD	MMO_SO_QPSO	MMEAWI	MPMMEA	MMONBSA	MMOEA/DC	CEA-LES	TADE_DDS
MMF16_l2	2.01E-01 (3.23E-03) +	1.95E-01 (4.68E-03) +	1.96E-01 (7.84E-03) +	2.30E-01 (2.43E-03) +	2.36E-01 (6.13E-03) +	2.06E-01 (4.78E-03) +	**1.10E-01 (1.22E-03)-**	1.88E-01 (8.85E-03) +	1.12E-01 (1.84E-03)
MMF16_l3	1.62E-01 (3.89E-03) +	1.59E-01 (3.44E-03) +	1.56E-01 (3.30E-03) +	1.82E-01 (2.46E-03) +	1.86E-01 (6.22E-03) +	1.63E-01 (3.76E-03) +	1.19E-01 (3.46E-02) +	**1.01E-01 (1.63E-03)-**	1.05E-01 (2.51E-03)
+/-/~	23/1/0	15/8/1	16/4/4	15/7/2	16/6/2	17/6/1	7/15/2	8/12/4	

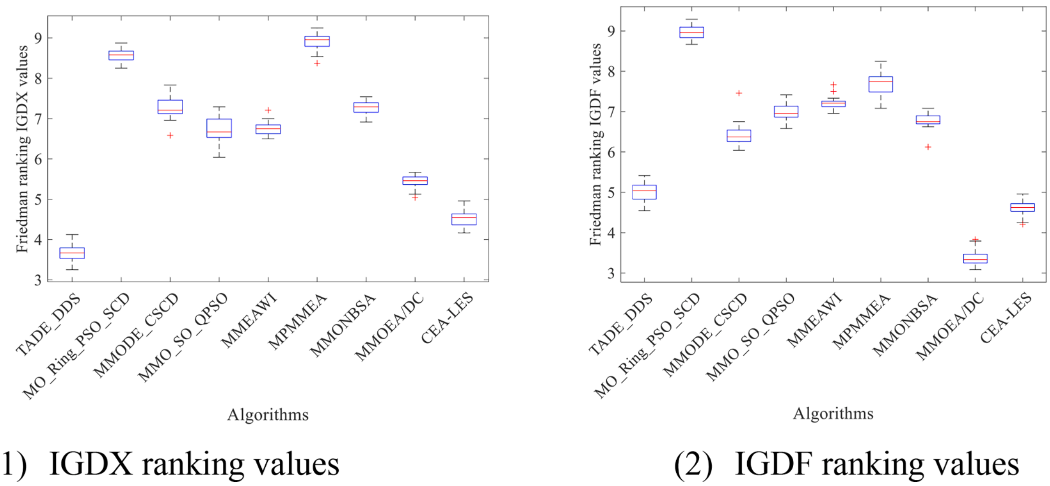


Fig. 4. The Friedman average ranking value.

4.5. Discussion on global and local environmental selection strategy

Environmental selection strategy plays an essential role in determining global individuals and local individuals from offspring and parent. To validate the influence of two environmental selections, a correlated experiment is intended in this section. Likewise, in this experiment, three environmental selection strategies, including local environmental selection (LES), global environmental selection (GES), and both global environmental selection and local environmental selection (GES + LES) are performed on 9 test problems. The IGDX and IGDF values of three unique environmental selection strategies are shown in Fig. 10.

In Fig. 10, it is insightful that GES + LES is better than the other two environmental selection strategies, and GES is also superior to LES in 9 cases. It doesn't demonstrate that the LES strategy is ineffective since LES primarily aims to find diverse local Pareto solutions. Therefore, we can deduce that the dual environment selection (GES + LES) in TADE_DDS can locate global PSs and local PSs, and it also balances the search procedure and distribution of global and local PSs.

Additionally, to analyze the influence of the parameters α and β involved in dual environmental selection in Eq. (12), three different parameter settings are set as comparison methods in the experiment. In TADE_DDS, the parameters are set to $\alpha = \beta = 1 - e^{(1-\lambda)}$ in the local environmental selection and $\alpha = 1, \beta = 1 - e^{(1-\lambda)}$ in the global environmental selection, respectively, and three scenarios are all set to $\alpha = \beta = 1$(Case 1), $\alpha = \beta = 1 - e^{(1-\lambda)}$(Case 2), and $\alpha = 1, \beta = 1 - e^{(1-\lambda)}$(Case 3) in two unique environmental selections, respectively. In Eq. (12), k is set to 5 from previous experience [38]. Table 5 shows the experimental results of four distinct parameter settings involved in TADE_DDS and demonstrates the Wilcoxon rank sum test results with four different parameter scenarios. The symbols '+', '-', and '~' show that TADE_DDS is better than, worse than, or no different from the three cases, respectively. In view of IGDX and IGDF, TADE_DDS is better than Case 1 on 18 and 19 test functions, respectively. The IGDX of TADE_DDS is better than Case 2 in 12 scenarios, and the IGDF of TADE_DDS is slightly poor than Case 2. TADE_DDS is better than Case 3 on the IGDX and IGDF values of only 5 and 4 test problems, respectively, and there are no significant differences between TADE_DDS and Case 3 in most scenarios according to the Wilcoxon rank sum test results. Generally, TADE_DDS is more feasible as opposed to the other three scenarios. Therefore, two parameters α and β involved in the global environmental selection and local environmental selection are reasonable for TADE_DDS.

5. Conclusion

MMOPs involve locating global PSs and local PSs. Traditional MMEAs concentrate on searching for equivalent global PSs in addressing MMOPs resulting in missing local PSs with acceptable quality. A two-stage adaptive differential evolution algorithm with a

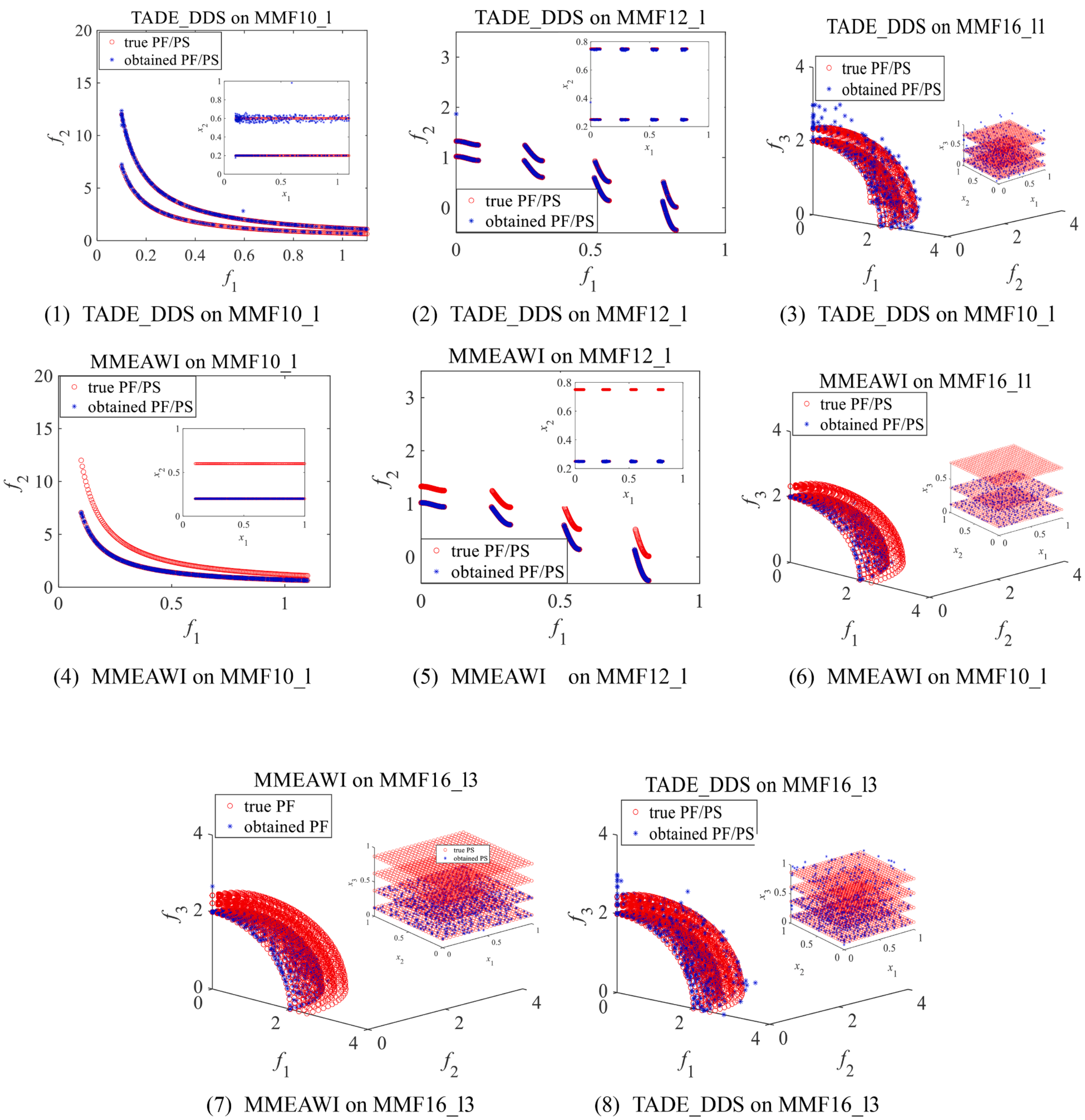
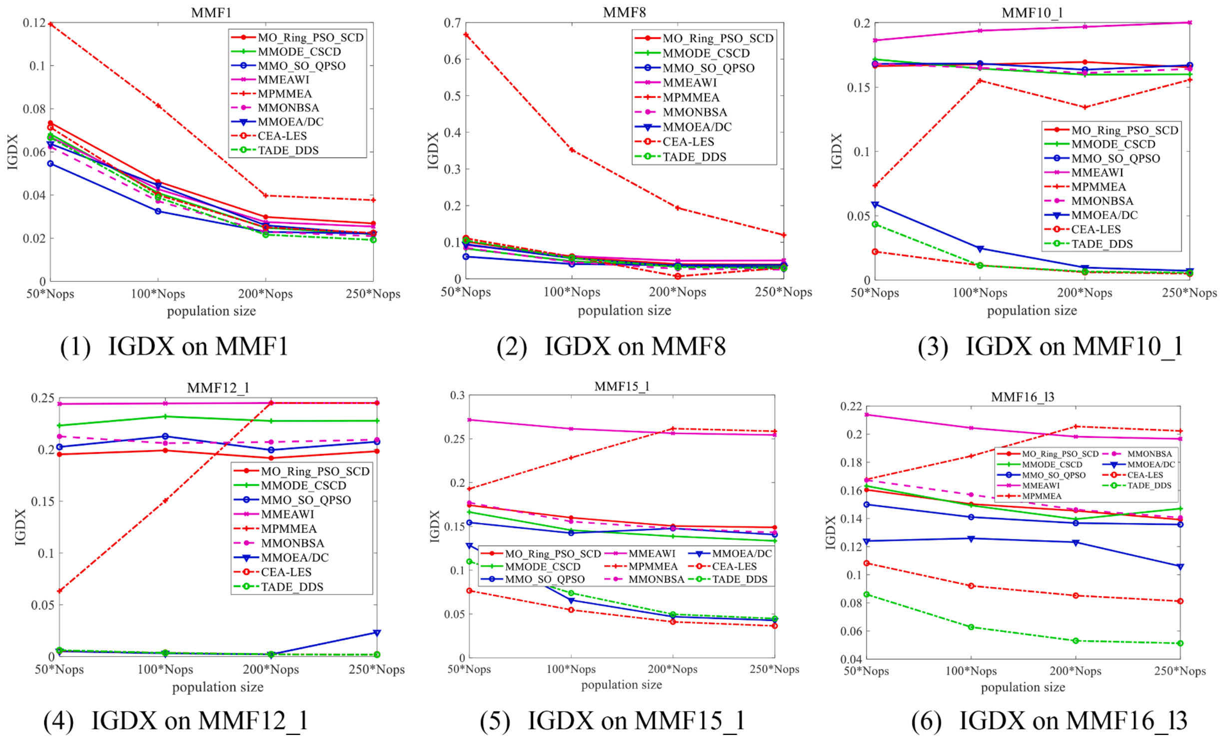


Fig. 5. PSs and PF obtained by several MMEAs.

dynamic dual-populations strategy is therefore presented in this paper. The global and local populations are dynamic in TADE_DDS. Likewise, the environmental selection is divided into a global environmental selection that focuses on diverse solutions and a local environmental selection that emphasizes diverse and convergent solutions in dual space. In the first-stage evolution, diverse local PSs are found associated with decreasing global population size and increasing local population size, and numerous local PSs with acceptable quality are maintained by local environmental selection. Immediately after, multiple equivalent global PSs are exploited and local PSs are explored and maintained in parallel with increasing global population size and decreasing local population size dynamically in the second-stage evolution. If so, diverse PSs including local PSs are located by the dynamic dual-populations strategy, and diverse solutions also are balanced by two unique environmental selections. The experimental results confirm that TADE_DDS is suitable for addressing MMOP with local Pareto solutions.

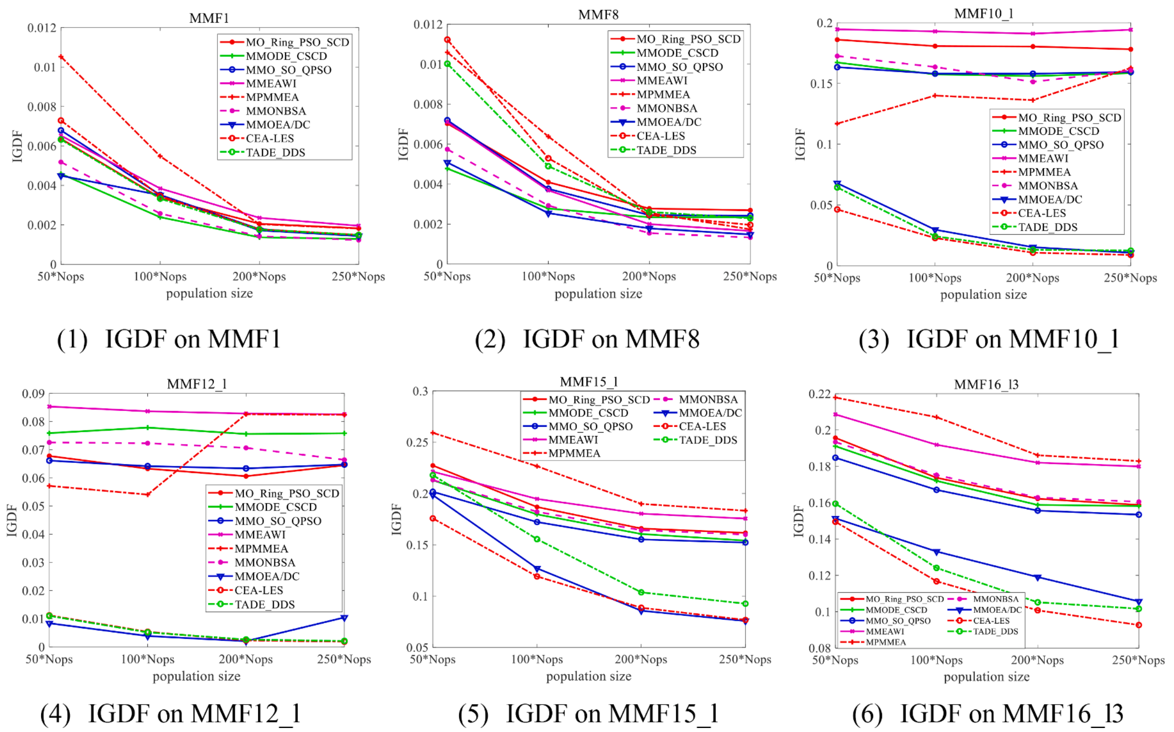
This proposed TADE_DDS is competent to find global PSs and local PSs and balance the distribution of Pareto solutions. Interestingly, compared with MMOEA/DC and CEA-LES, we can observe that the proposed TADE_DDS obtains good IGDX and IGDF without obvious advantages. On the contrary, they obtain good IGDF but poor IGDX. Therefore, the IGDX and IGDF performance may be



(1) IGDX on MMF1 (2) IGDX on MMF8 (3) IGDX on MMF10_l

(4) IGDX on MMF12_l (5) IGDX on MMF15_l (6) IGDX on MMF16_l3

Fig. 6. The IGDX of all MMEAs with various *N*.



(1) IGDF on MMF1 (2) IGDF on MMF8 (3) IGDF on MMF10_l

(4) IGDF on MMF12_l (5) IGDF on MMF15_l (6) IGDF on MMF16_l3

Fig. 7. The IGDF of all MMEAs with various *N*.

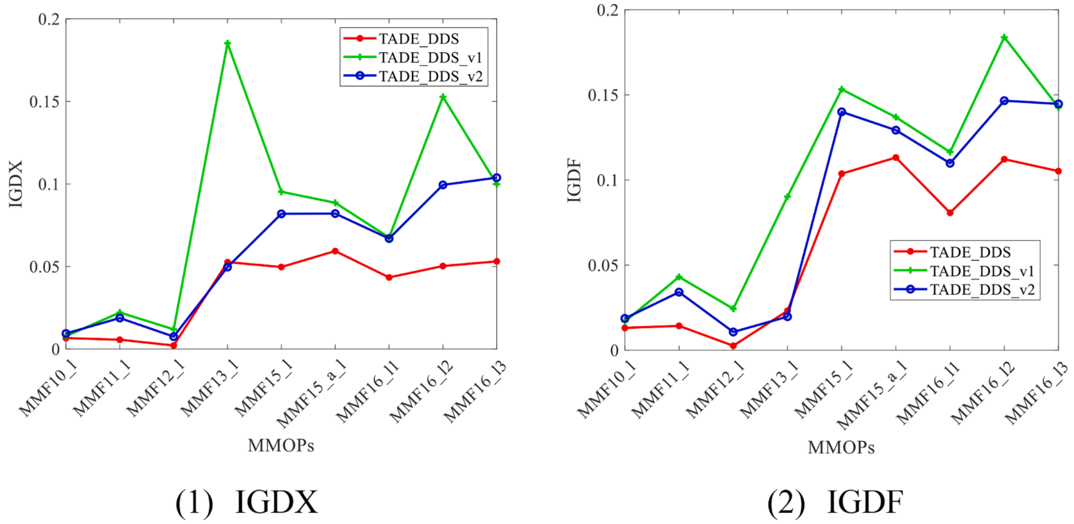


Fig. 8. The IGDX and IGDF of two variations of TADE_DDS.

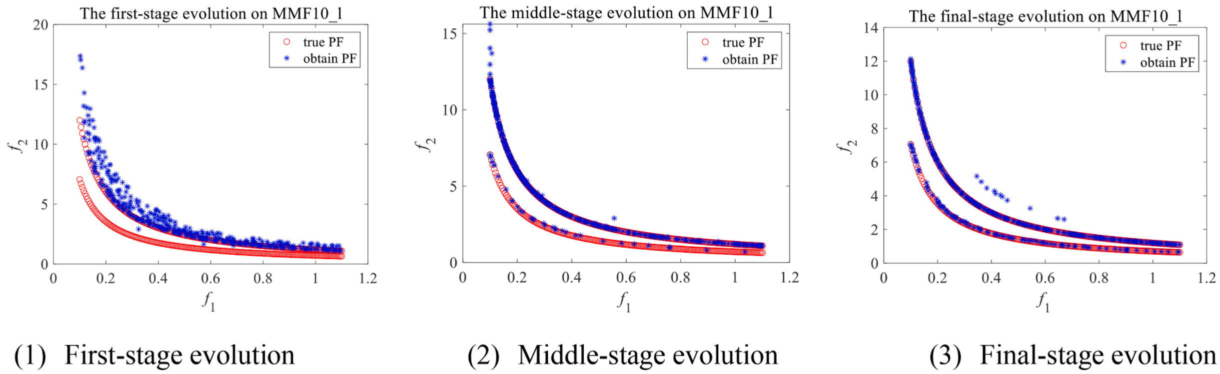


Fig. 9. Obtained PS and PF by TADE_DDS on three stages of evolution.

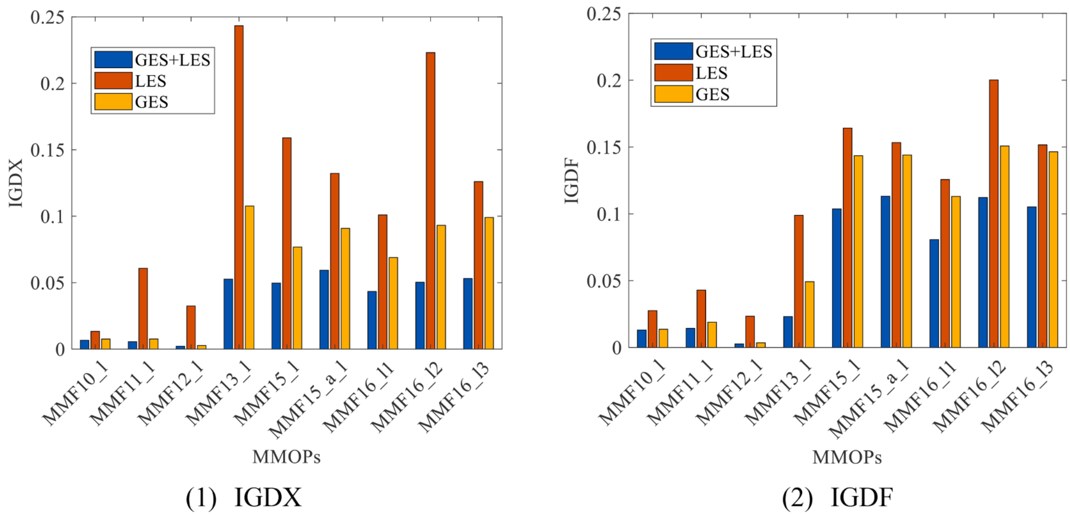


Fig. 10. IGDX and IGDF of three unique environmental selections.

Table 5
Experimental result of different parameters setting.

Parameters setting(α,β)	+/-/~(IGDX)	+/-/~(IGDF)
TADE_DDS vs Case 1	18/4/2	16/5/3
TADE_DDS vs Case 2	12/9/3	10/12/2
TADE_DDS vs Case 3	5/1/18	4/1/19

treated as MOPs to discuss and analyze in future work.

CRediT authorship contribution statement

Guoqing Li: Conceptualization, Methodology, Investigation, Writing – review & editing. **Wanliang Wang:** Investigation, Writing – review & editing. **Caitong Yue:** Investigation, Conceptualization. **Weiwei Zhang:** Writing – review & editing. **Yirui Wang:** Data curation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgment

This work is supported by the National Natural Science Foundation of China (No. 61873240, No. 62106230), the China Postdoctoral Science Foundation (2021 T140616, 2021 M692920), Zhejiang Provincial Natural Science Foundation of China under Grant (No. Q23F030020), and the Fundamental Research Funds for the Provincial Universities of Zhejiang in China (SJLY2023010).

References

[1] Y. Zhang, D.W. Gong, J.Y. Sun, B.Y. Qu, A decomposition-based archiving approach for multi-objective evolutionary optimization, Inf. Sci. 430 (2018) 397–413.
[2] Y. Tian, R. Cheng, X. Zhang, F. Cheng, Y. Jin, An indicator-based multiobjective evolutionary algorithm with reference point adaptation for better versatility, IEEE Trans. Evol. Comput. 22 (4) (2018) 609–622, https://doi.org/10.1109/TEVC.2017.2749619.
[3] K. Deb, A. Pratap, S. Agarwal, T. Meyarivan, A fast and elitist multiobjective genetic algorithm: NSGA-II, IEEE Trans. Evol. Comput. 6 (2) (2002, 2002,) 182–197.
[4] J. J. Liang, C. T. Yue, B. Y. Qu, Multimodal multi-objective optimization: A preliminary study, 2016 IEEE Congress on Evolutionary Computation (CEC), Vancouver, BC, Canada, 2016, 2454-2461.
[5] H. Ishibuchi, N. Akedo, Y. Nojima, A many-objective test problem for visually examining diversity maintenance behavior in a decision space, Conference on Genetic & Evolutionary Computation. ACM, 2011, 649-656.
[6] C. T. Yue, J. J. Liang, B. Y. Qu, K. J. Yu, H. Song, Multimodal multiobjective optimization in feature selection, 2019 IEEE Congress on Evolutionary Computation (CEC), Wellington, New Zealand, 2019, 302-309.
[7] K. Jha, S. Saha, Incorporation of multimodal multiobjective optimization in designing a filter based feature selection technique, Appl. Soft Comput. 98 (2021) 1–13.
[8] Y. Liu, L. T. Xu, Y. Y. Han, Multi-modal multi-objective traveling salesman problem and its evolutionary optimizer, 2021 IEEE International Conference on Systems, Man, and Cybernetics (SMC), Melbourne, Australia, 2021, 770-777.
[9] G. Rudolph, B. Naujoks, M. Preuss, Capabilities of EMOA to detect and preserve equivalent Pareto subsets, in: S. Obayashi, K. Deb, C. Poloni, T. Hiroyasu, T. Murata (Eds.), Lecture Notes in Computer ScienceEvolutionary Multi-Criterion Optimization, Springer, Berlin, Heidelberg, 2007, pp. 36–50.
[10] C. Yue, B. Qu, J. Liang, A multiobjective particle swarm optimizer using ring topology for solving multimodal multiobjective problems, IEEE Trans. Evol. Comput. 22 (5) (2018) 805–817.
[11] W.Z. Zhang, G.Q. Li, W.W. Zhang, J.J. Liang, G.Y. Gary, A cluster based PSO with leader updating mechanism and ring-topology for multimodal multi-objective optimization, Swarm Evol. Comput. 50 (2019) 1–13.
[12] Z.P. Li, J. Zou, S.X. Yang, J.H. Zheng, A two-archive algorithm with decomposition and fitness allocation for multi-modal multi-objective optimization, Inf. Sci. 574 (2021) 413–430.
[13] W. Li, T. Zhang, R. Wang, H. Ishibuchi, Weighted indicator-based evolutionary algorithm for multimodal multi-objective optimization, IEEE Transactions on Evolutionary Computation 25 (6) (2021) 1064–1078.
[14] K. Zhang, C. Shen, J. He, G.G. Yen, Knee based multimodal multi-objective evolutionary algorithm for decision making, Inf. Sci. 544 (2021) 39–55.
[15] Q. Yang, Z. Wang, J. Luo, Q. He, Balancing performance between the decision space and the objective space in multimodal multiobjective optimization, Memetic Computing 13 (1) (2021) 31–47.
[16] X.W. Zhang, H. Liu, L.P. Tu, A modified particle swarm optimization for multimodal multi-objective optimization, Eng. Appl. Artif. Intel. 95 (2020) 1–10.
[17] Y. Peng, H. Ishibuchi, A diversity-enhanced subset selection framework for multi-modal multi-objective optimization, IEEE Transactions on Evolutionary Computation, 10.1109/TEVC.2021.3117702.
[18] Y. Liu, H. Ishibuchi, G.G. Yen, Y. Nojima, N. Masuyama, Handling imbalance between convergence and diversity in the decision space in evolutionary multimodal multiobjective optimization, IEEE Trans. Evol. Comput. 24 (3) (2020) 551–565.
[19] Q. Fan, O.K. Ersoy, Zoning search with adaptive resource allocating method for balanced and imbalanced multimodal multi-objective optimization, IEEE/CAA J. Autom. Sin. 8 (6) (2021) 1163–1176.
[20] Y. Tian, R. Liu, X. Zhang, H. Ma, K.C. Tan, Y. Jin, A multipopulation evolutionary algorithm for solving large-scale multimodal multiobjective optimization problems, IEEE Trans. Evol. Comput. 25 (3) (2021) 405–418.

[21] Q. Lin, W. Lin, Z. Zhu, M. Gong, J. Li, C.A.C. Coello, Multimodal multiobjective evolutionary optimization with dual clustering in decision and objective Spaces, IEEE Trans. Evol. Comput. 25 (1) (2021) 130–144.
[22] Y. Liu, H. Ishibuchi, Y. Nojima, N. Masuyama, Y. Han, Searching for local Pareto optimal solutions: A Case Study on Polygon-Based Problems, 2019 IEEE Congress on Evolutionary Computation (CEC), Wellington, New Zealand, 2019, 896-903.
[23] W.L. Wang, G.Q. Li, Y.L. Wang, F. Wu, W.W. Zhang, L. Li, Clearing-based multimodal multi-objective evolutionary optimization with layer-to-layer strategy, Swarm Evol. Comput. 68 (2022) 1–25.
[24] R. Tanabe, H. Ishibuchi, A review of evolutionary multimodal multiobjective optimization, IEEE Trans. Evol. Comput. 24 (1) (2020) 193–200.
[25] S. Gupta, R. Su, An efficient differential evolution with fitness-based dynamic mutation strategy and control parameters, Knowl.-Based Syst. 251 (2022) 109280.
[26] J. Liang, Q.Q. Guo, C.T. Yue, B.Y. Qu, K.J. Yu, A self-organizing multi-objective particle swarm optimization algorithm for multimodal multi-objective problems, International Conference on Swarm Intelligence (2018) 550–560.
[27] B.Y. Qu, C. Li, J. Liang, L. Yan, K.J. Yu, Y.S. Zhu, A self-organized speciation based multi-objective particle swarm optimizer for multimodal multi-objective problems, Appl. Soft Comput. 86 (2020) 1–13.
[28] G.Q. Li, W.L. Wang, W.W. Zhang, W.B. You, F. Wu, H.Y. Tu, Handling multimodal multi-objective problems through self-organizing quantum-inspired particle swarm optimization, Inf. Sci. 577 (2021) 510–540.
[29] G.Q. Li, W.L. Wang, W.W. Zhang, Z. Wang, H.Y. Tu, W.B. You, Grid search based multi-population particle swarm optimization algorithm for multimodal multi-objective optimization, Swarm Evol. Comput. 62 (2021) 1–18.
[30] Q. Fan, X. Yan, Solving multimodal multiobjective problems through zoning search, IEEE Transactions on Systems, Man, and Cybernetics: Systems 51 (8) (2021) 4836–4847.
[31] J.J. Liang, W. Xu, C. Yue, K. Yu, H. Song, O.D. Crisalle, B.Y. Qu, Multimodal multiobjective optimization with diffierential evolution, Swarm Evol. Comput. 44 (2019) 1028–1059.
[32] J. Liang, K. Qiao, C. Yue, K. Yu, B. Qu, R. Xu, Z. Li, Y.i. Hu, A clustering-based differential evolution algorithm for solving multimodal multi-objective optimization problems, Swarm Evol. Comput. 60 (2021) 100788.
[33] C.T. Yue, P.N. Suganthan, J. Liang, B.Y. Qu, K.J. Yu, Y.S. Zhu, Y. Li, "Differential evolution using improved crowding distance for multimodal multiobjective optimization, Swarm Evol. Comput. 62 (2021) 1–14.
[34] R. Tanabe, H. Ishibuchi, A niching indicator-based multi-modal many-objective optimizer, Swarm Evol. Comput. 49 (2019) 134–146.
[35] R. Tanabe, H. Ishibuchi, A framework to handle multimodal multiobjective optimization in decomposition-based evolutionary algorithms, IEEE Trans. Evol. Comput. 24 (4) (2020) 720–734.
[36] M. Pal, S. Bandyopadhyay, Decomposition in decision and objective space for multi-modal multi-objective optimization, Swarm Evol. Comput. 62 (2021) 1–15.
[37] R. Wang, W. Ma, M. Tan, G.H. Wu, L. Wang, D.W. Gong, J. Xiong, Preference-inspired coevolutionary algorithm with active diversity strategy for multi-objective multi-modal optimization, Inf. Sci. 546 (2021) 1148–1165.
[38] G. Li, W. Wang, H. Chen, W. You, Y. Wang, Y. Jin, W. Zhang, A SHADE-based multimodal multi-objective evolutionary algorithm with fitness sharing, Appl. Intell. 51 (12) (2021) 8720–8752.
[39] R. Tanabe, A. Fukunaga, Success-history based parameter adaptation for Differential Evolution, 2013 IEEE Congress on Evolutionary Computation, Cancun, Mexico, 2013, 71-78.
[40] B.J. Frey, D. Dueck, Clustering by passing messages between data points, Science 315 (5814) (2007) 972–976.
[41] J. Liang, P.N. Suganthan, B.Y. Qu, Problem Definitions and Evaluation Crite- ria for the CEC 2020 Special Session on Multimodal Multiobjective Optimization, Zhengzhou University, 2020.
[42] Z.B. Hu, T. Zhou, Q.H. Su, M.F. Liu, A niching backtracking search algorithm with adaptive local search for multimodal multiobjective optimization, Swarm Evol. Comput. 69 (2022) 1–17.
[43] B. Rosner, J.G. Robert, M.L. Lee, Incorporation of clustering effects for the Wilcoxon rank sum test: a large-sample approach, Biometrics 59 (4) (2013) 1089–1098.
[44] M.R. Sheldon, J.F. Michael, W.D. Thompson, The use and interpretation of the Friedman test in the analysis of ordinal-scale data in repeated measures designs, Physiother. Res. Int. 1 (4) (1996) 221–228.