

2016/10/19

Ex1. Derive

$$(9+7)*(5-6)$$

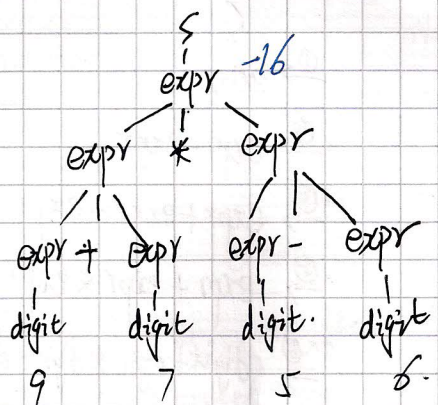
from G_3 and construct a parse tree.

G_3 :

- $S \rightarrow \text{expr} \mid (\text{expr})$
- $\text{expr} \rightarrow \text{expr} + \text{expr}$
- $\text{expr} \rightarrow \text{expr} - \text{expr}$
- $\text{expr} \rightarrow \text{expr} * \text{expr}$
- $\text{expr} \rightarrow \text{digit}$
- $\text{digit} \rightarrow 0 \mid 1 \mid \dots \mid 9$

Start S

- ① ~~(~~expr~~)~~
- ⑤ $\text{expr} * \text{expr}$
- ②② $(\text{expr}) * (\text{expr})$
- ③④ $(\text{expr} + \text{expr}) * (\text{expr} - \text{expr})$
- ⑧ $\times 4$ $(\text{digit} + \text{digit}) * (\text{digit} - \text{digit})$
- ⑦ $\times 4$ $(9 + 7) * (5 - 6)$



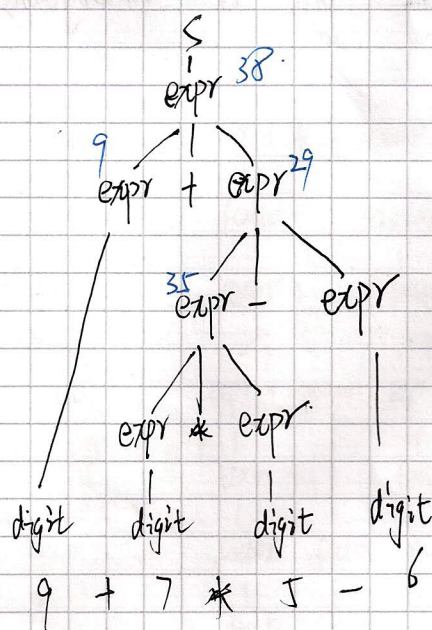
There is only one parse tree for $(9+7)*(5-6)$.

Nevertheless the grammar is still ambiguous on the second example. show as:

Ex2. Derive $9+7*5-6$ from G_3 and construct a parse tree.

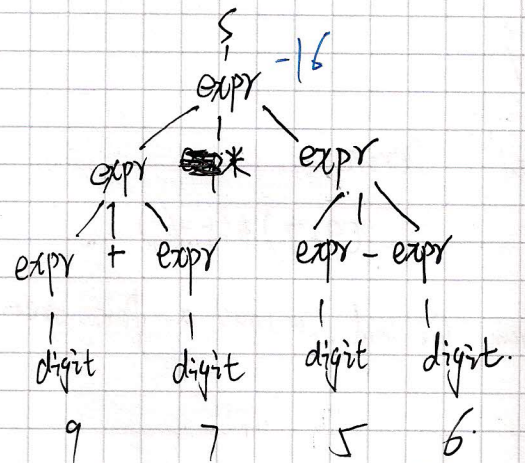
First,

- S
- ① expr
- ③ $\text{expr} * \text{expr}$
- ④ $\text{expr} + \text{expr} - \text{expr}$
- ⑤ $\text{expr} + \text{expr} * \text{expr} - \text{expr}$
- ⑥ $\times 4$ $\text{digit} + \text{digit} * \text{digit} - \text{digit}$
- ⑦ $\times 4$ $9 + 7 * 5 - 6$



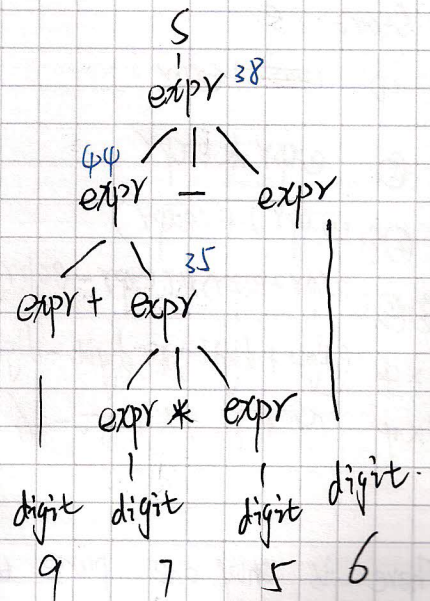
Second,

S
 $\rightarrow \text{expr}$
 $\rightarrow \text{expr} * \text{expr}$
 $\rightarrow \text{expr} + \text{expr} * \text{expr}$
 $\rightarrow \text{expr} + \text{expr} * \text{expr} - \text{expr}$
 $\rightarrow \text{digit} + \text{digit} * \text{digit} - \text{digit}$
 $\rightarrow 9 + 7 * 5 - 6$



Third,

S
 $\rightarrow \text{expr}$
 $\rightarrow \text{expr} - \text{expr}$
 $\rightarrow \text{expr} + \text{expr} - \text{expr}$
 $\rightarrow \text{expr} + \text{expr} * \text{expr} - \text{expr}$
 $\rightarrow \text{digit} + \text{digit} * \text{digit} - \text{digit}$
 $\rightarrow 9 + 7 * 5 - 6$



There are five situation for ~~this~~ this string
 This is ambiguous.

strategy

	operator	nonterminal
level I	+ -	expr, term
level II	* /	term, factor

define new non-terminals.

grammar for ~~arith~~ arithmetic expressions II.

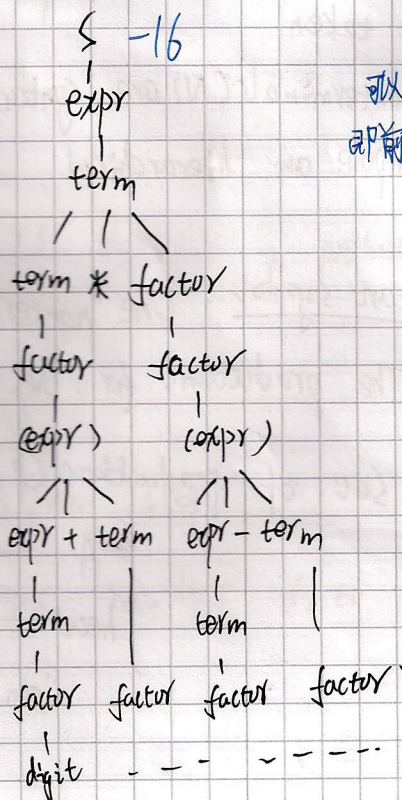
G#

$$\begin{aligned}
 S &\rightarrow \text{expr} \quad ① \\
 \text{expr} &\rightarrow \text{expr} + \text{term} \quad ② \\
 &\quad | \text{expr} - \text{term} \quad ③ \\
 &\quad | \text{term} \quad ④ \\
 \text{term} &\rightarrow \text{factor} * \text{factor} \quad ⑤ \\
 &\quad | \text{term} / \text{factor} \quad ⑥ \\
 &\quad | \text{factor} \quad ⑦ \\
 \text{factor} &\rightarrow \text{digit} \quad ⑧ \\
 &\quad | (\text{expr}) \quad ⑨ \\
 \text{digit} &\rightarrow 0111 \dots 9 \quad ⑩
 \end{aligned}$$

Notice:

- ① ambiguity = more than 1 parse trees for one string.
- ② more than 1 ~~derivation~~ derivation with one parse tree \rightarrow not ambiguous.

Ex1. $(9+7)*(5-6)$

$$\begin{aligned}
 &\overset{①}{\rightarrow} \text{expr} \\
 &\overset{②}{\rightarrow} \text{term} \\
 &\overset{⑤}{\rightarrow} \text{term} * \text{factor} \\
 &\overset{⑦}{\rightarrow} \text{factor} * \text{factor} \\
 &\overset{⑨}{\rightarrow} (\text{expr}) * (\text{expr}) \\
 &\overset{②}{\rightarrow} (\text{expr} + \text{term}) * (\text{expr}) \\
 &\overset{③}{\rightarrow} (\text{expr} + \text{term}) * (\text{expr} - \text{term}) \\
 &\overset{④}{\rightarrow} (\text{term} + \text{term}) * (\text{term} - \text{term}) \\
 &\overset{⑦}{\rightarrow} (\text{factor} + \text{factor}) * (\text{factor} - \text{factor}) \\
 &\overset{⑧}{\rightarrow} (\text{digit} + \text{digit}) * (\text{digit} - \text{digit}) \\
 &\overset{⑩}{\rightarrow} (9 + 7) * (5 - 6)
 \end{aligned}$$


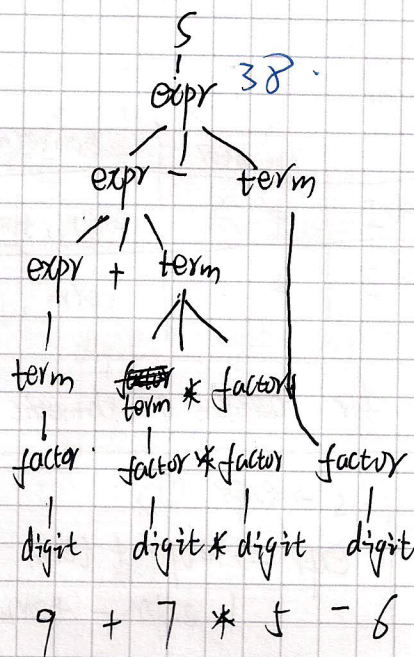
可以有多种分解方式
即前缀表达式

- ①
- ④
- ⑤
- ⑦
- ⑦
- ⑦

Result: There is more than one derivation, but all the derivations generate the same parse tree \Rightarrow The grammar is not ambiguous.

Ex 2. $9 + 7 * 5 - 6$.

- ① $S \rightarrow \text{expr}$
- ② $\text{expr} \rightarrow \text{expr} - \text{term}$
- ③ $\text{expr} \rightarrow \text{expr} + \text{term} - \text{term}$
- ④ $\text{term} \rightarrow \text{term} * \text{factor} - \text{term}$
- ⑤ $\text{term} \rightarrow \text{term} + \text{term} * \text{factor} - \text{term}$
- ⑥ $\text{factor} \rightarrow \text{factor} * \text{factor} - \text{factor}$
- ⑦ digit — — — — —
- ⑧ $9 + 7 * 5 - 6$



20/6/10/25

Definition context-free grammar

A context-free grammar consists of a start symbol, nonterminals, terminals and a set of productions.

1. Terminals (T) are the basic symbols from which strings are formed.
e.g. "a", "b", "if", "else", "for"
= token
2. Nonterminals (N) are syntactic variables that denote sets of strings impose an hierarchical structure on the language (expr, term, factor)
3. Start symbol. One nonterminal is distinguished as start symbol (S).
The productions for the start symbol, conventionally, are listed first.
4. Set of productions (P)

$A \rightarrow B$
head \rightarrow body
"can have the form"

Notice:

$A \rightarrow B$
 $| C$

(two alternatives)