Statistical Methods in Finance Project

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Executive Summary

- the assets & the S&P 500 are roughly normal distributed with some indications of stationarity
- the expected return of the tangency portfolio is around 17.43%, and its standard deviation is around 30.87%
- VaR is 49021.71 and ES is 24429.62. Short-sold VaR is 17,960.20 and short-sold ES is -30,164.01.
- the tangency portfolio and risk-free asset subject to 6% returns are $1051.57 and $9134.59, respectively
    - value-at-risk and expected shortfall in this configuration are considerably lower than those of the minimum variance portfolio with only risky assets
- including only the first four principal components can explain the majority of the variance in the returns of this selection of 15 stocks
- the highest VaR is TSLA at $84715.87; the stock with the lowest VaR is KO at $26945.52
- the normal copula fits the joint distribution of the 15 selected stock returns the best

Descriptive Statistics

Looking over the sample statistics of the assets, it is determined that the rest of the assets and the S&P 500 (SNP) means is around 1 or below 1, other than the asset NFLX at around 2. For the standard deviations of the assets, the majority of them are around 1, except the S&P, KO, and JNJ having a standard deviation around 0.5. Looking at the equity curves for each asset and the S&P 500 in the appendix, we can see that these assets have mostly positive upward trends, with a few exceptions. It is notable that NFLX, TSLA, and BMY seem to have more data points close to 0, which reflects observations from other statistics. The highest Sharpe ratio is with AMZN at 1.4044, the next highest being MSFT at 1.3861, while the rest of the assets and the S&P 500 are mostly within the range of 0.6 to 1.4.

Descriptive Statistics of the Assets & S&P 500

ASSETS	MEAN_RETURN	STD_DEV	SHARPE	SKEWNESS_COEF	KURTOSIS_COEF	BETA
JNJ	0.4825	0.7023	0.6736	-0.5569	4.2880	0.7826
ABBV	0.5351	1.2669	0.4149	0.0012	3.4268	0.8213
AMZN	1.8665	1.3223	1.4044	-0.1173	3.8709	1.3919
DIS	0.4287	1.0083	0.4158	0.5578	5.2029	0.8524
GE	-0.5109	1.4112	-0.3687	0.0418	4.3035	1.1723
GOOG	1.0354	0.9102	1.1272	0.5138	4.0094	0.9470
INTU	1.2823	1.0061	1.2652	-0.6363	4.6606	1.0713
KO	0.4848	0.6060	0.7845	-0.7128	3.5487	0.3983
MSFT	1.4755	1.0577	1.3861	-0.2163	3.7931	1.1795
NFLX	2.0927	2.0199	1.0314	0.3131	2.9684	1.1542
SBUX	1.0098	0.9577	1.0446	-0.4460	3.2394	0.5198
TSLA	1.2503	1.8478	0.6716	0.0778	2.8312	0.8685
SONY	1.4925	1.3461	1.1018	0.1721	2.7154	1.2908
BMY	0.1362	1.2719	0.0997	-0.9681	4.7988	0.7032
CRM	1.1002	1.0309	1.0581	0.2993	4.4868	0.9858
SNP	0.4476	0.5704	0.7682	-0.8985	5.5405	1.0000

Overall the returns for each asset and the S&P 500 look roughly normally distributed, with minor skewness most notably in assets BMY and TSLA. There are no strong indicators that there are outliers within the data. The majority of the assets after undergoing the Augmented Dickey-Fuller Test show that there is evidence of stationarity at the 0.05 significance level. Assets TSLA, NFLX, and ABBV fail the null hypothesis of non-stationarity at the 0.1 significance level. This is not surprising with stock returns, as there may be elements of the random walk and financial time series that can be exhibited, leading to stationarity.

Portfolio Theory

In computing the minimum variance portfolio (MVP), we find the assets' weights have a fairly even spread across them, all roughly around 0.1, though AMZN, GOOG, MSFT, and SONY have negative weights. KO has the largest weight at 0.4310 in this portfolio. In comparison to the annual mean return of the assets, the mean return of the MVP has around the same value as a third of the assets' mean return, while the remaining 66% are quite higher than the MVP's return.

expected return	**std. dev.**	**VaR**	**ES**	**short-sold VaR**	**short-sold ES**
0.4068	0.1298	49021.71	24429.62	17,960.20	-30,164.01

Comparing the VaR of the MVP to the VaR of the individual assets, we can see from the table that there is a wide spread of the individual assets but the VaR of the MVP would be the smallest compared to the individual assets.

The plot of the efficient portfolio frontier for the risky assets using the Markowitz approach is attached below in the appendix.

The Sharpe ratio of the tangency portfolio computed from the current risk-free annual Treasury bill rate illustrated in the efficient portfolio frontier plot is around 0.42, which is not greater than many of the included assets.

The expected return of the tangency portfolio is around 17.43%, and its standard deviation is around 30.87%.

The weights of the assets in the tangency portfolio are as follows:

Tangency Portfolio Weights

Stock ticker	**Weights**
JNJ	-0.1
ABBV	-0.1

AMZN	0.317144232
DIS	-0.1
GE	-0.1
GOOG	0.002206201
INTU	0.292640790
KO	-0.1
MSFT	0.287900193
NFLX	0.060360574
SBUX	0.316280415
TSLA	0.170881736
SONY	0.197915643
BMY	-0.045329785
CRM	-0.1

The weights of the assets in the minimum variance portfolio are as follows:

Minimum Variance Portfolio Weights

Stock ticker	**Weights**
JNJ	0.128121716
ABBV	0.109817828
AMZN	-0.052129244
DIS	-0.002802821
GE	0.069334355
GOOG	-0.014880029
INTU	-0.027568941
KO	0.499474572
MSFT	-0.1
NFLX	0.059163726
SBUX	0.169429416
TSLA	0.016848937

SONY	-0.022673826
BMY	0.075517689
CRM	0.092346620

Asset Allocation

The monthly risk (i.e., standard deviation) of the minimum variance portfolio subject to 6% returns with no short sales is about 14.51%

The weights of each of the assets in this portfolio are as follows:

Minimum Variance Portfolio Subject to 6% Return; No Short Sales

Stock ticker	**Weights**
JNJ	0.0329
ABBV	0.0628
AMZN	0.0244
DIS	0
GE	0
GOOG	0
INTU	0.0580
KO	0.3927
MSFT	0
NFLX	0.0639
SBUX	0.1976
TSLA	0.0592
SONY	0.0253
BMY	0.0698
CRM	0.0133

The monthly 5% value-at-risk and expected shortfall based on an initial $100,000 investment in the MVP subject to 6% returns with no short selling are $17862.67 and $23924.75, respectively.

When computing the weights of the combination of risk-free T-Bills and the tangency portfolio for the expected return of 6%, the tangency portfolio would have a weight of -0.05425916, while the weight of the amount invested in the risk-free asset would be 1.054259.

As such, the monthly risk of this combination of the tangency portfolio and the risk-free asset is about 1.67%.

The weights of each of the assets in this portfolio are as follows:

Tangency Portfolio and Treasury Bills Subject to 6% Return

Asset	**Weights**
JNJ	0.0054259165
ABBV	0.0054259165
AMZN	-0.0172079812
DIS	0.0054259165
GE	0.0054259165
GOOG	-0.0001197066
INTU	-0.0158784449
KO	0.0054259165
MSFT	-0.0156212240
NFLX	-0.0032751143
SBUX	-0.0171611112
TSLA	-0.0092719003
SONY	-0.0107387375
BMY	0.0024595563
CRM	0.0054259165
T-Bill	1.054259

The monthly 5% value-at-risk and expected shortfall based on an initial $100,000 investment in the tangency portfolio and risk-free asset subject to 6% returns are $1051.57 and $9134.59, respectively. The value-at-risk and expected shortfall in this configuration are considerably lower than those of the minimum variance portfolio with only risky assets.

Principal Component Analysis

From the sample correlation heatmap and sample pairwise correlation matrix attached in the appendix, it seems to appear that the three most highly correlated pairs of stocks are Netflix and Amazon, Microsoft and Google, and Microsoft and Salesforce. This follows intuition, as all of these stocks are software companies, and their corresponding stock performances should behave similarly. As such, further diversification (outside of the tech and software industry) may reduce risk with these assets since many of the chosen stocks are highly correlated with each other.

The loadings of the principal components, principal components analysis biplot, and scree plot are attached in the appendix.

From the attached results, it seems to appear that all the loadings of the first principal component are positive since the stocks are almost all positively correlated with each other. The principal components biplot seems to suggest that Starbucks and Coca-Cola behave similarly; Microsoft, Google, and Sony behave similarly; and Amazon and Salesforce behave similarly. These interpretations are intuitive, since Starbucks and Coca-Cola are both in the food and beverage industry; Microsoft, Google and Sony are all tech companies; and Amazon and Salesforce are also in the tech/software industry.

Lastly, it seems that including only the first four principal components can explain the majority of the variance in the returns of this selection of 15 stocks.

Risk Management

Given an initial $100,000 to invest and assuming the stock returns follow a normal distribution, the estimated 5% Value-at-Risk and Expected Shortfall for each individual stock are as follows, respectively:

5% Value-at-Risk ($)

JNJ	**ABBV**	**AMZN**	**DIS**	**GE**	**GOOG**	**INTU**	
30828.64	57677.55	47233.24	44879.72	72388.26	34589.27	37445.84	
KO	**MSFT**	**NFLX**	**SBUX**	**TSLA**	**SONY**	**BMY**	**CRM**
26044.7	39062.13	78469.83	37517.16	77320.13	51477.19	60662.67	39782.47

5% Expected Shortfall ($)

JNJ	**ABBV**	**AMZN**	**DIS**	**GE**	**GOOG**	**INTU**	
39388.82	73046.46	63183.78	57049.23	89413.46	45568.26	49564.94	
KO	**MSFT**	**NFLX**	**SBUX**	**TSLA**	**SONY**	**BMY**	**CRM**

33343.66	51878.04	102834.6	48999.75	99609.39	67714.1	76118.98	52217.92

The nonparametrically estimated 5% Value-at-Risk and Expected Shortfall for each individual stock are given below, as well:

5% Nonparametric Value-at-Risk ($)

JNJ	**ABBV**	**AMZN**	**DIS**	**GE**	**GOOG**	**INTU**	
31724.6	53891.98	34273.02	38047.49	77486.31	34050.68	32087.91	
KO	**MSFT**	**NFLX**	**SBUX**	**TSLA**	**SONY**	**BMY**	**CRM**
26945.52	37448.28	74448.06	40002.39	84715.87	49934.05	84558.36	29915.21

5% Nonparametric Expected Shortfall ($)

JNJ	**ABBV**	**AMZN**	**DIS**	**GE**	**GOOG**	**INTU**	
47342.88	78459.89	73322.5	54944.56	98594.79	39998.79	56544.52	
KO	**MSFT**	**NFLX**	**SBUX**	**TSLA**	**SONY**	**BMY**	**CRM**
39179.94	58752.43	89341.28	53919.47	100468.3	59715.27	101830.8	52816.62

The stock with the highest VaR is TSLA at $84715.87; the stock with the lowest VaR is KO at $26945.52. This result makes sense intuitively, given Tesla's infamous volatility and the fact that Coca-Cola is a blue-chip stock famous for its stability.

The stock with the highest ES is NFLX at $101830.80; the stock with the lowest ES is CRM at $39179.94.

The corresponding 95% confidence intervals and standard deviations for the Value-at-Risk and Expected Shortfall calculations above are given as follows, respectively:

95% Confidence Intervals for Value-at-Risk ($)

Stock ticker	**VaR Lower bound**	**VaR Upper bound**	**VaR Standard error**
JNJ	17534.87	41171.23	7878.66
ABBV	44081.06	86626.43	13251.49

AMZN	17324.43	85650.33	20976.04
DIS	23341.27	45262.27	8456.3
GE	36372.07	93255.89	16096.56
GOOG	19609.76	38512.37	5438.4
INTU	25023.76	44763.86	9484.22
KO	18161.14	38012.99	5939.24
MSFT	19618.51	53323.40	9906.45
NFLX	45429.36	91587.13	12928.11
SBUX	24858.69	47860.18	8395.75
TSLA	37481.01	103439.44	20422.23
SONY	29957.27	61222.49	9304.47
BMY	25377.04	95568.83	23136.3
CRM	21954.75	63589.25	13298.25

95% Confidence Intervals for Expected Shortfall ($)

Stock ticker	**ES CI lower bound**	**ES CI upper bound**	**ES standard error**
JNJ	28510.40	65103.57	11461.49
ABBV	51220.35	87903.30	11699.51
AMZN	27686.75	95945.00	21586.92
DIS	33457.31	81868.61	15887.36
GE	67162.24	115880.96	13615.82
GOOG	29886.99	44065.19	3872.8
INTU	30467.16	93797.42	21590.25
KO	24944.38	50580.96	7824.79
MSFT	35717.70	72977.45	10903.31

NFLX	66506.76	97151.09	8747.91
SBUX	34112.73	73224.03	11957.87
TSLA	66232.76	106581.57	11278.45
SONY	43875.09	61691.68	4889.97
BMY	58592.58	121070.59	16921.36
CRM	25595.65	64266.74	13052.5

Copulas

The AIC values of the normal, Frank, Clayton, Gumbel, and Joe copulas fitted to the joint distribution of the monthly stock returns are as follows:

Gaussian	**Frank**	**Clayton**	**Gumbel**	**Joe**
-155.5956	-90.33394	-120.237	-95.84345	-60.11087

Based on the AIC, it seems to appear that the normal copula fits the joint distribution of the 15 selected stock returns the best. As such, this suggests that the dependence structure between our monthly stock return data can be described by a normal distribution. The implication of this result is that the marginal distributions of each stock return are normally distributed, which seems to follow our findings in the Descriptive Statistics section.

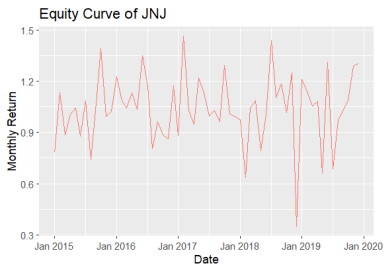
Conclusion

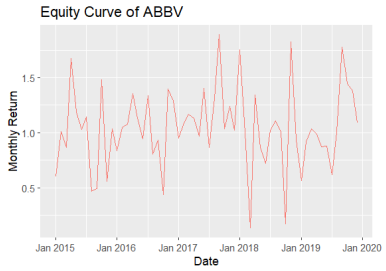
Considering that the asset returns had a roughly normal distribution with some indication of stationarity, this follows our analysis of how the best-fit copula was normal using the AIC method. With this collection of 15 assets with the minimum variance portfolio, the value-at-risk would be 49021.71 not short-selling and 17,960 when short-selling. Notably, in the tangency portfolio, the value-at-risk and expected shortfall are considerably lower than the minimum variance portfolio with only risky assets. We can see common trends through our analysis with certain stocks like Tesla, as it is known for its infamous volatility. Naturally, certain stocks in the same or similar industries follow similar trends and are accordingly correlated, as shown in the sample correlations and sample correlation biplot.

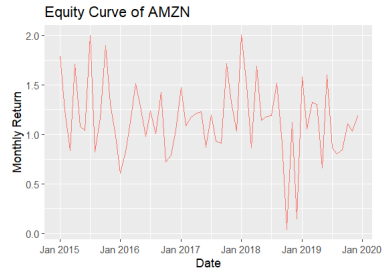
Overall, a well-diversified portfolio looks into the individual investor's risk profile which can help mitigate risks and maximize returns over the long term. It is essential to stay informed and vigilant, as market conditions and asset performance can change quickly and unpredictably.

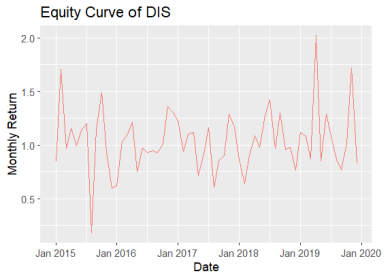
Appendix

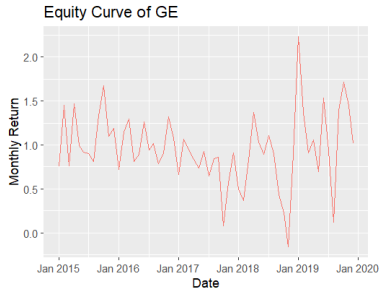
Kurtosis Curves for Each Asset

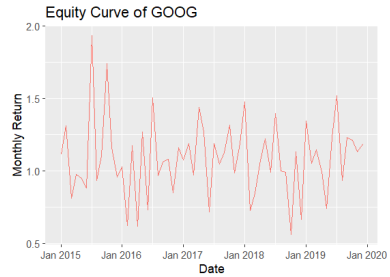


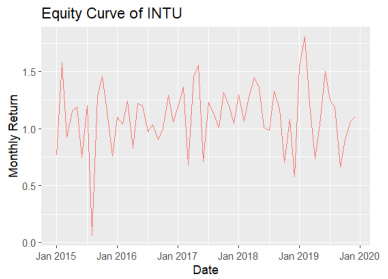


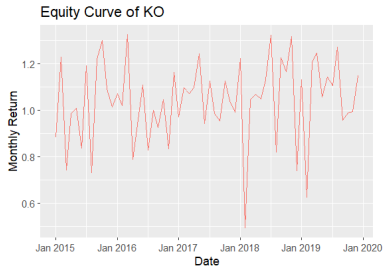


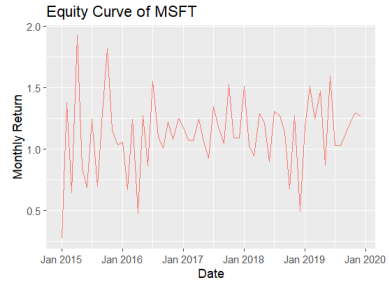


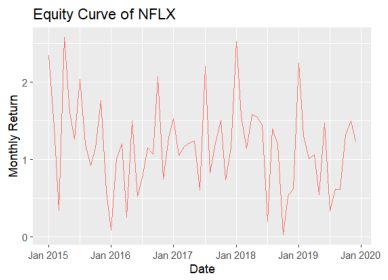


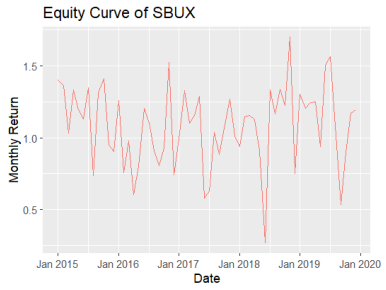


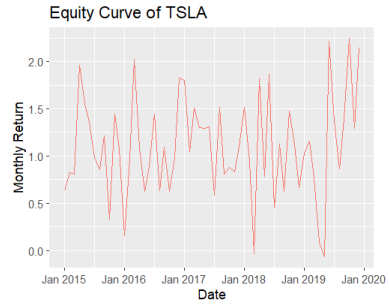


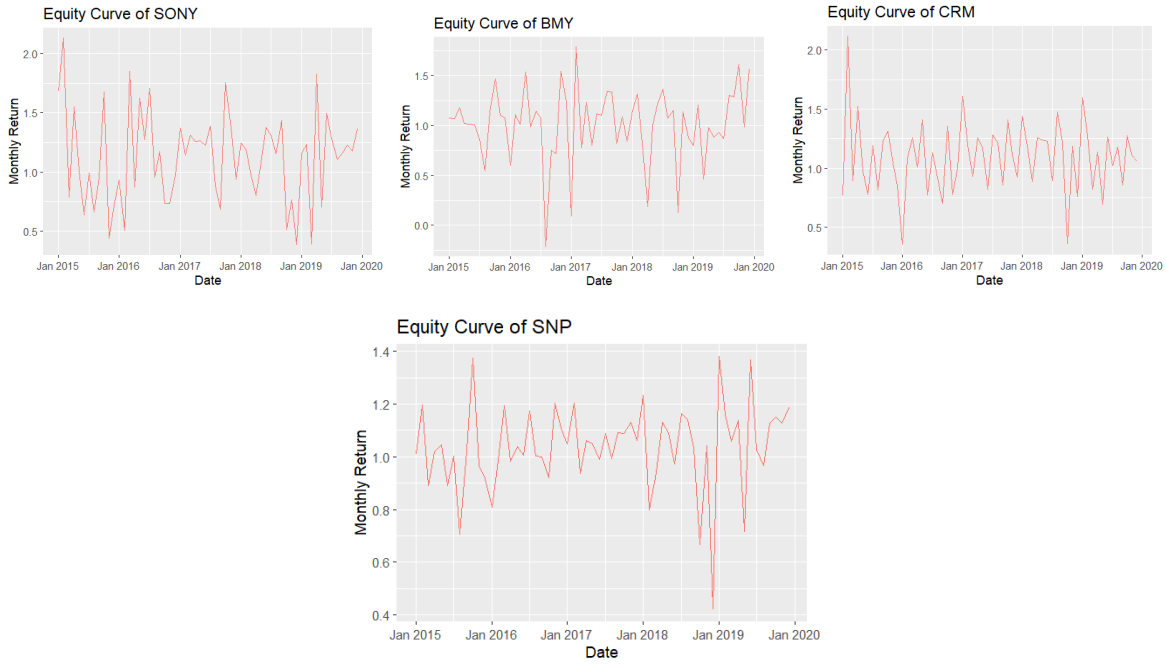




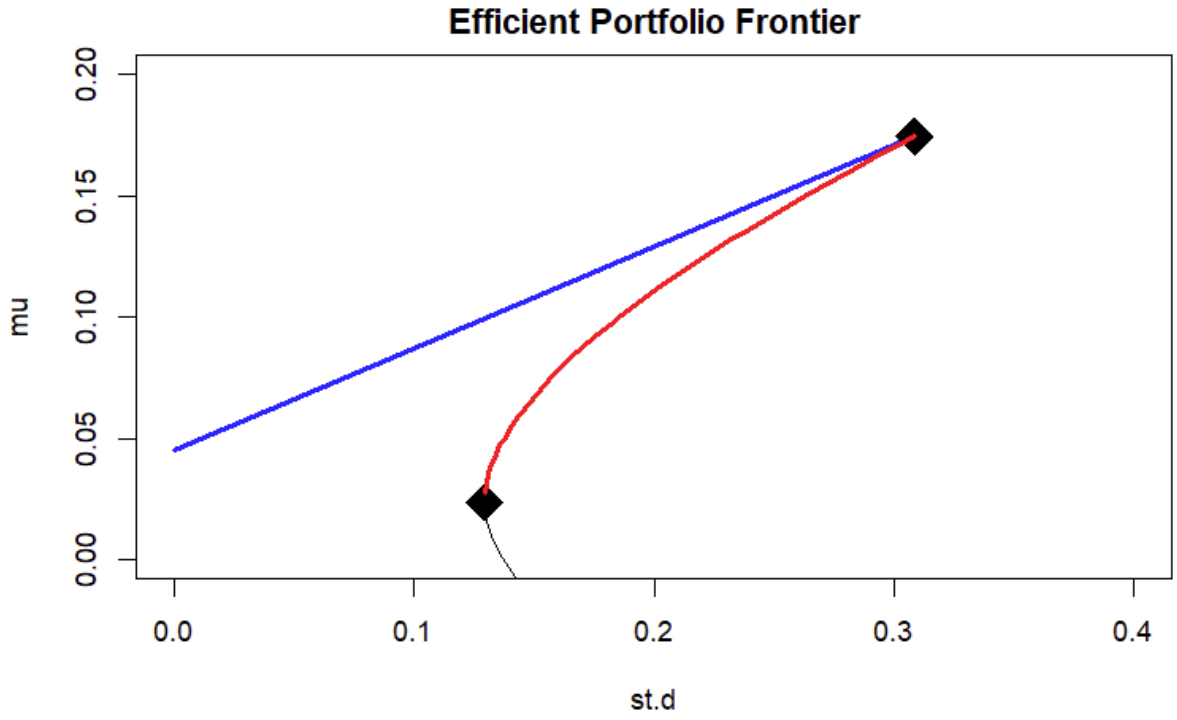








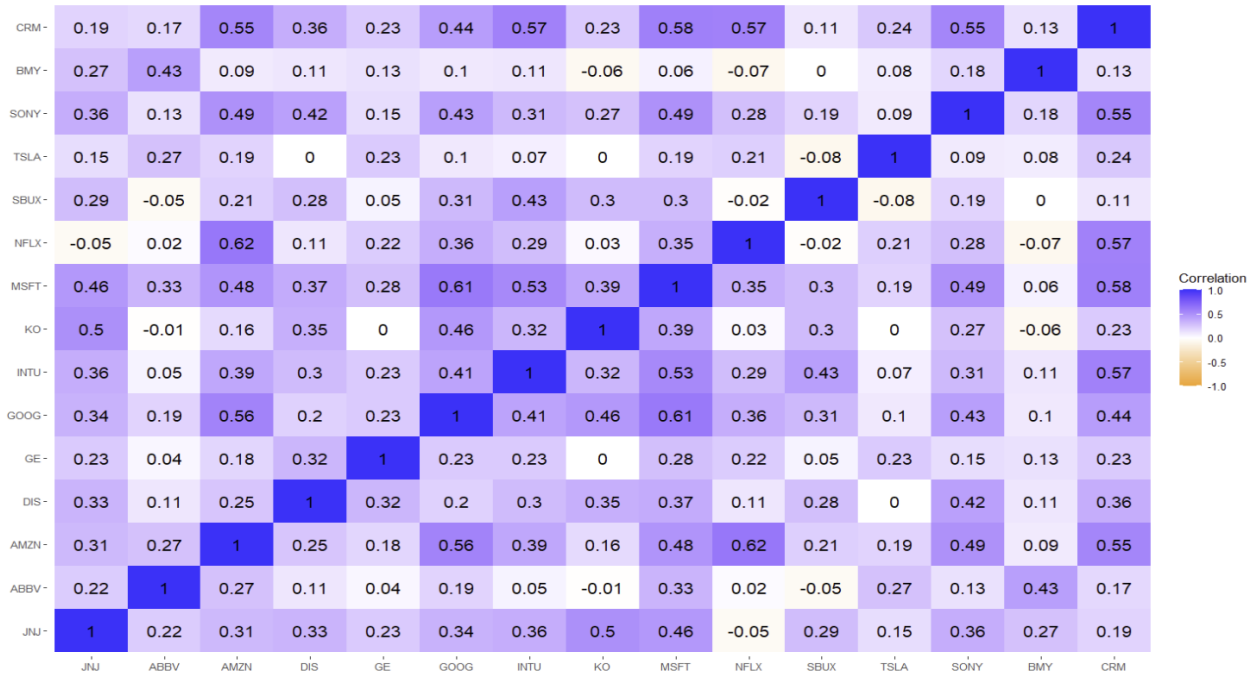
Efficient Portfolio Frontier



Sample Pairwise Correlation Matrix of the Assets

	JNJ	ABBV	AMZN	DIS	GE	GOOG	INTU	KO	MSFT	NFLX	SBUX	TSLA	SONY	BMY	CRM
JNJ	1	0.22	0.31	0.33	0.23	0.34	0.36	0.5	0.46	-0.05	0.29	0.15	0.36	0.27	0.19
ABBV	0.22	1	0.27	0.11	0.04	0.19	0.05	-0.01	0.33	0.02	-0.05	0.27	0.13	0.43	0.17
AMZN	0.31	0.27	1	0.25	0.18	0.56	0.39	0.16	0.48	0.62	0.21	0.19	0.49	0.09	0.55
DIS	0.33	0.11	0.25	1	0.32	0.2	0.3	0.35	0.37	0.11	0.28	0	0.42	0.11	0.36
GE	0.23	0.04	0.18	0.32	1	0.23	0.23	0	0.28	0.22	0.05	0.23	0.15	0.13	0.23
GOOG	0.34	0.19	0.56	0.2	0.23	1	0.41	0.46	0.61	0.36	0.31	0.1	0.43	0.1	0.44
INTU	0.36	0.05	0.39	0.3	0.23	0.41	1	0.32	0.53	0.29	0.43	0.07	0.31	0.11	0.57
KO	0.5	-0.01	0.16	0.35	0	0.46	0.32	1	0.39	0.03	0.3	0	0.27	-0.06	0.23
MSFT	0.46	0.33	0.48	0.37	0.28	0.61	0.53	0.39	1	0.35	0.3	0.19	0.49	0.06	0.58
NFLX	-0.05	0.02	0.62	0.11	0.22	0.36	0.29	0.03	0.35	1	-0.02	0.21	0.28	-0.07	0.57
SBUX	0.29	-0.05	0.21	0.28	0.05	0.31	0.43	0.3	0.3	-0.02	1	-0.08	0.19	0	0.11
TSLA	0.15	0.27	0.19	0	0.23	0.1	0.07	0	0.19	0.21	-0.08	1	0.09	0.08	0.24
SONY	0.36	0.13	0.49	0.42	0.15	0.43	0.31	0.27	0.49	0.28	0.19	0.09	1	0.18	0.55
BMY	0.27	0.43	0.09	0.11	0.13	0.1	0.11	-0.06	0.06	-0.07	0	0.08	0.18	1	0.13
CRM	0.19	0.17	0.55	0.36	0.23	0.44	0.57	0.23	0.58	0.57	0.11	0.24	0.55	0.13	1

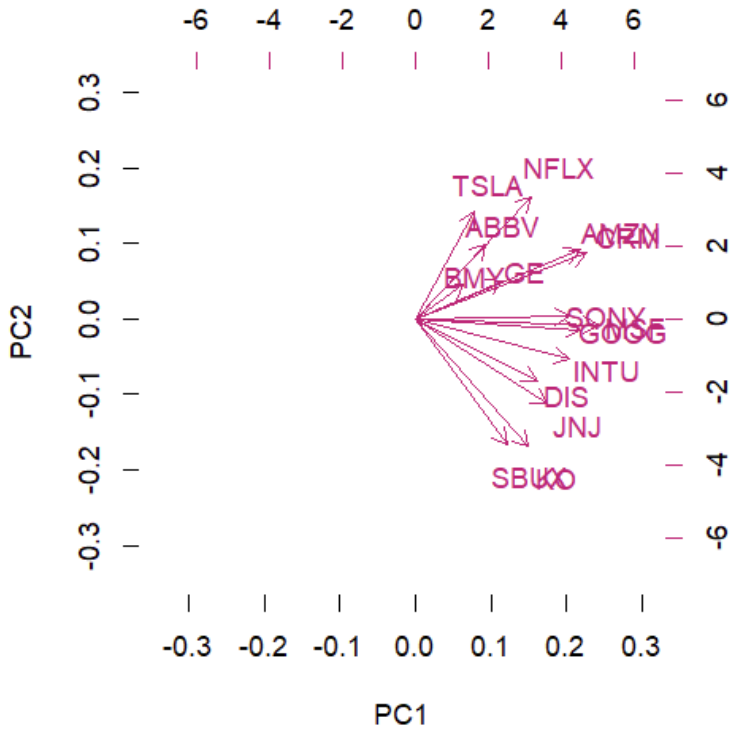
Pearson Correlation Coefficient Heatmap of the Assets



Principal Components Loadings

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12	PC13	PC14	PC15
JNJ	0.262466	-0.28536	0.337575	-0.04372	0.219056	0.089648	-0.10152	0.197382	-0.26333	0.176726	0.571839	0.141099	-0.10046	0.169952	-0.37692
ABBV	0.140039	0.250231	0.53158	0.295663	0.037269	-0.03749	0.049496	-0.41808	0.379634	0.028514	0.060497	-0.08969	0.397406	-0.09282	-0.21109
AMZN	0.329306	0.232193	-0.13264	0.22788	-0.06032	-0.03676	-0.18161	-0.23468	-0.30314	0.317424	0.283206	-0.3225	-0.01239	0.141661	0.527249
DIS	0.24438	-0.2064	0.071379	-0.39958	-0.31879	0.358349	0.209897	-0.39732	0.137156	0.299719	-0.14803	-0.15509	-0.36517	-0.06127	-0.09617
GE	0.174716	0.128322	0.077591	-0.75514	0.055642	-0.15702	-0.44636	-0.01817	0.058148	-0.11879	-0.00011	-0.0188	0.345169	0.032932	0.106613
GOOG	0.331623	-0.03623	-0.08345	0.236958	0.172059	-0.07493	-0.49975	0.004785	0.024351	-0.19347	-0.4025	-0.33629	-0.31111	0.068764	-0.35493
INTU	0.307946	-0.13512	-0.11947	-0.05806	-0.07543	-0.45156	0.309475	0.362741	0.253018	0.033515	0.199603	-0.39731	0.014201	-0.41073	-0.0241
KO	0.225622	-0.42827	-0.02709	0.12357	0.343842	0.339141	-0.03952	0.214298	0.155532	0.315262	-0.32354	0.070345	0.387908	-0.08281	0.280019
MSFT	0.367062	-0.02143	0.012668	0.0622	0.147538	-0.00618	-0.01063	-0.11295	0.365535	-0.3944	0.175422	0.473676	-0.37059	-0.07294	0.373004
NFLX	0.231762	0.41076	-0.39224	0.024628	-0.03667	-0.0135	-0.0985	0.008587	-0.0295	0.402501	-0.0359	0.473553	0.047875	-0.33569	-0.32924
SBUX	0.182764	-0.42199	-0.07294	0.034916	-0.0565	-0.55442	0.174375	-0.40852	-0.32419	-0.02803	-0.21538	0.263123	0.180988	0.130776	-0.05241
TSLA	0.117183	0.361283	0.18859	-0.19746	0.61099	-0.01556	0.448038	-0.00794	-0.32402	-0.02411	-0.24793	-0.08521	-0.16007	-0.07024	0.044474
SONY	0.30775	0.007334	-0.00673	0.069131	-0.30063	0.404447	0.087707	0.042478	-0.42683	-0.51985	0.008637	-0.03447	0.279369	-0.32058	-0.04956
BMY	0.096802	0.115855	0.575497	0.067009	-0.40886	-0.18848	-0.1028	0.386018	-0.14105	0.178369	-0.3375	0.201331	-0.17025	-0.04566	0.205037
CRM	0.342692	0.220745	-0.16323	-0.01025	-0.17772	0.068271	0.323519	0.254676	0.192335	-0.06578	-0.08247	0.030073	0.163781	0.715827	-0.11427

Principal Components Biplot



Principal Components Scree Plot