Fast Characterization of Inducible Regions of Atrial Fibrillation Models with Multi-Fidelity Gaussian Process Classification

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Center for Computational Medicine in Cardiology CCMC
Euler Institute

Gander L.(1), Pezzuto S.(1), Gharaviri A.(1), Krause R.(1), Perdikaris P.(2), Sahli Costabal F.(3)

(1) Center for Computational Medicine in Cardiology, Euler Institute, Università della Svizzera italiana, Lugano.
(2) Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia.
(3) Department of Mechanical and Metallurgical Engineering, School of Engineering, Pontificia Universidad Católica de Chile, Santiago de Chile.

Goal

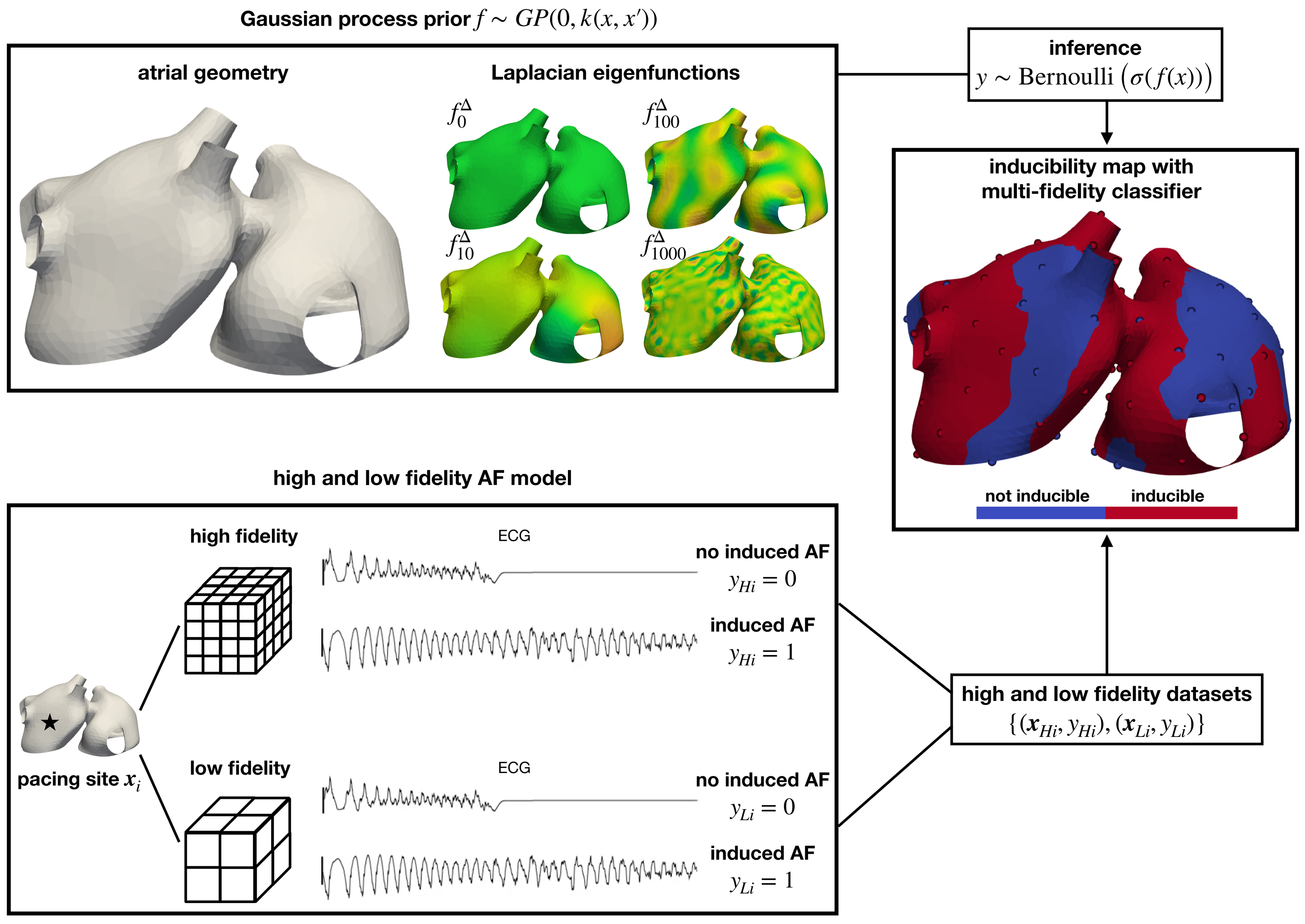
Given: atrial model on the geometry $\Omega \subset \mathbb{R}^3$ and atrial surface $\mathcal{S} \subset \Omega$.
For any $\boldsymbol{x} \in \mathcal{S}$, determine if a pacing protocol applied at $\boldsymbol{x}$ induces an Atrial Fibrillation event ($y = 1$) or not ($y = 0$). The mapping $F\colon \mathcal{S} \to \{0, 1\}$, $\boldsymbol{x} \mapsto y$ is called **inducibility map** [2].

Method

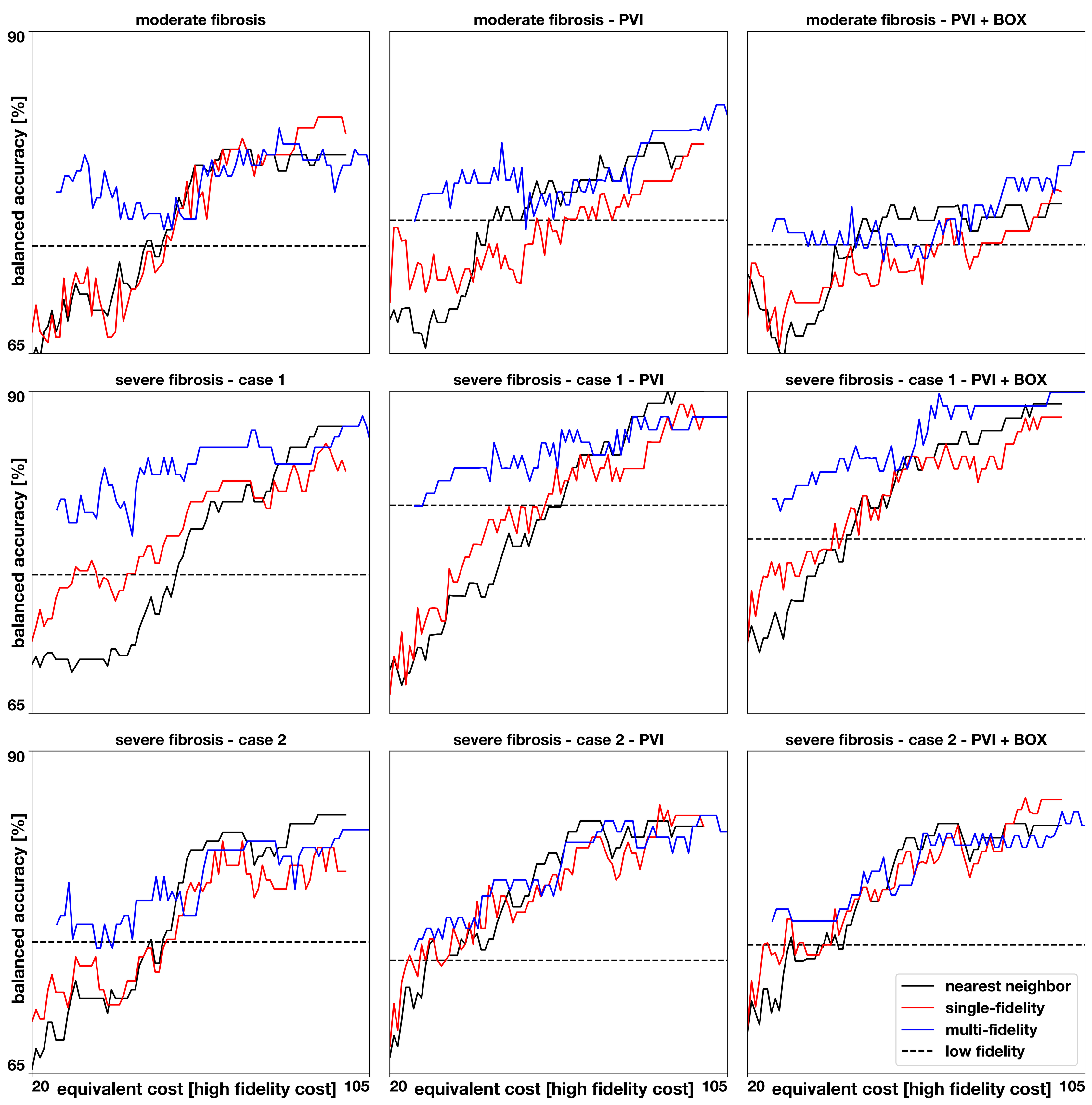
- High fidelity model and **low fidelity model based on a coarser discretization** of the monodomain equation in space and time. The low fidelity model is **16 times faster than the high fidelity one**
- Multi-fidelity Gaussian Process classification [3]
  - Train set $\{(\boldsymbol{x}_{Hi}, y_{Hi})_{i=1}^{N_H}, (\boldsymbol{x}_{Li}, y_{Li})_{i=1}^{N_L}\}$
  - Latent functions $f_H(\boldsymbol{x}) = \rho f_L(\boldsymbol{x}) + \delta(\boldsymbol{x})$
  - Gaussian Process (GP) priors on f_L and δ, and prior distributions over ρ, $\boldsymbol{\theta}_L$ and $\boldsymbol{\theta}_H$

$$f_L \sim \mathcal{GP}(0, k(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{\theta}_L))$$
$$\delta \sim \mathcal{GP}(0, k(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{\theta}_H))$$

  - Matérn kernel k on the manifold based on the eigenpairs of the Laplacian [1]
  - Bayesian inference of the posterior distributions of ρ, $\boldsymbol{\theta}_L$, $\boldsymbol{\theta}_H$ and $\boldsymbol{f} = \begin{bmatrix} f_L \\ f_H \end{bmatrix}$
  - Posterior distribution of the predictive latent function $\boldsymbol{f}^*(\boldsymbol{x}^*)$ at a new location $\boldsymbol{x}^*$ obtained by conditioning on the train data
  - Prediction $y^* = \sigma(\boldsymbol{f}^*(\boldsymbol{x}^*))$, where $\sigma\colon \mathbb{R} \to [0, 1]$ is the sigmoid function

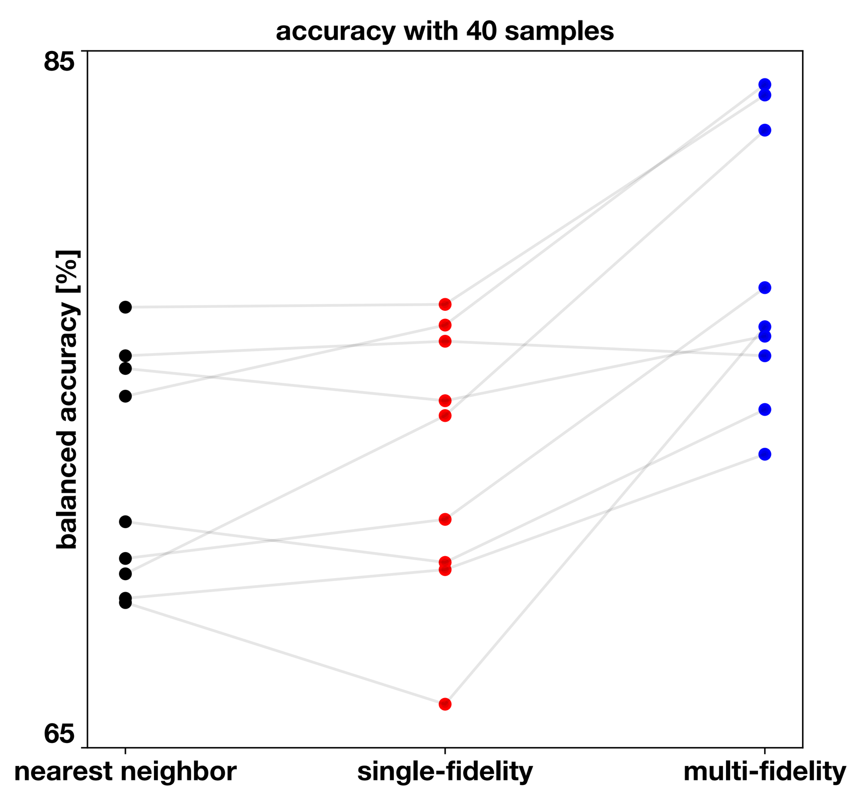


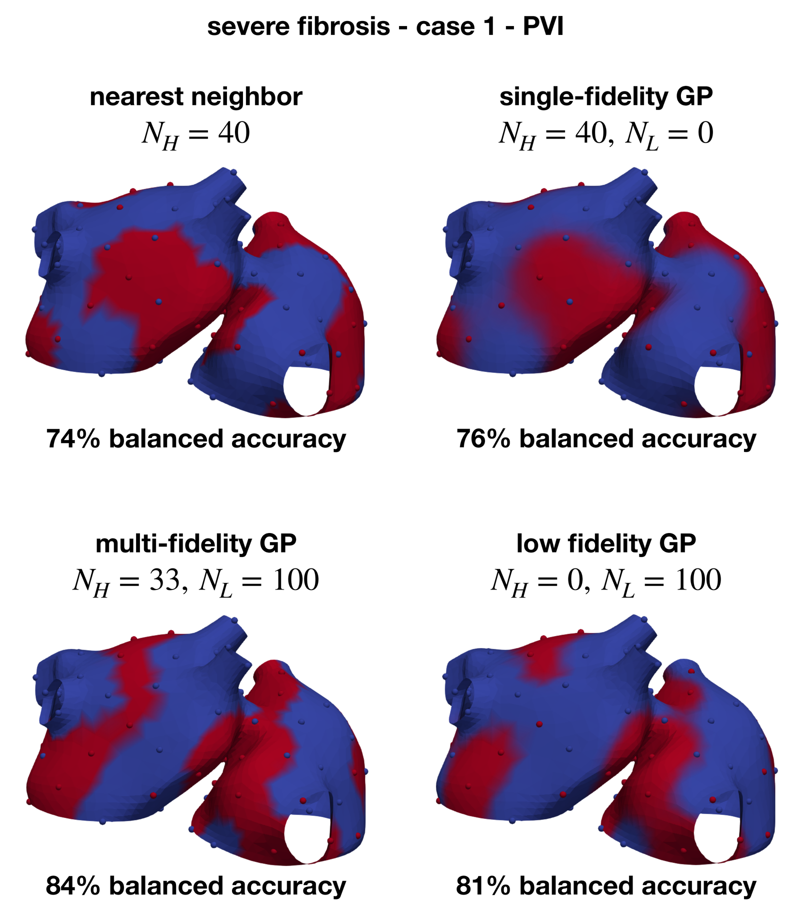
Numerical Results



- 9 atrial models
- Comparison of 4 classifiers for N_H ranging from 20 to 100
  - nearest neighbor
  - single-fidelity GP
  - multi-fidelity GP with $N_L = 100$
  - low fidelity GP with $N_H = 0$ and $N_L = 100$
- Balanced accuracy computed on a test set

For small train sets the multi-fidelity GP classifier performs better than the single-fidelity GP and the nearest neighbor classifiers.





- In clinical studies a typical choice is $N_H = 40$ high fidelity simulations
- The multi-fidelity classifier with $N_H = 33$ has equivalent cost

On average, **with the multi-fidelity GP classifier we gain 5.4% accuracy compared to the single-fidelity GP classifier and 5.7% accuracy compared to the nearest neighbor classifier.**

Conclusion

- It is worth taking advantage of the cheap low fidelity model in a multi-fidelity framework
  - higher accuracy can be achieved with fixed computational cost
  - a target accuracy can be achieved with lower computational cost
- The multi-fidelity strategy is interesting for clinical applications, where patient-specific results need to be delivered within practical time constraints
- **The multi-fidelity GP classifier is efficient in evaluating the effect of personalized ablation therapies**

References

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[3] F. Sahli Costabal, P. Perdikaris, E. Kuhl, and D. E. Hurtado. Multi-Fidelity Classification using Gaussian Processes: Accelerating the Prediction of Large-Scale Computational Models. *Computer Methods in Applied Mechanics and Engineering*, 357:112602, 2019.