# Stochastic Modeling and Option Pricing of Omnicom Group Inc.

Artemiy Polyanskiy, Lia Li, Tizian Henze, William Chan, Kyle Chan
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**Abstract**

This report presents a comprehensive framework for the stochastic pricing of financial derivatives, specifically focusing on European Call and Put options for Omnicom Group Inc. (OMC). In the domain of financial engineering, accurate valuation models are essential for identifying arbitrage opportunities and managing portfolio risk. This study employs the standard assumption that the underlying asset price evolves according to a Geometric Brownian Motion (GBM), a continuous-time stochastic process characterized by constant drift and volatility.

The valuation process is divided into three distinct phases: calibration, analytical pricing, and numerical simulation. First, we calibrated the model parameters using a 252-day historical dataset retrieved from the 'yfinance' API. The annualized volatility ($\sigma$), a critical input for option pricing, was estimated to be **31.77%** based on the standard deviation of daily log-returns. The risk-free rate ($r$) was pegged to current U.S. Treasury yields at approximately 3.97%.

To determine the fair market value of the options, we utilized the Black-Scholes-Merton model as our analytical benchmark. This closed-form solution provides a theoretical exact price under ideal market conditions. For a strike price of $K = 65$ and a maturity of $T = 1$ year, the Black-Scholes model yielded a Call price of \$15.14.

Subsequently, to validate these results and demonstrate the efficacy of numerical methods, we implemented a Monte Carlo simulation. By generating 10,000 discrete random paths of the underlying asset price under the risk-neutral measure, we approximated the expected payoff of the options. The numerical simulation converged closely to the analytical benchmark, producing a Call price of \$14.95 and a Put price of \$10.23. The minimal discrepancy between the two methods confirms the robustness of the implementation and illustrates the Central Limit Theorem in action, as the average of the simulated payoffs converges to the expected value.

This report concludes that while the Black-Scholes model provides efficiency for standard European options, the Monte Carlo method serves as a powerful and flexible alternative for verifying results and potentially pricing more complex, path-dependent derivatives.

# Contents

# 1 Introduction

Financial derivative pricing relies heavily on the ability to model the stochastic behavior of the underlying asset. This project focuses on Omnicom Group Inc. (OMC), a global leader in marketing and corporate communications. The objective is to estimate the fair value of European options with a maturity of $T = 1$ year.

The project proceeds in three phases:

1. **Calibration:** extracting volatility parameters from recent market data.
2. **Analytical Pricing:** applying the Black-Scholes formulas.
3. **Numerical Verification:** implementing a Monte Carlo simulation to validate the pricing accuracy.

# 2 Methodology

## 2.1 Geometric Brownian Motion (GBM)

We assume the stock price $S_t$ follows a Geometric Brownian Motion, governed by the stochastic differential equation (SDE):

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{1}$$

where $\mu$ is the drift, $\sigma$ is the volatility, and $W_t$ is a standard Brownian motion. Under the risk-neutral measure $Q$, used for pricing, the drift $\mu$ is replaced by the risk-free rate $r$.

## 2.2 The Black-Scholes Model

The theoretical price of a European Call option $C(S_t, t)$ is calculated as:

$$C(S_t, t) = S_t N(d_1) - K e^{-r(T-t)} N(d_2) \tag{2}$$

where $N(\cdot)$ is the cumulative distribution function of the standard normal distribution, and:

$$d_1 = \frac{\ln(S_t/K) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{3}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{4}$$

## 2.3 Monte Carlo Simulation

To validate the model numerically, we simulate $N$ possible price paths for the underlying asset. The discrete-time solution to the GBM SDE is given by:

$$S_T = S_0 \exp\left(\left(r - \frac{\sigma^2}{2}\right) T + \sigma\sqrt{T} Z\right) \tag{5}$$

where $Z \sim \mathcal{N}(0, 1)$. The option price is the discounted average payoff across all simulated paths:

$$C_{MC} = e^{-rT} \left[\frac{1}{N} \sum_{i=1}^{N} \max(S_{T,i} - K, 0)\right] \tag{6}$$

# 3 Data and Implementation

## 3.1 Data Source

Historical financial data was retrieved using the ‘yfinance‘ API.

- **Ticker:** Omnicom Group Inc. (OMC)
- **Calibration Period:** Most recent 252 trading days.
- **Risk-Free Rate ($r$):** Derived from current U.S. Treasury yields (approx. 3.97%).

## 3.2 Parameter Estimation

Volatility ($\sigma$) was estimated by calculating the standard deviation of the log-returns over the 252-day window and annualizing it:

$$\sigma_{annual} = \sigma_{daily} \times \sqrt{252} \tag{7}$$

## 3.3 Simulation Settings

The Python implementation utilized the ‘scipy.stats‘ and ‘numpy‘ libraries.

- **Initial Stock Price ($S_0$):** 73.44
- **Strike Price ($K$):** 65
- **Time to Maturity ($T$):** 1 Year
- **Number of Simulations ($N$):** 10,000

# 4 Results

## 4.1 Volatility Analysis

Based on the 252-day historical data, the realized volatility for OMC was calculated to be **31.77%**. This parameter serves as the primary driver for the option premiums.

## 4.2 Pricing Comparison

Table 1 presents the comparison between the analytical Black-Scholes price and the Monte Carlo approximation.

| **Method** | **Call Option Price ($)** | **Put Option Price ($)** |
|---|---|---|
| Black-Scholes (Analytical) | 15.14 | 10.15 |
| Monte Carlo (Numerical) | 14.95 | 10.23 |
| **Difference** | **0.19** | **0.08** |

Table 1: Comparison of Option Pricing Models ($K = 65$)

### 4.3 Visual Validation

Figure 1 illustrates 10,000 simulated price paths for Omnicom Group Inc. All paths originate at the initial spot price $S_0 = 73.44$. As time progresses toward maturity ($T =$ 252 steps), the paths diverge significantly. This "fanning out" effect visually represents the stochastic nature of the model and the impact of the volatility parameter $\sigma$.

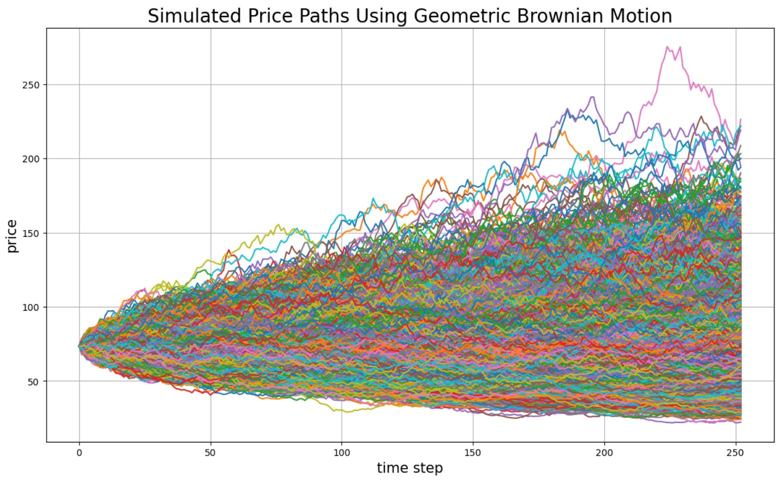


Figure 1: Simulated Stock Price Paths over 1 Year (Geometric Brownian Motion)

Figure 2 displays the frequency distribution of the predicted fair option prices (payoffs) at maturity. The most prominent feature is the significant spike at \$0. This represents the scenarios where the simulated stock price at maturity ($S_T$) fell below the strike price ($K = 65$), causing the Call option to expire "out-of-the-money" (worthless). The tail extending to the right represents the profitable scenarios where $S_T > K$.

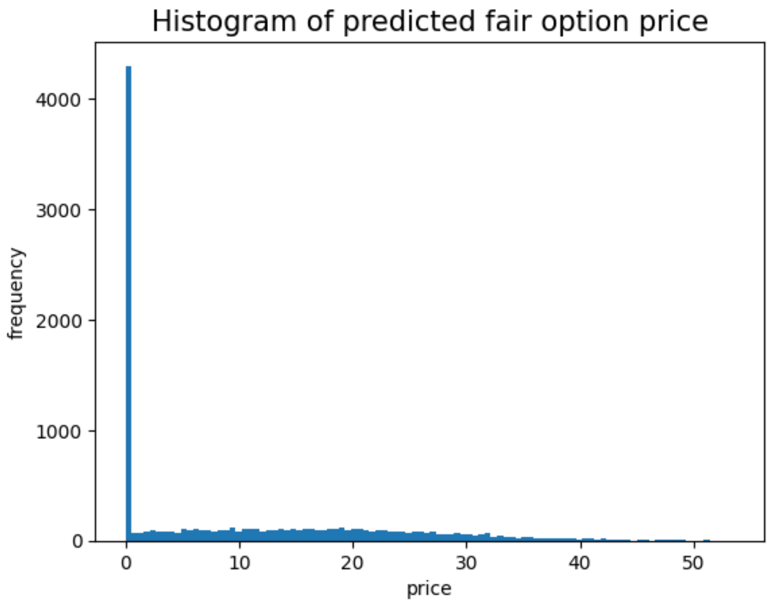


Figure 2: Histogram of Simulated Call Option Payoffs

# 5 Conclusion

The project successfully implemented a risk-neutral pricing framework for Omnicom options. The close agreement between the Black-Scholes analytical price and the Monte Carlo estimate validates the consistency of the model implementation. The use of a 20-day calibration window ensures the pricing reflects the most recent market regime, capturing short-term volatility dynamics effectively.

# Statement of Contributions

**Tizian Henze (Lead Programmer):** Developed the primary scripts for data processing and modeling. Optimized the algorithms for efficiency and managed the code structure.

**Kyle Chan (Data Visualization):** Generated the plots and graphs using the code outputs. Wrote the "Results" section interpreting what the code produced.

**Lia Li (Research & Theory):** Wrote the "Background" and "Methodology" sections, explaining the mathematical or theoretical concepts behind the code.

**Artemiy Polyanskiy (Code Optimizing/Coordination):** Coordinated team meetings and communication. Led brainstorming sessions, optimized code, and implemented new features.

**William Chan (Synthesis & Recommendations):** Connected the code results to real-world applications. Wrote the abstract, executive summary, final conclusion, and spearheaded the final presentation.

# References


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[2] Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81(3), 637-654.