Computational Optimization - MSc AIDA UoM

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The objectives of scaling techniques are the following:

1. To produce compact bounds on decision variables
2. To reduce the condition number of matrix A
3. To improve the numerical behavior of solution algorithms
4. To reduce the number of iterations required to solve a linear problem
5. To simplify the initialization/setting of tolerances.

Several techniques have been developed to transform a matrix into a well-scaled matrix.

Tasks:

[**A.**] Write code in Python that reads a file in matrix storage format according to the following form:

$$\begin{aligned} \min(\max) \quad & c^T x \\ \text{s.t.} \quad & Ax \otimes b \\ & x \geq 0 \end{aligned}$$

where $\otimes = \{\leq, =, \geq\}$, $c, x \in \mathbb{R}^n$, $b \in \mathbb{R}^m$ and $A \in \mathbb{R}^{m \times n}$.

- A: Dimensions $m \times n$. Matrix A stores the coefficients of the technological constraints.
- b: Dimensions $m \times 1$. Vector b stores the right-hand sides of the technological constraints.
- c: Dimensions $1 \times n$. Vector c stores the coefficients of the objective function.
- $Eqin$: Dimensions $m \times 1$. Vector $Eqin$ stores the types of constraints. If $Eqin(i) = -1$, then the i^{th} constraint is of type $\leq$. If $Eqin(i) = 1$, then it is of type $\geq$. If $Eqin(i) = 0$, then it is of type $=$.
- $MinMax$: Dimensions 1×1. This variable specifies the type of the problem. If $MinMax = -1$, then the problem is a minimization. If $MinMax = 1$, then it is a maximization.
- R: Dimensions (number of ranges in the MPS)x4. This matrix stores the ranges of the constraints as follows:
  - 1st column: constraint name
  - 2nd column: The RHS of the constraint
  - 3rd column: RHS+—range— 1, if range ¿ 0; -1, if range ¡ 0; 0, if range = 0
- BS: Dimensions (number of bounds in the MPS)x3. This matrix stores the bounds of the constraints as follows:
  - 1st column: variable name

2$^{\text{nd}}$ Column	**3$^{\text{rd}}$ Column**
Type of Bound	Value of Bound
LO: Lower Bound	Value
UP: Upper Bound	Value
FX: Fixed	Value
FR: Free	None
MI: Minus Infinity	None
PL: Plus Infinity	None

[**B.**] Write Python code that implements the Arithmetic Mean scaling technique, as presented in lecture 4.