MTHE 332/393 Lab Manual (Solutions)

December 9, 2020

Lab 1

Intro

1.1 Deliverables

1. Plots of motor position (θ) and velocity (ω) with respect to time
2. What is the steady state angular velocity (in rads/s) of the motor? Does this correlate to the vlaue obtained using the encoder plot?
3. Determine the motor constant τ. Include a plot at t=τ (use proper units)
4. Determine the motor torque constant, K_E (include units). Show your calculations.

1.2 Solutions

1. The plots for position and angular velocity of the motor can be found in figures 1.1, 1.2.

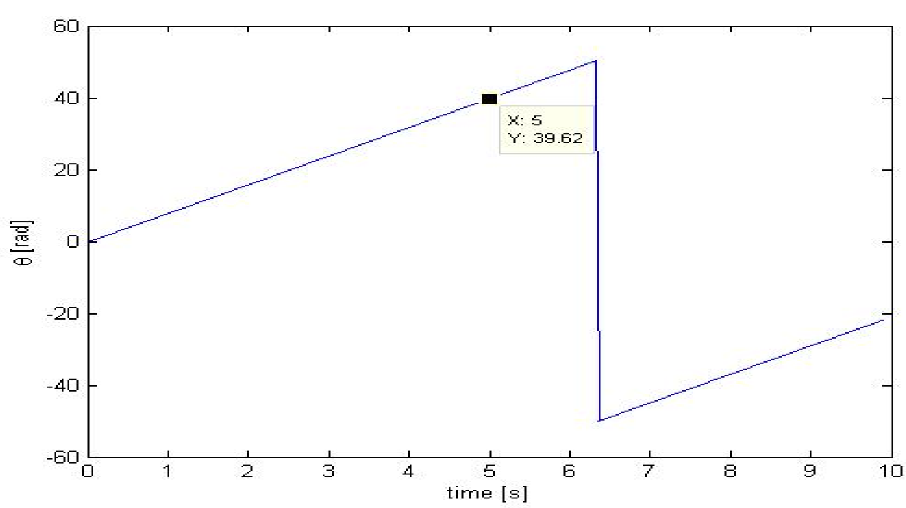

Figure 1.1: Plot for theta vs time for Lab 1

2. Steady state angular velocity is roughly 8.25 rads/s

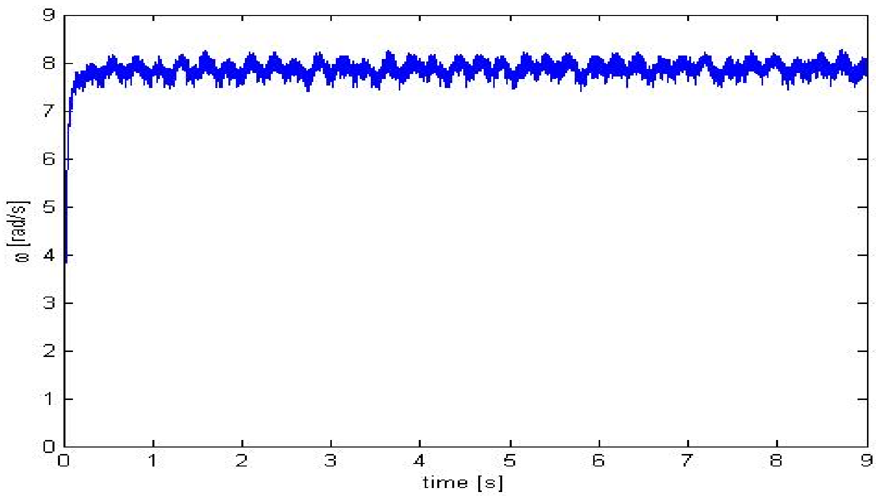

Figure 1.2: Plot of omega vs time for Lab 1

3. By manipulating the solution to equation The motor constant τ was found to be roughly 0.036. A zoomed in plot of the angular velocity at $t = \tau$ can be found in figure 1.3

4. By using the value of τ and rearranging equation K_E was found to be 229.16.

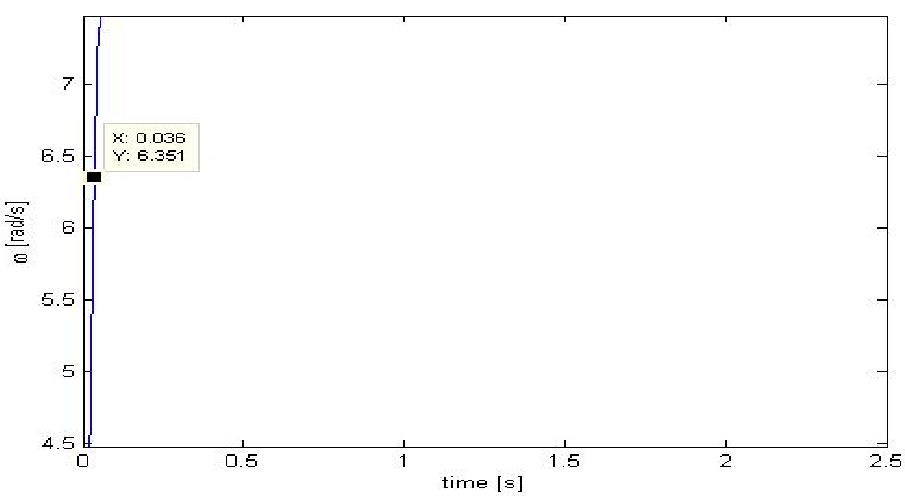

Figure 1.3: Zoomed in plot of omega vs time at $t = \tau$

Lab 2
Frequency Response

2.1 Deliverables

1. Include the Matlab generated bode plots for θ and ω. Use your K_E and τ values from lab 1

2. Plots of your motor angular position and velocity and the input function $u = 5\sin(t^2)$. Comment on the general trend of the magnitude and phase shift of the outputs. Does this agree with the bode plots you generated?

3. Include the tabulated data gathered from your experimental Bode plots.

4. Include experimental bode plots for both magnitude and phase difference.

2.2 Solutions

1. Figures 2.1 and 2.2 show the Matlab generated Bode plots for the systems angular position and velocity.

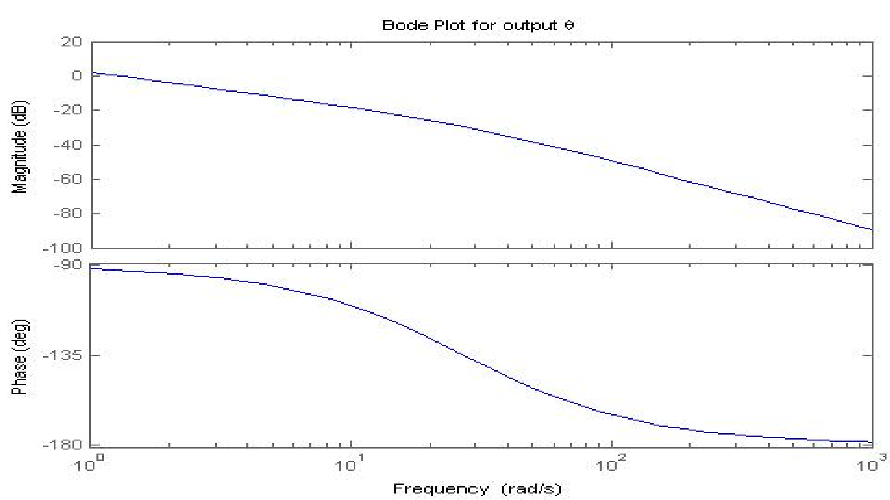

Figure 2.1: Matlab generated Bode plot for output θ

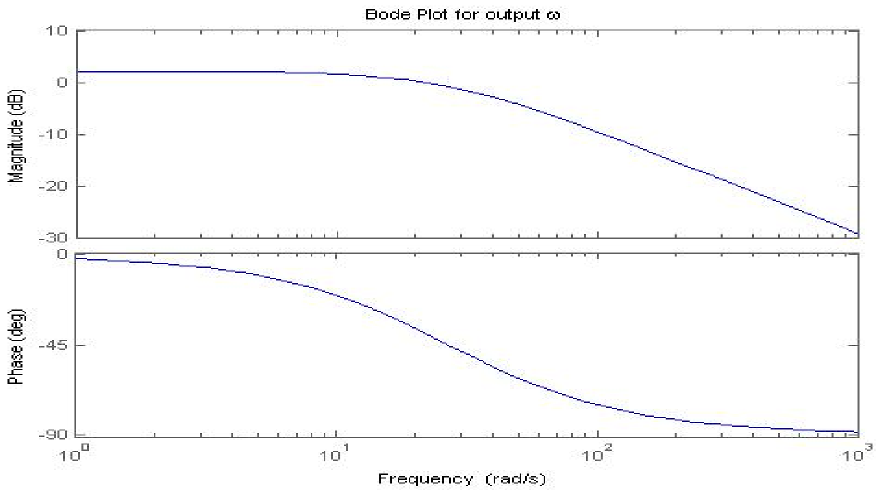

Figure 2.2: Matlab generated Bode plot for output ω

2. Figures 2.3 and 2.4 show the systems response to an input of a harmonic signal with increasing frequency. As you can see, the magnitude of the output decreases as the frequency increases, which agrees with the Bode plot generated in Matlab.

3. Table 2.1 includes data for the experimental Bode plots.

4. Figures 2.5 and 2.6 show the experimental bode plots for magnitude and phase difference when the output is ω

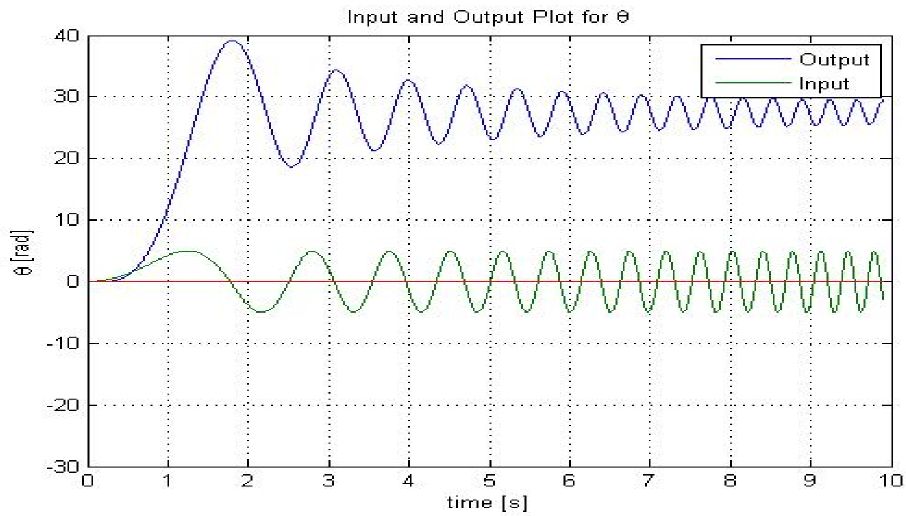

Figure 2.3: Input and output for θ for the input $5\sin(t^2)$

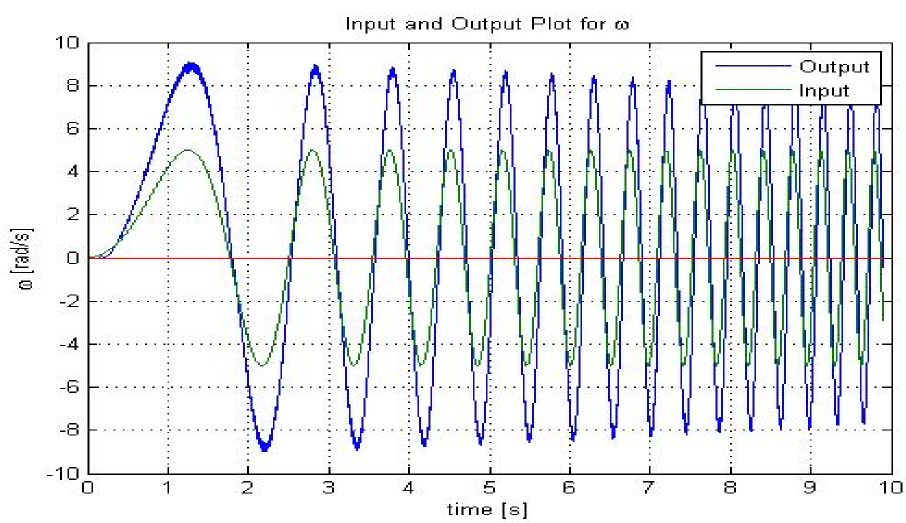

Figure 2.4: Input and output for ω for the input $5\sin(t^2)$

Table 2.1: Tabulated data for experimental Bode plots.

Omega (rad/s)	Magnitude, M	Gain (dB)	Peak-time Diff (input-output)	Phase (rad)	Phase (degree)
0.5	1.40	2.92	0	0	0.00
1	1.30	2.28	0	0	0.00
3	1.18	1.44	0	0	0.00
5	1.17	1.36	0	0	0.00
10	1.16	1.29	-0.01	-0.1	-5.73
20	1.05	0.42	-0.018	-0.36	-20.63
30	0.87	-1.26	-0.014	-0.42	-24.06
60	0.65	-3.74	-0.012	-0.72	-41.25
80	0.53	-5.60	-0.012	-0.96	-55.00
100	0.42	-7.54	-0.01	-1	-57.30
150	0.33	-9.76	-0.009	-1.35	-77.35
200	0.28	-11.06	-0.008	-1.6	-91.67
300	0.22	-13.15	-0.006	-1.8	-103.13

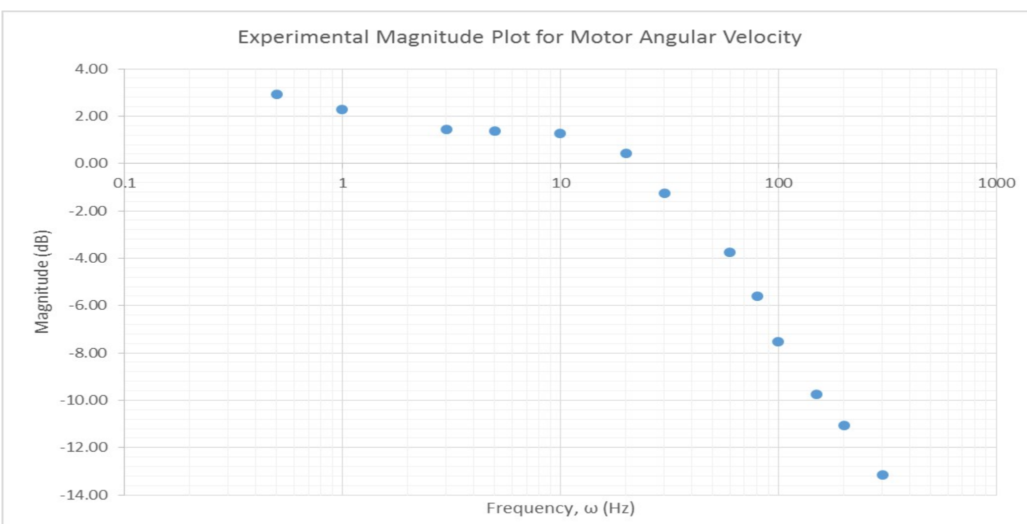

Figure 2.5: Experimental magnitude Bode plot for ω

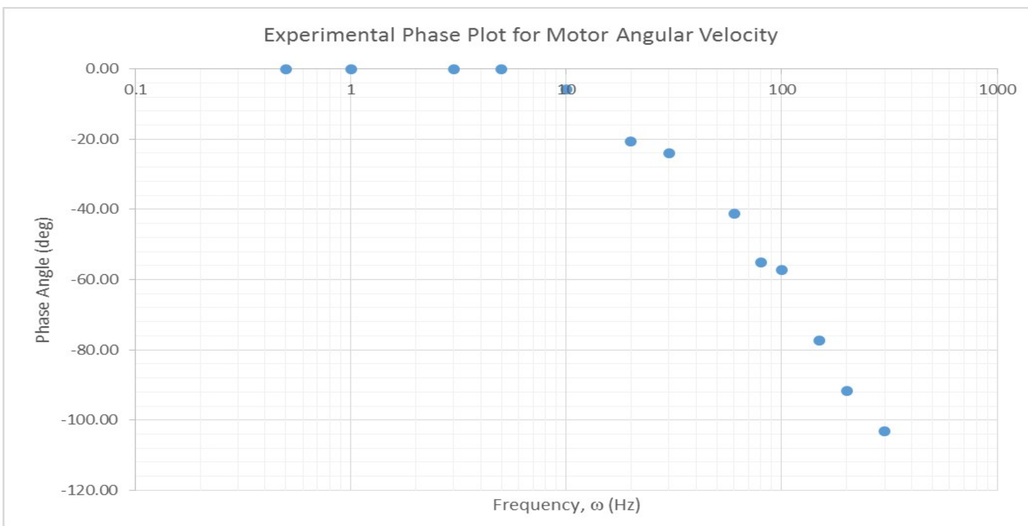

Figure 2.6: Experimental phase Bode Plot for ω

Lab 3
PID

3.1 Deliverables

1. Pick a good K value (without P or D values), comment on how different values of K affects overshoot, rise time, settling time, and steady state error. Note that a "good" K value is subjective, i.e. do you want the system to respond quickly at the expense of accuracy, or slowly but extremely precisely? Justify your decision.
2. Include tabulated data of the response characteristics from steps **??**, **??**, and **??**.
3. Include the plot of the output that oscillates consistently about the reference trajectory of 10 radians generated in step **??**. Why does higher K values increase the oscillation of the output? What is happening to the location of the closed loop poles?
4. Include the plot of the output when using only integral control. Using the transfer function, determine the roots of the characteristic polynomial. What does this tell you about the behaviour of a system using only integral control?
5. Include a plot of the output using your PID controller when the desired angle is generated by the multi-input switch. Be sure to specify your P, I, and D values.

3.2 Solutions

1. For increasing K values, overshoot and settling time will increase, but rise time and steady state error decreased.
2. Tables 3.1, 3.2, and 3.3 contain numerical values for rise time, settling time, overshoot, and steady state error for various values of K_P, K_I, and K_D.

   Table 3.1: Response characteristics for various K_P with $K_I = 0, K_D = 0$

K_P	Overshoot	Rise Time	Settling Time	S.S. Error
2	0.5	0.214	0.374	0.06
5	1.07	0.188	0.39	0.03
20	1.08	0.172	0.614	0.005

3. Figure 3.1 demonstrates how significantly increasing the proportional control term can cause the system to oscillate about its reference trajectory. This is because increasing the K value moves the poles of the closed loop transfer funciton closer to the imaginary axis.

Table 3.2: Response characteristics for various K_D with $K_P = 5, K_I = 0$

K_D	**Overshoot**	**Rise Time**	**Settling Time**	**S.S. Error**
0.75	0	1.702	1.702	0.06
0.5	0	1.1	1.1	0.04
0.25	0	0.752	0.75	0.034
0.05	0.27	0.182	0.27	0.01

Table 3.3: Response characteristics for various K_I with $K_P = 5, K_D = 0.25$

K_I	**Overshoot**	**Rise Time**	**Settling Time**	**S.S. Error**
1	0.17	0.62	¿10	?
0.75	0.11	0.8	¿10	?
0.005	0	0.69	0.69	0.038
0.001	0	0.684	0.684	0.046
0.0005	0	0.666	0.666	0.034

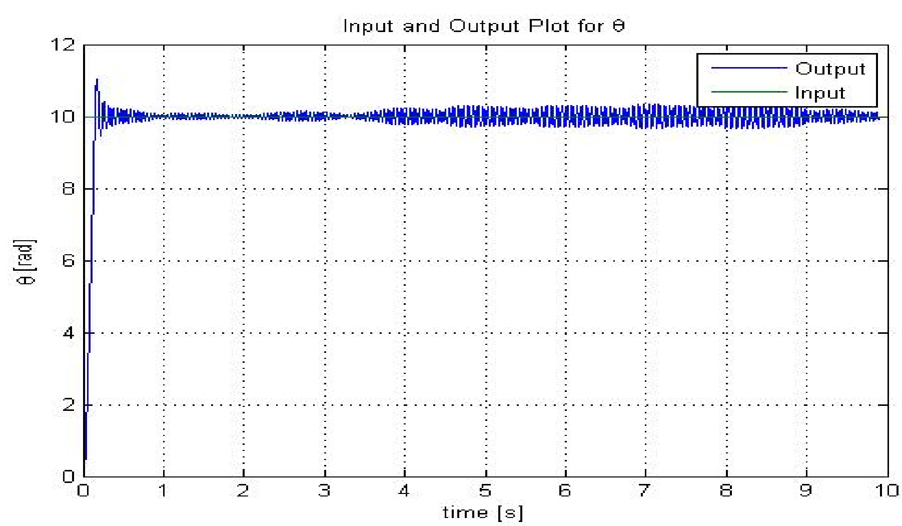

Figure 3.1: Input of 10 rads, K = 50, I = 0, D = 0

4. Using only integral control puts a pole in the right half of the complex plane, making it no longer BIBO stable. Therefore, the system can never be controlled by only integral control. Figure 3.2 demonstrates this property.

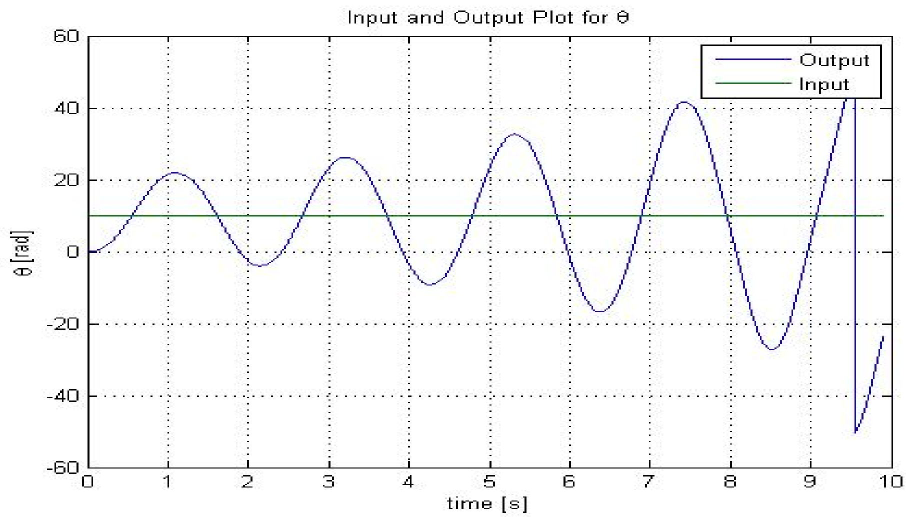

Figure 3.2: Input of 10 rads, K = 0, I = 1, D = 0

5. Figure 3.3 uses a PID controller with the optimal values for K_P, K_I, and K_D, and the reference trajectory is generated by the `Multiport Switch` in Simulink.

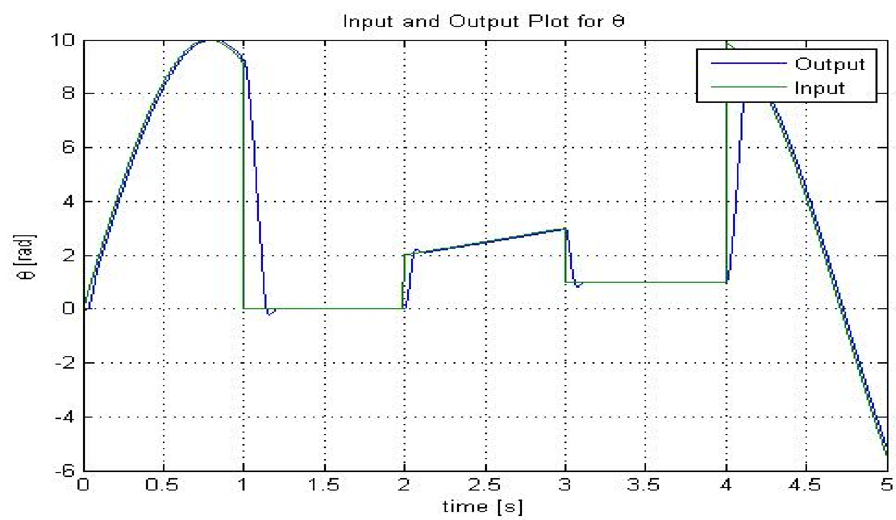

Figure 3.3: Input and output of the system for a square wave function, with $K_P = 7$, $K_I = \frac{1}{125}$, and $K_D = \frac{1}{93}$

Lab 4
Performance Specs

4.1 Deliverables

1. Include plots of the systems response with varying ζ and ω_0 values, comment on the effect off varying these constants.

2. For both positive and negative α values, what is the effect of increasing the magnitude of α on the rise time, settling time, overshoot/undershoot, and peak time? Include plots.

3. Create the following table for the system type section. Be sure to include and reference all necessary plots for the justification section.

System No.	Response to Constant Input	Response to Ramp Input	Response to Quadratic Input	Type?	Justification
System 1					
System 2					
System 3					

4.2 Solutions

1. Figure 4.1 shows the step response of the system with varying values of ω_0. It is clear that as ω_0 increases, rise time, peak time, and settling time all decrease, but the overshoot remains constant.
   Similarly, as ζ increases, both overshoot and settling time decrease, but rise time and peak time both increase. This can be seen in Figure 4.2.

2. Figure 4.3 shows the effect of adding a zero (α) to the systems transfer function. In this case, α varies from 1 to 9. As you can see, for positive values of α, increasing the magnitude increases rise time and peak time, while settling time and overshoot decrease. Similarly for negative α shown in 4.4, increasing the magnitude reduces overshoot, rise time, settling time, and peak time. Furthermore, adding a negative zero creates what is called undershoot, and the closer α is to the imaginary axis, the larger the undershoot becomes.

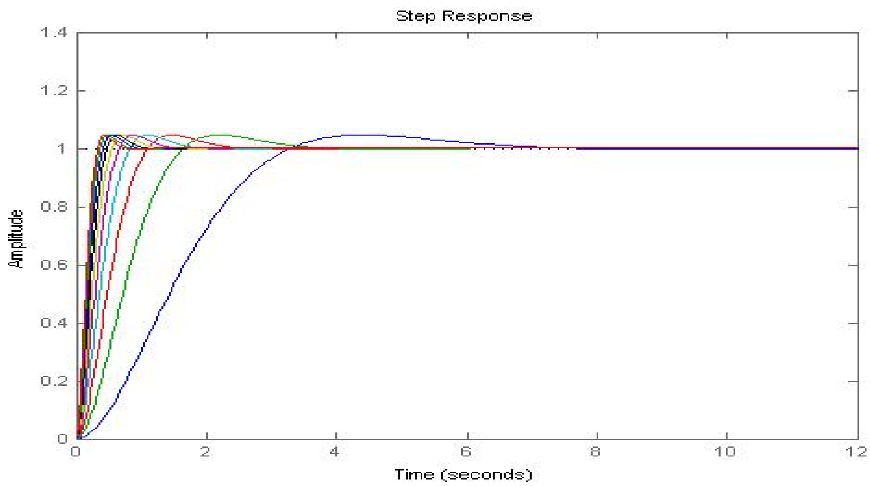

Figure 4.1: Step response and impulse response of the system for $1 \leq \omega_0 \leq 10, \zeta = 0.7$

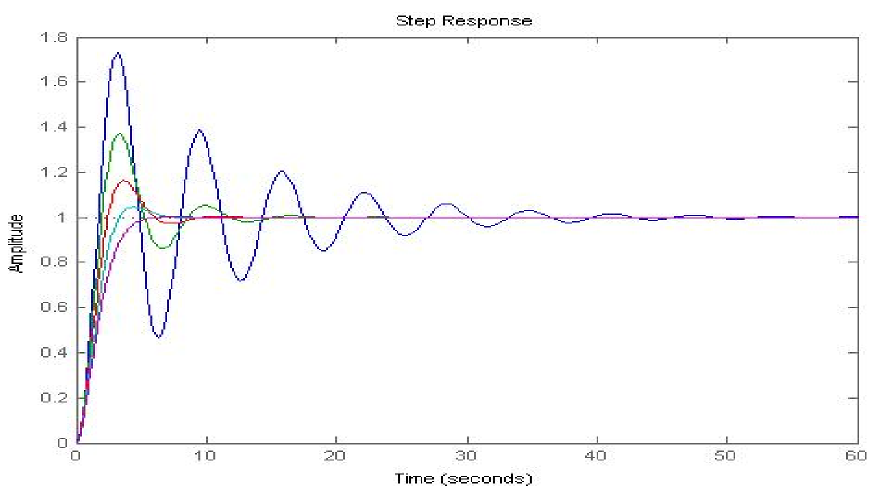

Figure 4.2: Step response and impulse response of the system for $0.1 \leq \zeta \leq 0.9, \omega_0 = 1$

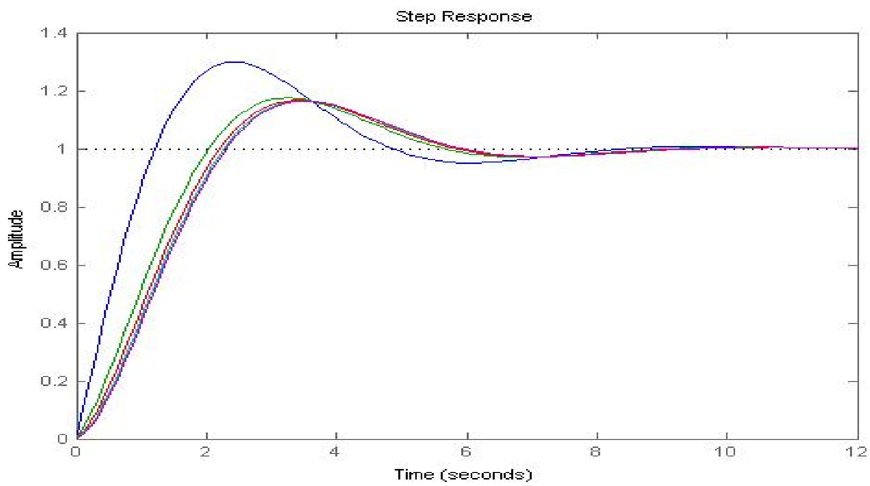

Figure 4.3: Step response and impulse response of the system with added zero ($1 \leq \alpha \leq 9$)

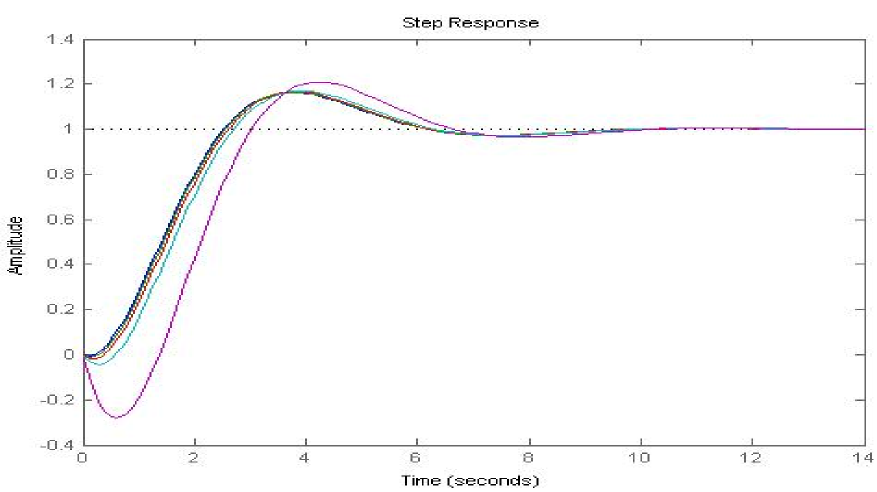

Figure 4.4: Step response and impulse response of the system with added zero ($-9 \leq \alpha \leq -1$)

3. Figures 4.5- 4.22 are plots of the systems response and error signal for 3 different controllers (proportional, integral, and derivative) with 3 different inputs (constant, ramp, and quadratic).

   (a) System 1: For this system, the error is bounded on a constant input, however it is unbounded for any polynomial with degree greater than 1. Therefore this system is type 1.

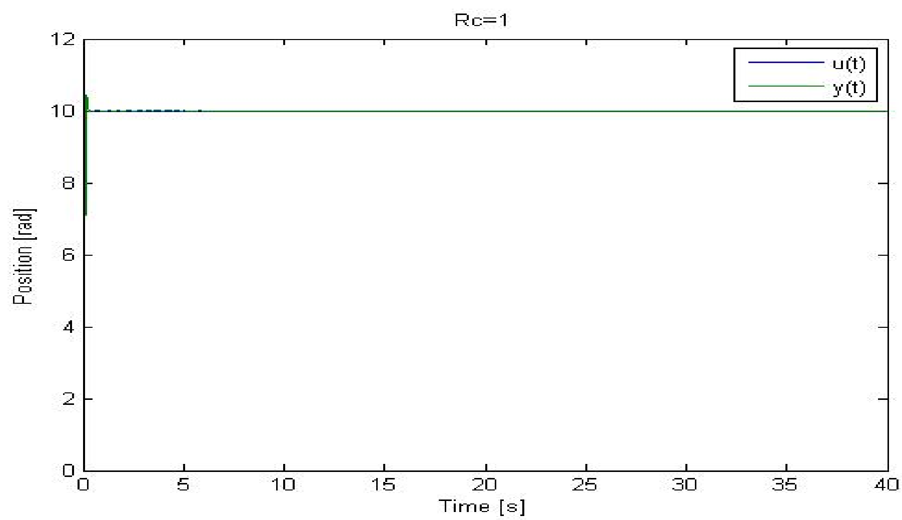

Figure 4.5: Systems response to constant input with $Rc = 1$

   (b) System 2: This system is type 0 because every input had unbounded error.

   (c) System 3: Lastly, this system is type 2, because the constant and ramp inputs had bounded error, and only the quadratic input had unbounded error.

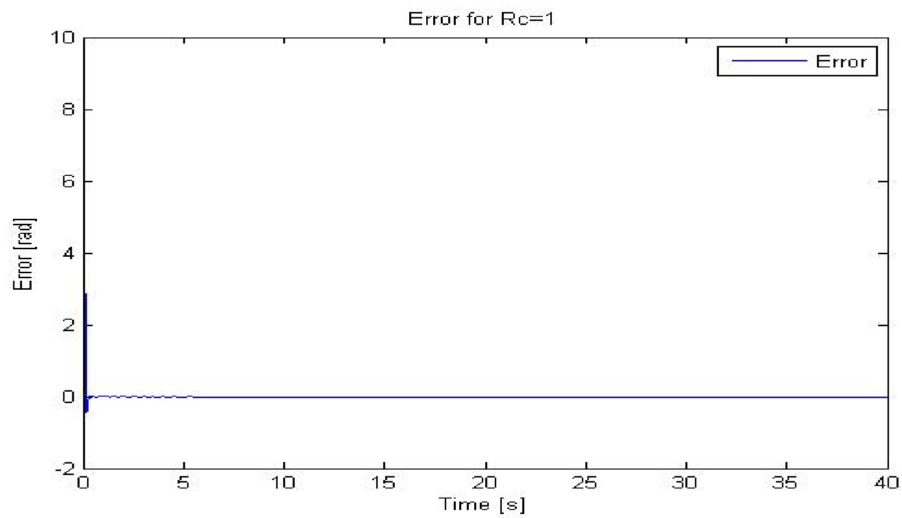

Figure 4.6: Systems error signal for constant input with $Rc = 1$

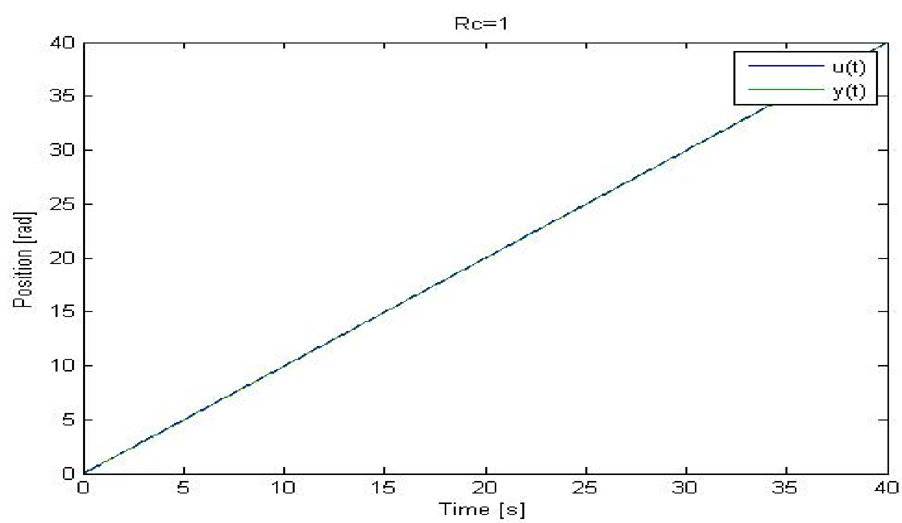

Figure 4.7: Systems response to a ramp input with $Rc = 1$

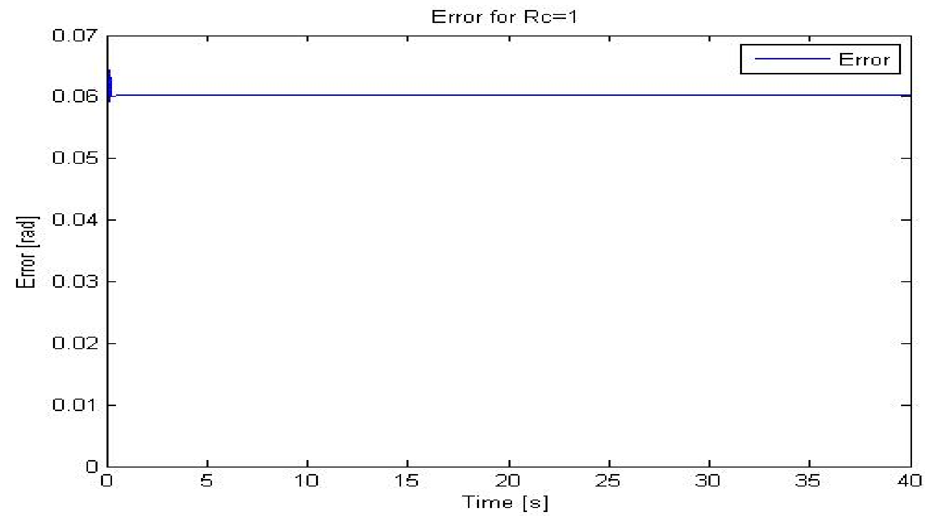

Figure 4.8: Systems error signal for a ramp input with $Rc = 1$

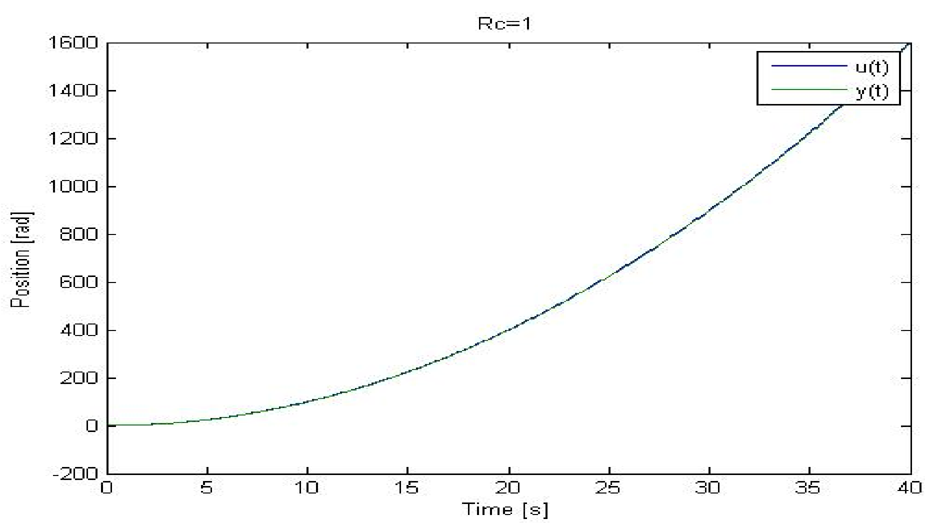

Figure 4.9: Systems response to quadratic input with $Rc = 1$

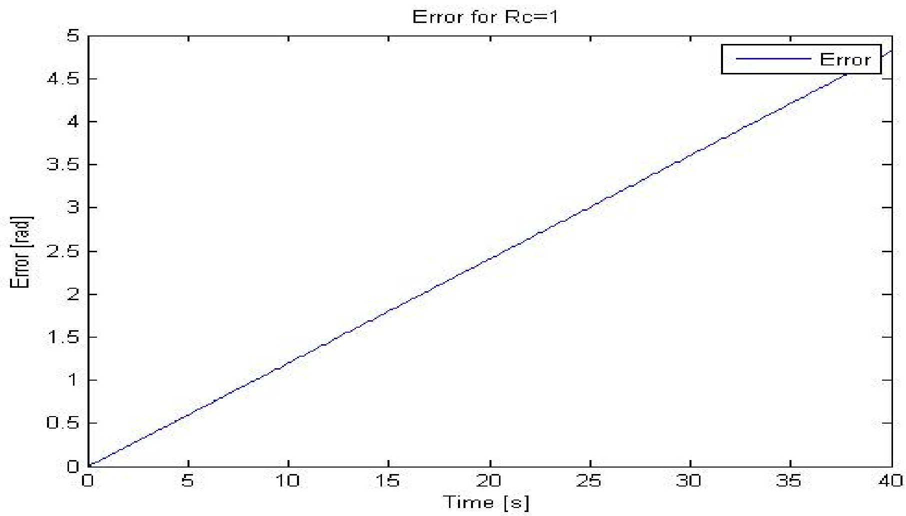

Figure 4.10: Systems error signal for quadratic input with $Rc = 1$

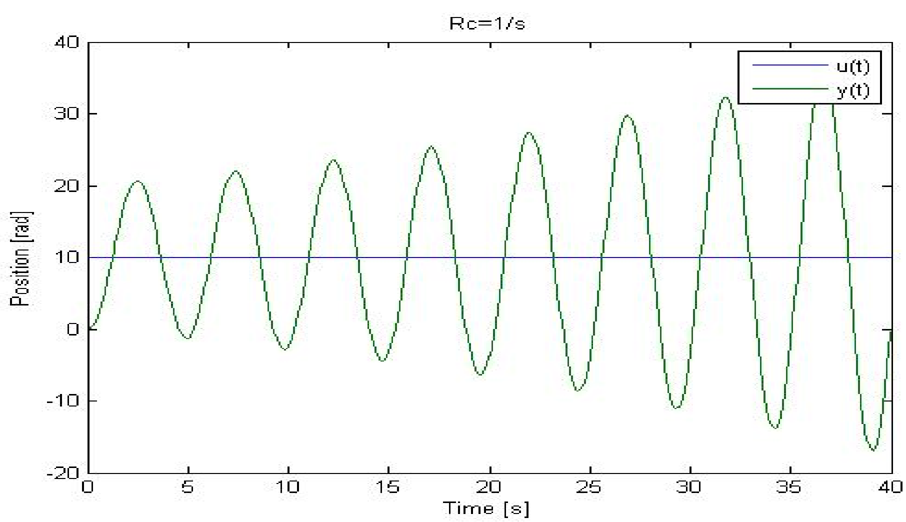

Figure 4.11: Systems response to constant input with $R_c = \frac{1}{s}$

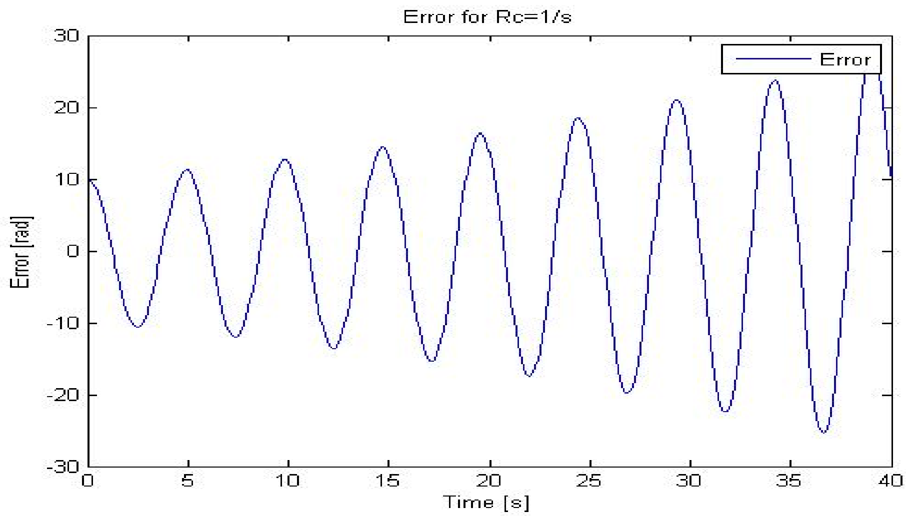

Figure 4.12: Systems error signal for constant input with $R_c = \frac{1}{s}$

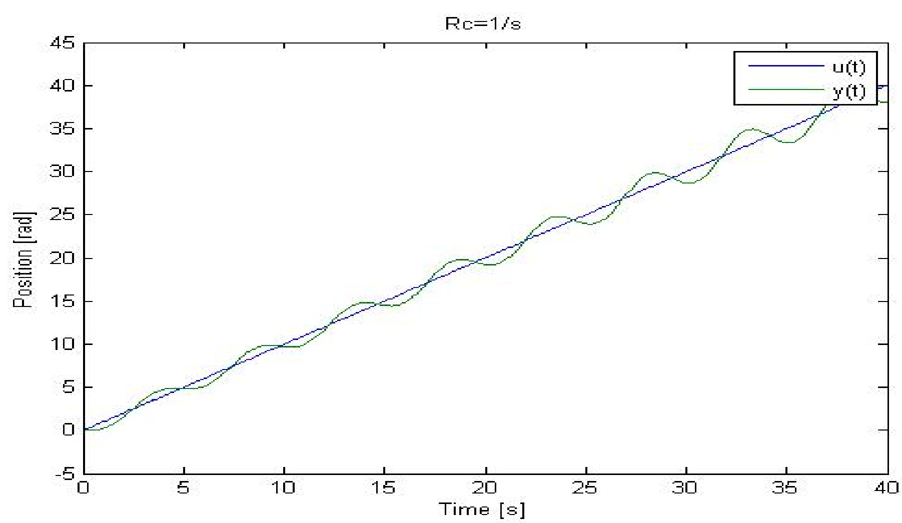

Figure 4.13: Systems response to a ramp input with $R_c = \frac{1}{s}$

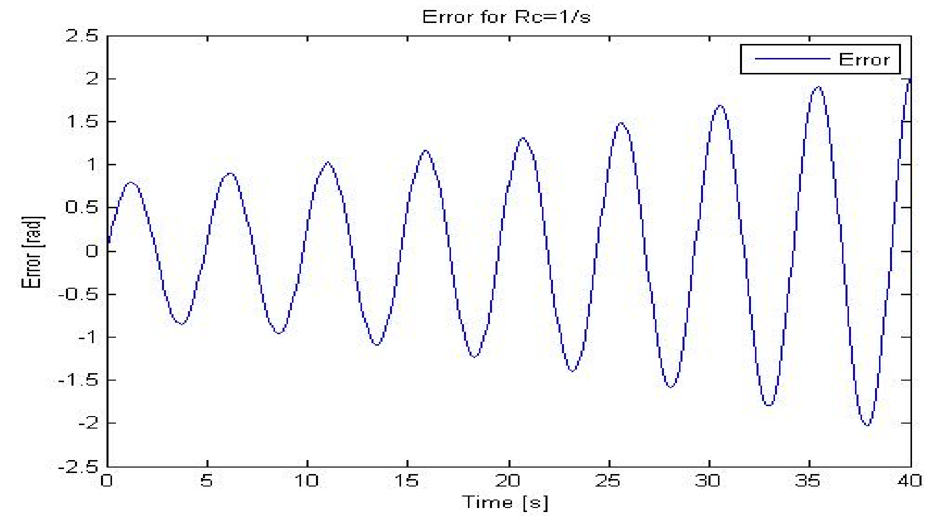

Figure 4.14: Systems error signal for a ramp input with $R_c = \frac{1}{s}$

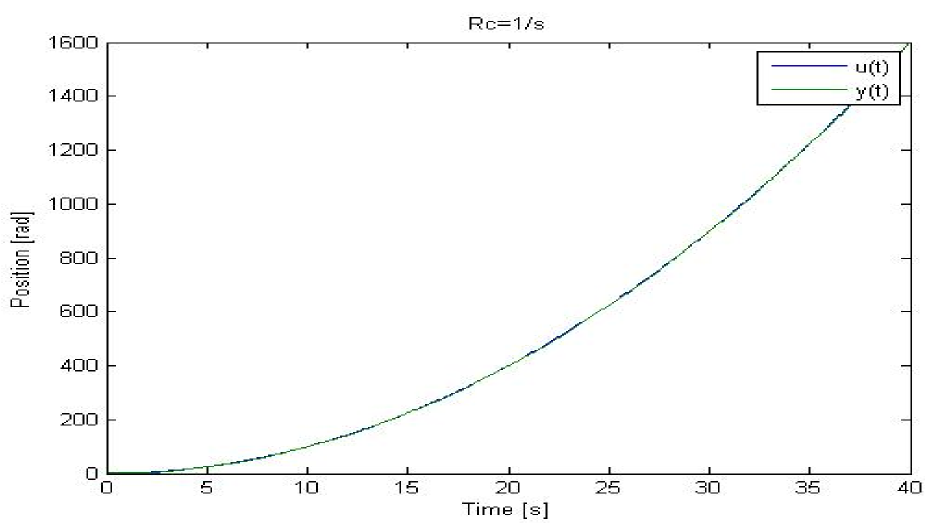

Figure 4.15: Systems response to quadratic input with $R_c = \frac{1}{s}$

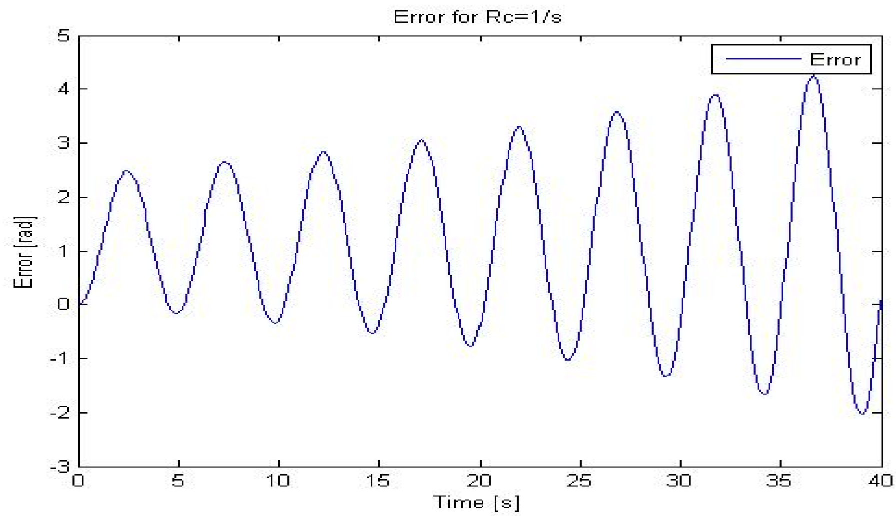

Figure 4.16: Systems error signal for a quadratic input with $R_c = \frac{1}{s}$

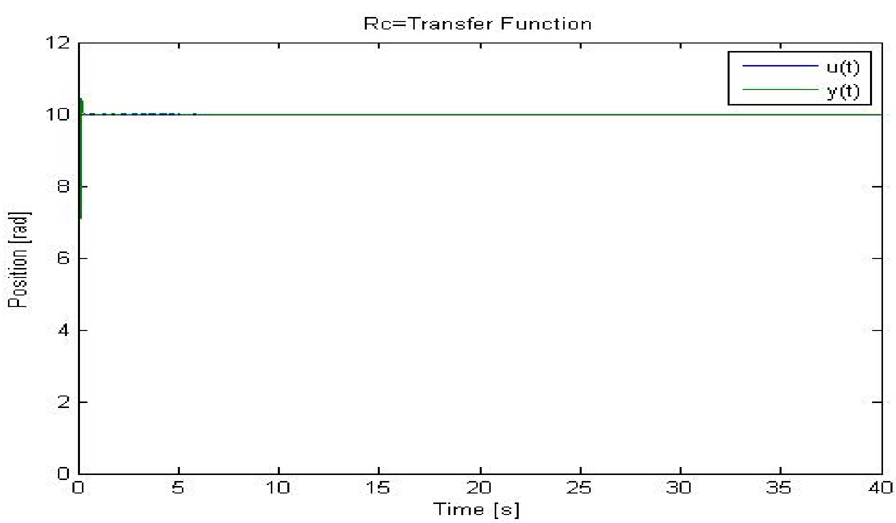

Figure 4.17: Systems response for a constant input with $R_c = s$

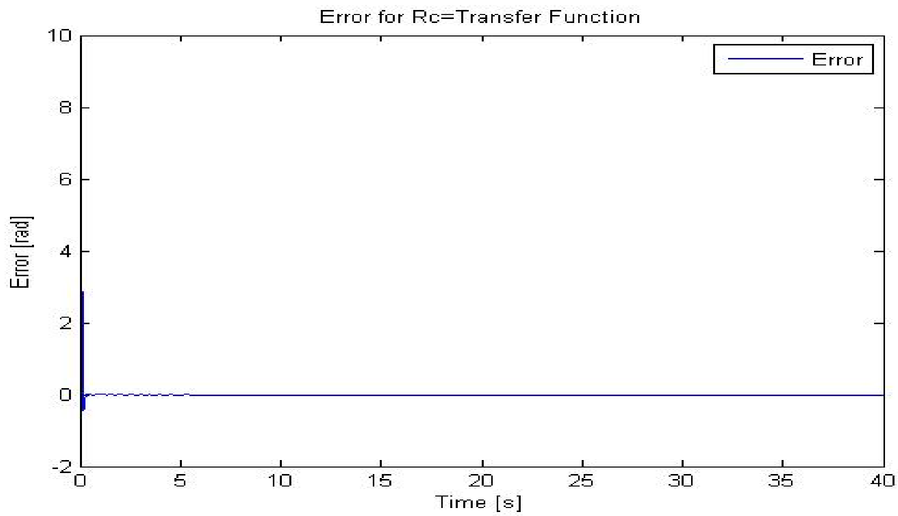

Figure 4.18: Systems error signal for for constant input with $R_c = s$

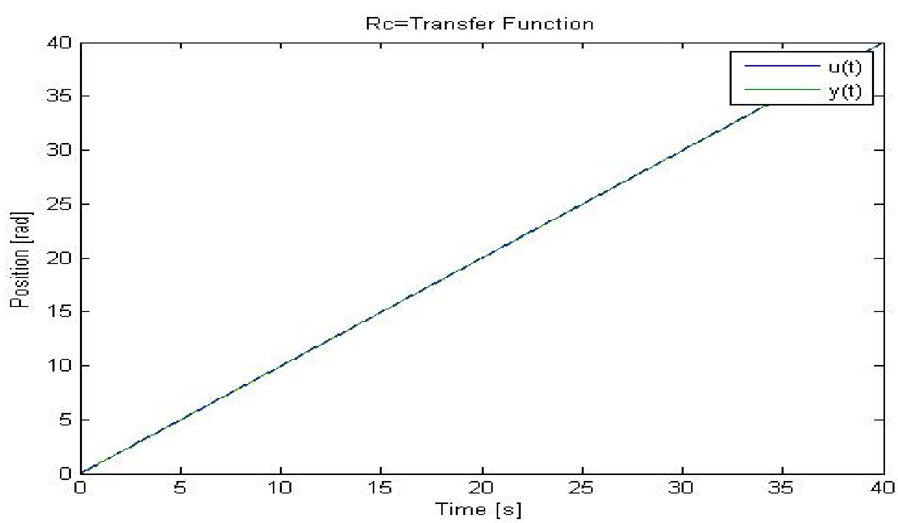

Figure 4.19: Systems response for a ramp input with $R_c = s$

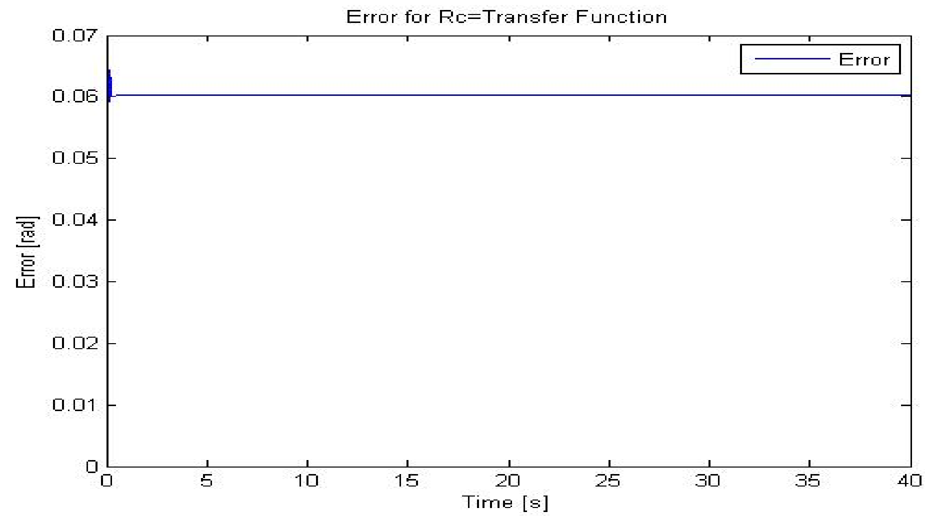

Figure 4.20: Systems error signal for a ramp input with $R_c = s$

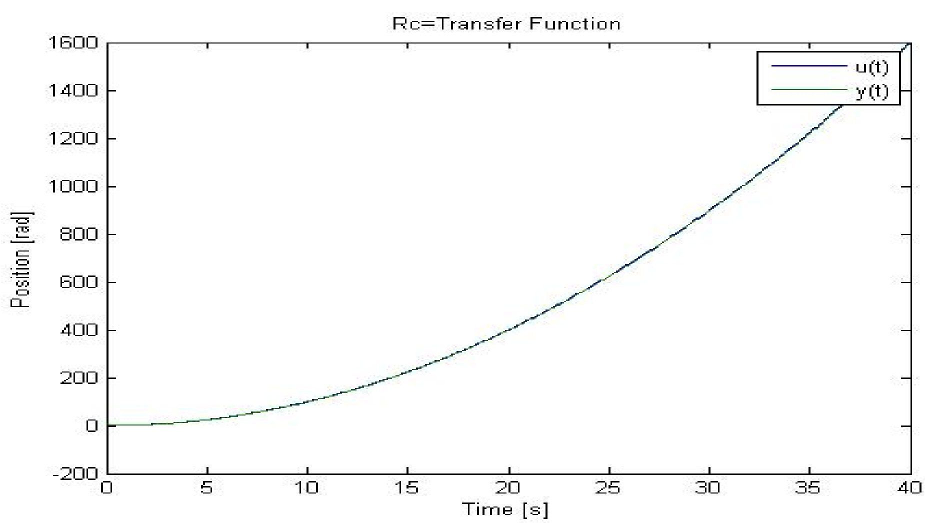

Figure 4.21: Systems response to a quadratic input with $R_c = s$

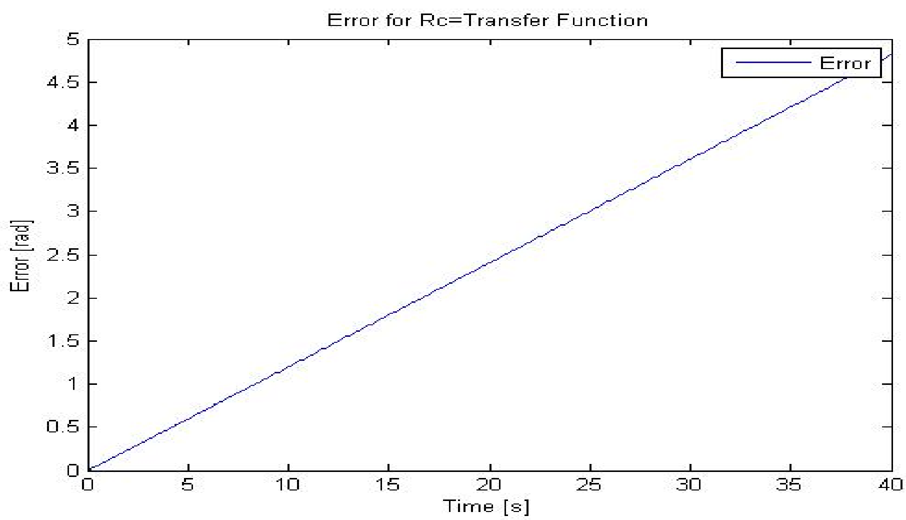

Figure 4.22: Systems error signal for a quadratic input with $R_c = s$