Contents

Liam Jackson HW1 BE700 ML

Question 1

```
%{
Ok, honestly this code is overcomplicated. I had to rewrite it 5
times because MATLAB didn't save my changes a couple days in a row. So
in the interest of time, I tried writing functions to accomplish the
analysis for question 1 that I could then recycle for question 2. It's
ugly, I switch data structure types all over the place. But hopefully I
approached the correct answers in the end.
%}
```

Part 1

```
close all, clear all, clc;
warning('off','MATLAB:polyfit:RepeatedPointsOrRescale')
warning('off','MATLAB:nearlySingularMatrix')
```

Importing / Sorting Data

```
[x1, x2, y] = textread('besseldata.txt', ' %f%f%f', 'headerlines', 1);
r = sqrt(x1.^2 + x2.^2);
r_norm = normalize(r);

data_arr = sortrows([x1, x2, r, r_norm, y], 3);
data_table = array2table(data_arr,...
    'VariableNames', {'x1','x2','r','r_norm','y'});
```

Bessel Approx

```
k_bes = 1;
bes_approx = besselj(0, k_bes*data_table.r);

fig1 = figure(1);
dot_sz = 0.2;
line_w = 2.5;
scatter(data_table.r, data_table.y, dot_sz, '.');
hold on;
plot(data_table.r, bes_approx, 'LineWidth', line_w);
hold off;
title({'Timpanic Memb Displacement', 'approximated by Bessel Fxn (J_0)'});
xlabel('r');
ylabel('Intensity');
legend({'Real Data', 'J_0'});
```

Polynomial Approximations

```
max_poly_order = 14;
model_data = poly_model_vals(data_table, max_poly_order);
ry_polyvals_table = model_data.ry_polyvals_table;
```

Calculate Residuals

```
residuals_table = res_table(ry_polyvals_table)
```

Plotting LS Poly fits

```
data_labels = ry_polyvals_table.Properties.VariableNames;

fig2 = figure(2);
sgtitle({'Membrane Displacement Data', 'vs. OLS Polynomial Fits'});
number_of_plots = max_poly_order;

for plot_id = 1:number_of_plots
    subplot(number_of_plots / 2, 2, plot_id);
    scatter(ry_polyvals_table.r, ry_polyvals_table.y, dot_sz, '.');
    hold on;
    plot(ry_polyvals_table.r, ry_polyvals_table.(string(data_labels(plot_id + 2))), 'LineWidth', line_w)
    hold off;
    xlabel('r');
    ylabel('Displacement');
    legend({'y real', string(data_labels(plot_id + 2))});
end
```

Part 2

20 rounds of (k = 5) Cross Validation

```
cv_rounds = 20;
k_cv = 5;

PE_arr = zeros([cv_rounds, max_poly_order]);
MSE_arr = zeros([max_poly_order, k_cv, cv_rounds]);
all_cv_poly_coeffs = zeros([max_poly_order + 1, max_poly_order, k_cv, cv_rounds]);

for cv_round = 1:cv_rounds
    binned_data_struct = bin_this_data(data_arr, k_cv);
    binned_data_cell = binned_data_struct.cell;
    bin_indices = 1:k_cv;

    for test_bin = 1:k_cv
        train_bins = bin_indices(1:end ~= test_bin);
        train_data_cell = binned_data_cell(train_bins);

        train_data_arr = sortrows(cat(1, train_data_cell{:}), 3);
        test_data_arr = sortrows(cell2mat(binned_data_cell(test_bin)), 3);

        train_data_table = array2table(train_data_arr,...
            'VariableNames', {'x1','x2','r','r_norm','y'});
        test_data_table = array2table(test_data_arr,...
            'VariableNames', {'x1','x2','r','r_norm','y'});

        model_train_struct = poly_model_vals(train_data_table, max_poly_order);
        cv_ry_polyvals_table = model_train_struct.ry_polyvals_table;
        cv_coeffs_arr = model_train_struct.coeffs_arr;
        all_cv_poly_coeffs(:, :, test_bin, cv_round) = cv_coeffs_arr;

        poly_zeros_pad = zeros([size(test_data_arr, 1), max_poly_order]);
        model_poly_vals = [test_data_arr, poly_zeros_pad];
        for poly_ord_ind = 1:max_poly_order
            n_coeffs = poly_ord_ind + 1;
            temp_coeffs = cv_coeffs_arr(1:n_coeffs, poly_ord_ind);
            model_poly_vals(:, poly_ord_ind + 5) = polyval(temp_coeffs, model_poly_vals(:, 4));
        end

        temp_ry_polyvals_table = array2table([model_poly_vals(:, 3), model_poly_vals(:, 5), model_poly_vals(:,6:end)],...
            'VariableNames', cv_ry_polyvals_table.Properties.VariableNames);
        temp_residuals_table = res_table(temp_ry_polyvals_table);
        MSE_arr(:, test_bin, cv_round) = temp_residuals_table.MSE;

    end
    PE_col = mean(squeeze(MSE_arr(:, :, cv_round)), 2);
    PE_arr(cv_round, :) = PE_col';
end

PE_var_labels = cv_ry_polyvals_table.Properties.VariableNames;
PE_var_labels = PE_var_labels(3:end);
PE_row_nums = 1:cv_rounds;
PE_row_labels = "rnd" + PE_row_nums;

PE_table = array2table(PE_arr,...
    'VariableNames', PE_var_labels,...
    'RowNames', PE_row_labels)
```

Plotting PE values for each CV Round

```
fig3 = figure(3);
plot(PE_table{:,:}.');
title('PE values for 20 rounds of (k=5)-CV');
xlabel('Polynomial Model Order');
ylabel('Predictive Error');
legend(PE_table.Properties.RowNames, 'location', 'eastoutside');
```

Part 3

```
%{
A polynomial OLS-fit of order 10 seems to have the best compromise of
accuracy and economy of variables. A substantial reduction in error
occurs from order-9 to order-10, with no substantial decrease with
additional (11, 12, 13, 14) order terms.
%}

char({'A polynomial OLS-fit of order 10 seems to have the best',...
    'compromise of accuracy and economy of variables. A substantial',...
    'reduction in error occurs from order-9 to order-10, with no',...
    'substantial decrease with additional (11, 12, 13, 14) order terms.'})
```

Part 4

```
data_table_opt = data_table;

x1 = data_table_opt.x1;
x2 = data_table_opt.x2;
r = data_table_opt.r;
y = data_table_opt.y;

opt_ord = 10;
beta_deg10 = ols_coeffs_data(r, y, opt_ord).beta;
beta_ud = flipud(beta_deg10);

[X1, X2] = meshgrid(-20:.2:20);
Y_opt = polyval(beta_ud, sqrt(X1.^2 + X2.^2));

% I'm removing the "wall" of the surf that approaches inf so the figure is easier to see
Y_opt(1:75, 1:75) = NaN;

J0 = besselj(k_bes, sqrt(X1.^2 + X2.^2));

fig4 = figure(4);
scatter3(x1, x2, y, 15, '.');
hold on;
opt = surf(X1, X2, Y_opt, 'EdgeColor', 'none');
colorbar
colormap(spring)
caxis([-1 1.5])
hold off;
title({'Polynomial (p=10) Model', 'vs. Real Displacement Data'});
xlabel('x1');
ylabel('x2');
zlabel('Displacement');
zlim([-.8, 1.5]);


fig5 = figure(5);
scatter3(x1, x2, y, 15, '.');
hold on;
surf(X1, X2, J0, 'EdgeColor', 'none');
title({'Bessel Fxn J_0', 'vs. Real Displacement Data'});
xlabel('x1');
ylabel('x2');
zlabel('Displacement');
```

```
residuals_table =

  14×3 table
```

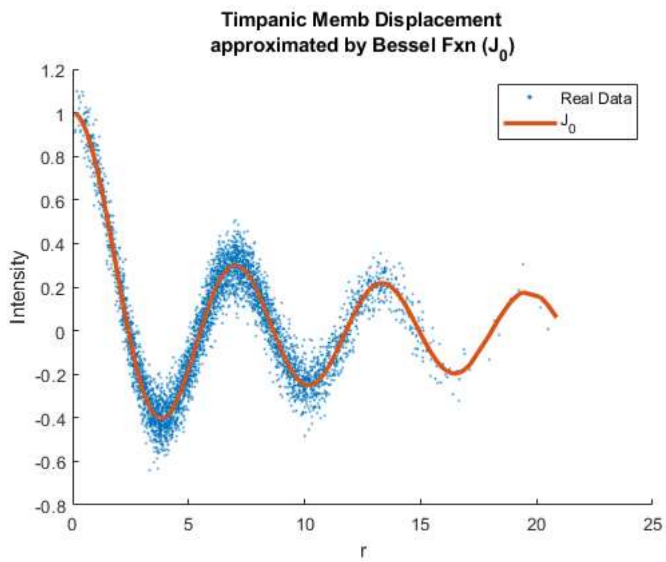
Polynomial_Order	Residual Sum	MSE
1	425.17	0.085034
2	412.42	0.082483
3	369.4	0.073879
4	268.61	0.053722
5	168.66	0.033733
6	164.73	0.032945
7	105.34	0.021069
8	47.384	0.0094768
9	43.922	0.0087844
10	28.828	0.0057656
11	28.809	0.0057619
12	27.775	0.0055551
13	27.718	0.0055437
14	27.697	0.0055394

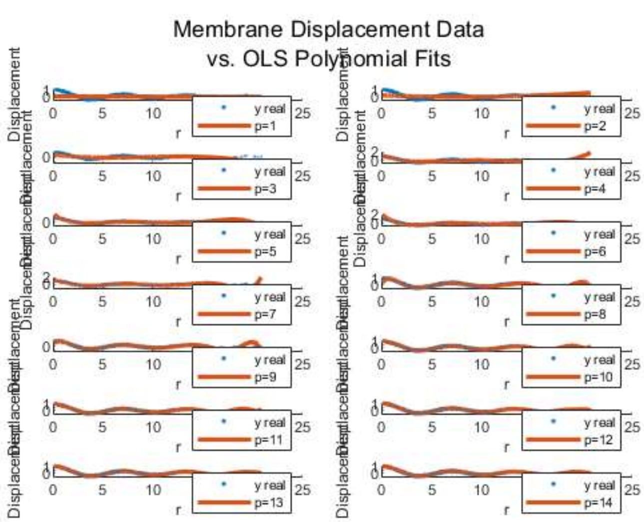
```
PE_table =

  20×14 table
```

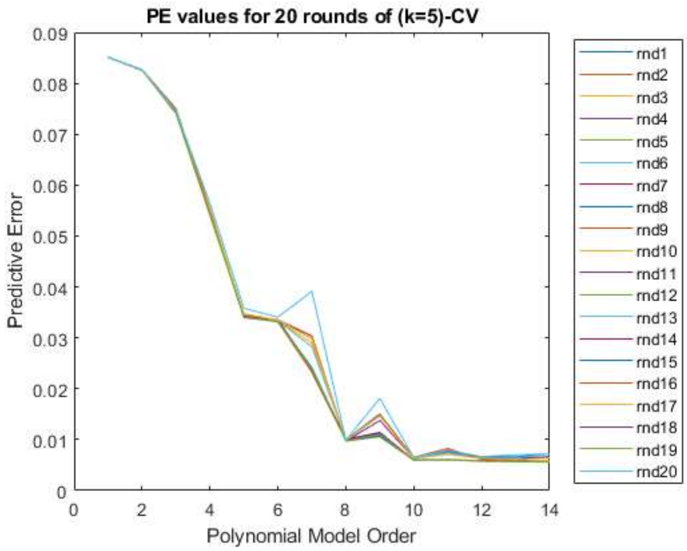
	p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=8	p=9	p=10	p=11	p=12	p
rnd1	0.085065	0.082577	0.074357	0.054969	0.034563	0.033381	0.028947	0.0098362	0.01501	0.006425	0.0076866	0.0065539	

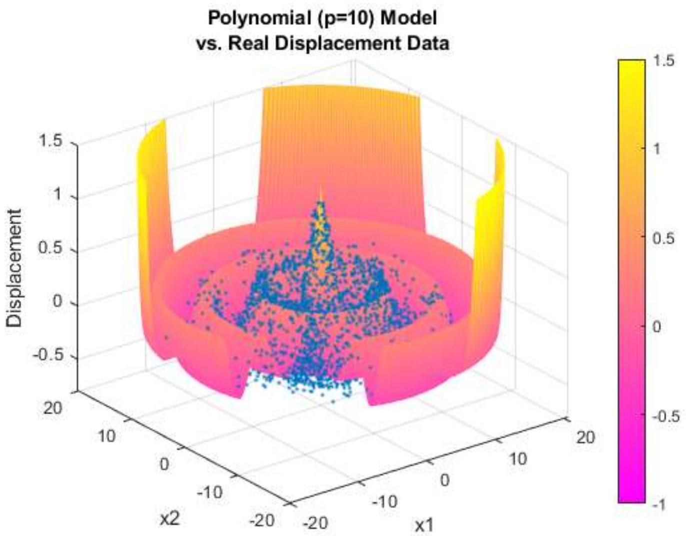
rnd2	0.085108	0.082569	0.074034	0.054003	0.034081	0.033134	0.02323	0.0098505	0.010745	0.0059828	0.0059781	0.0058127	0.0
rnd3	0.085127	0.082699	0.074457	0.054531	0.034317	0.03327	0.023955	0.0097887	0.010689	0.005926	0.0060101	0.0057978	0.0
rnd4	0.085134	0.082691	0.074525	0.054411	0.034338	0.033372	0.024019	0.010069	0.011438	0.0059981	0.0060173	0.0057865	0.0
rnd5	0.085151	0.082621	0.074146	0.054444	0.0343	0.033315	0.023954	0.0096891	0.010574	0.0059484	0.0061251	0.0057965	0.0
rnd6	0.085094	0.082574	0.07405	0.053968	0.034146	0.033214	0.024222	0.0098553	0.01086	0.0059703	0.0060338	0.0057492	0.0
rnd7	0.085065	0.082532	0.074127	0.054713	0.034718	0.033477	0.023915	0.010057	0.011002	0.0060462	0.0060709	0.0058321	0.0
rnd8	0.085165	0.08267	0.074137	0.054209	0.034092	0.033178	0.023494	0.0098238	0.010947	0.0059741	0.0060757	0.0058071	0.0
rnd9	0.085079	0.082579	0.074176	0.054251	0.034168	0.033249	0.023309	0.0098935	0.01112	0.0059834	0.0060462	0.0058289	0.0
rnd10	0.085113	0.08259	0.074421	0.05493	0.034551	0.03328	0.028994	0.0099285	0.015095	0.0063835	0.0071666	0.0062601	0.0
rnd11	0.085114	0.082641	0.074214	0.054661	0.034494	0.033468	0.030297	0.0097173	0.013768	0.0063214	0.0076979	0.0062456	0.0
rnd12	0.08514	0.082657	0.074335	0.054417	0.034287	0.03327	0.023885	0.0097416	0.010559	0.0059619	0.0060269	0.0057999	0.0
rnd13	0.085102	0.082591	0.074369	0.055	0.034708	0.033352	0.028202	0.010054	0.015138	0.006425	0.0074468	0.0063672	0.0
rnd14	0.085142	0.082635	0.074044	0.054013	0.034197	0.033606	0.023737	0.0099251	0.011152	0.0059168	0.0060737	0.0057321	0.0
rnd15	0.085182	0.082649	0.074122	0.054086	0.034096	0.033247	0.024024	0.0097745	0.01078	0.0059868	0.006066	0.0058092	0.0
rnd16	0.085159	0.082704	0.075006	0.055209	0.034545	0.033553	0.030413	0.0098368	0.014896	0.0065433	0.0082615	0.0060834	0.0
rnd17	0.08509	0.082586	0.074257	0.05483	0.034678	0.033489	0.029756	0.009869	0.014629	0.0062817	0.0070891	0.006335	0.0
rnd18	0.085052	0.082557	0.074247	0.05421	0.033971	0.033215	0.02378	0.0098048	0.011342	0.0059427	0.0060055	0.0058236	0.
rnd19	0.085087	0.082559	0.07409	0.054043	0.034102	0.033235	0.023818	0.0097457	0.010996	0.0059363	0.0060449	0.0057481	0.0
rnd20	0.085162	0.082648	0.074607	0.056379	0.035793	0.034067	0.039151	0.0099011	0.018097	0.0064823	0.0079107	0.0066796	0.0

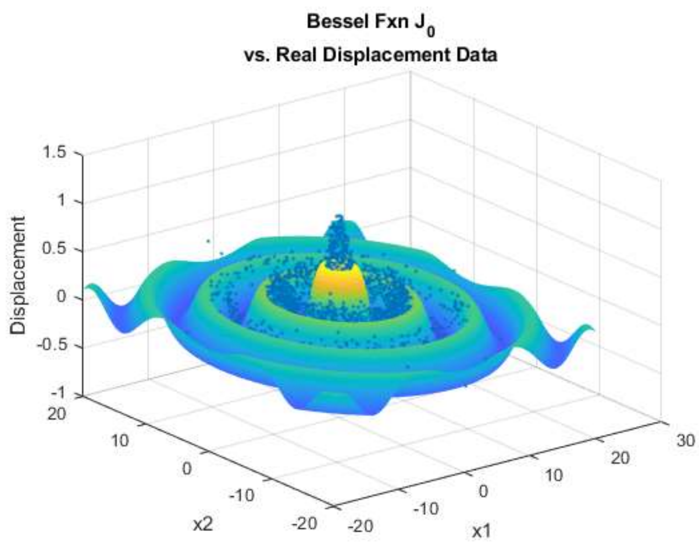
```
ans =

  4×66 char array

    'A polynomial OLS-fit of order 10 seems to have the best           '
    'compromise of accuracy and economy of variables. A substantial    '
    'reduction in error occurs from order-9 to order-10, with no       '
    'substantial decrease with additional (11, 12, 13, 14) order terms.'
```











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