

# COM3026: Distributed Systems

---

## Paxos

Gregory Chockler

# Consensus with Partial Synchrony

---

- Partially (Eventually) Synchronous model:
  - ▶ The timing assumptions hold eventually
  - ▶ Captures the idea that realistic systems are synchronous most of the time
- Abstracted via an Eventually Perfect Failure Detector ( $\diamond P$ )
  - ▶ Strong Completeness and Eventual Strong Accuracy
- $\diamond P$  makes it possible to eventually elect a stable leader
  - ▶ A correct process that is permanently trusted by all correct processes
  - ▶ Eventual Leader Detector  $\Omega$

# Eventual Leader Detector ( $\Omega$ )

- **Module:**
  - ▶ Name: EventualLeaderDetector, instance  $\Omega$
- **Events:**
  - ▶ **Indication:**  $\langle \Omega, \text{Trust} \mid p \rangle$  : Indicates that process  $p$  is **trusted** to be leader
- **Properties:**
  - ▶ **ELD1** (*Eventual Accuracy*)
  - ▶ **ELD2** (*Eventual Agreement*)

# Eventual Leader Detector ( $\Omega$ )

- **Properties:**

- ▶ **ELD1** (*Eventual Accuracy*)

- There is a time after which every correct process trusts some correct process

- ▶ **ELD2** (*Eventual Agreement*)

- There is a time after which no two correct processes trust different correct processes

# Problem Statement

---

- Solve Uniform Consensus
- **Fail-noisy** model
  - ▶ Crash-stop failure model for processes
  - ▶ Eventual Leader Detector  $\Omega$
  - ▶ Perfect Point-to-Point links
- **N** processes, up to a minority **f**  $< N/2$  can fail
  - ▶ There is a **correct majority** (Majority Quorum System)

# The Paxos Algorithm

---

- Solves Uniform Consensus in a fail-noisy model with a Eventual Leader Detector and correct majority
- Arguably, the most important algorithm in distributed computing
- Was initially proposed by Leslie Lamport in late 80s and further developed by Lamport and others over several decades
  - ▶ Hugely influential in practice — virtually all modern data replication systems have a variant of Paxos in their core
- Our presentation follows “Paxos Made Simple” (Lamport, 2001) with some further simplifications



Leslie Lamport

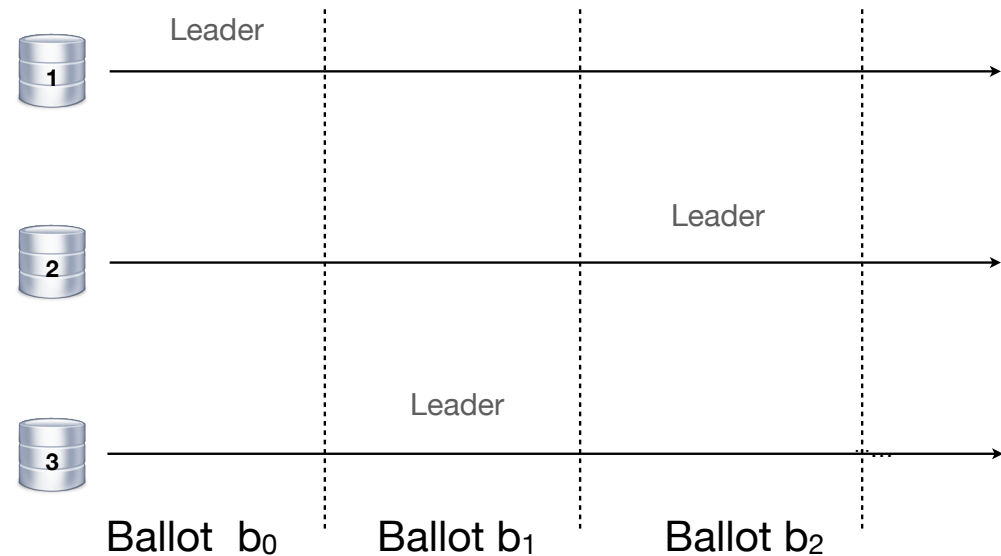
# Paxos Consensus (Synod, Single-Decree)

Execution is divided into a series of **ballots**

Each ballot has a **unique number** and is associated with a **unique leader**

At each ballot, the ballot leader tries to reach a decision

If a value  $V$  is decided at ballot  $b$ , any decision at ballot  $b' > b$  must be  $V$



$$b_0 < b_1 < b_2 < \dots$$

# Ballot $b_0$

---

- All processes start in a default ballot  $b_0$  with a default leader (e.g.,  $p_1$ )



# Ballot $b_0$

---

- Leader of ballot  $b_0$  broadcasts a value *val* to all processes and decides *val*
  - ▶ Does this work?

# Ballot $b_0$

---

- Leader of ballot  $b_0$  broadcasts a value  $val$  to all processes and decides  $val$ 
  - ▶ Does this work? - No
  - ▶ If the leader fails, may not reach the leader of ballot  $b_1$

# Ballot $b_0$

---

- Leader of ballot  $b_0$  broadcasts a value *val* to all processes and **waits for acks**
  - ▶ How many acks should the leader wait for?

# Ballot $b_0$

---

- Leader of ballot  $b_0$  broadcasts a value *val* to all processes and **waits for acks**
  - ▶ How many acks should the leader wait for?

# Ballot $b_0$

---

- Leader of ballot  $b_0$  broadcasts a value to all processes and waits for acks
- $N - f$  is the **maximum** number of acks the leader may hope to receive
- The leader of ballot  $b_1$  will be able to discover  $val$  by contacting  $N - f$  nodes, if any two sets of  $N - f$  nodes intersect (i.e.,  $N - f$  is a **quorum**)
- $N - 2f > 0 \implies N > 2f$
- This is a necessary condition for solving crash fault-tolerant consensus under partial synchrony and crash failures [DLS88]
- We assume for simplicity:  $N = 2f + 1$

# Ballot $b_0$ : Accepting a Value

---

1. Leader of ballot  $b_0$  broadcasts  $\langle \text{ACCEPT}, b_0, val \rangle$  to all processes and waits for acks  $\langle \text{ACCEPTED}, b_0 \rangle$
2. If a process receives  $\langle \text{ACCEPT}, b_0, val \rangle$ , while in ballot  $b_0$ , it stores
  - $a\_bal := b_0$
  - $a\_val := val$
3. and responds with  $\langle \text{ACCEPTED}, b_0 \rangle$  to the leader
4. Once the leader receives  $f + 1$   $\langle \text{ACCEPTED}, b_0 \rangle$ , it **decides**  $val$ , and broadcasts  $\langle \text{COMMITTED}, val \rangle$
5. Whenever a process receives  $\langle \text{COMMITTED}, val \rangle$ , it decides  $val$

# Ballot $b > b_0$ : General Case

---

- The processes store the current ballot in **bal**
- The leader of ballot  $b$  broadcasts  $\langle \text{PREPARE}, b \rangle$
- If a process receives  $\langle \text{PREPARE}, b \rangle$ , and its current ballot  $\text{bal} < b$ :
  - ▶  $\text{bal} := b$  //join ballot  $b$
  - ▶ send  $\langle \text{PREPARED}, a_{\text{bal}}, a_{\text{val}} \rangle$  to the leader of  $b$  //to allow the leader to learn about values accepted in the prior ballots
  - ▶ From this point onward, the process will **not** be participating in any **ballot  $< b$**
- If the leader receives  $f + 1$   $\langle \text{PREPARED}, a_{\text{bal}_i}, a_{\text{val}_i} \rangle$  responses, it selects the value  $a_{\text{val}_j}$  associated with the highest ballot  $a_{\text{bal}_j}$ , and broadcasts  $\langle \text{ACCEPT}, b, \text{val} \rangle$  where  $\text{val} = a_{\text{val}_j}$ 
  - ▶ If all  $a_{\text{val}_i} = \text{null}$ , the leader broadcasts the value provided in its propose request
- Proceed with the steps 2 - 4 of the accept protocol with  **$b_0$  replaced with  $b$**

# Paxos Consensus: Agreement

---

- Sequence of ballots in the execution:  $b_0 < b_1 < b_2 < \dots$
- Let  $b_k$  be the **minimum** ballot in which some value  $val_k$  is decided
- Prove that for all  $l \geq k$ : if the leader of  $v$  adopts a value  $val$  at ballot  $b_l$ , then  $val = val_k$
- By induction on  $l \geq k$ :
  - ▶  $l = k$ : the ballot leader can commit at most one value

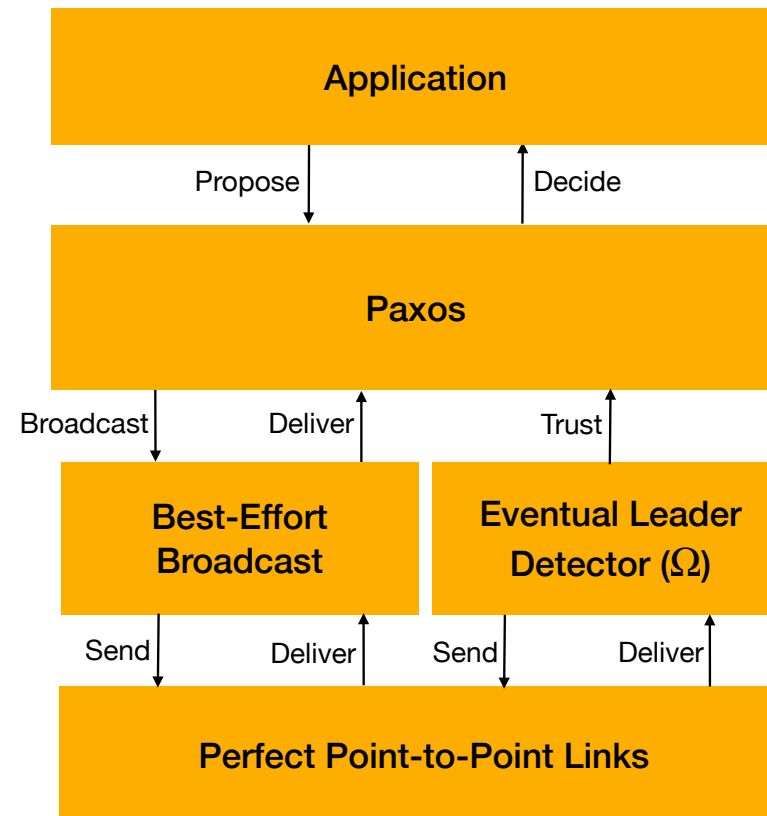


# Paxos Consensus: Agreement

---

- By induction on  $l \geq k$ :
  - ▶ Assume that the result holds for all  $l$  up to  $l = i$ , and consider  $l = i + 1$
  - ▶ The leader of  $b_{i+1}$  will hear from at least one process  $p^*$  that committed  $val_k$  in  $b_k$  (quorum intersection)
  - ▶  $\implies$  the highest ballot  $b' \geq b_k$
  - ▶ By the inductive hypothesis,  $val'$  associated with  $b'$  satisfies  $val' = val_k$

# Component Structure



# Sketch of the Leader Protocol

**upon event**  $\langle \Omega, Trust \mid p \rangle$  **do**

$leader := p$

**while**  $self = leader \wedge decided = FALSE$  **do**

- ▶ **Prepare** phase: choose a value  $V$ , lock majority
- ▶ **Accept** phase: convince a majority to **accept**  $V$
- ▶ If successful: **trigger**  $\langle Decide \mid V \rangle$ ;  $decided := TRUE$

Paxos core  
ABORTABLE  
CONSENSUS

**if**  $decided = TRUE$  **then**

**trigger**  $\langle beb, Broadcast \mid V \rangle$

# Abortable Consensus Properties

---

- At all times, **at most one** proposed value can be **chosen** by any process
  - **Safety**: **Agreement** and **Validity** (**Integrity** is achieved via *decided*)
- Succeeds once  $\Omega$  converges to a permanent correct leader
  - **Liveness**: **Termination**
- **Common pattern** followed by most **fail-noisy** (**partially synchronous**) algorithms:
  - Guarantee **safety at all times**
  - Guarantee **liveness once the system becomes stable**

# Abortable Consensus: Ballot $b$

## Leader

(1) Broadcast (prepare,  $b$ ) to all processes

(3) **upon** quorum  $S$  of (prepared,  $b$ ,  $a\_bal$ ,  $a\_val$ ) **do**:  
 If all  $a\_val = \text{null}$ :  
      $V = v_0$ ;  
 else  
      $V := a\_val$  with the highest  $a\_bal$  in  $S$ ;  
 Broadcast (accept,  $b$ ,  $V$ )

(5) **upon** quorum  $S$  of (accepted,  $b$ ) **do**:  
     **return**  $V$

If received (nack,  $b$ ) while waiting for a quorum in either (3) or (5),  
**return** ABORT

## All Processes

(2) **upon** (prepare,  $b$ ) from  $p$  **do**  
     **if**  $b > bal$  **then**  
          $bal := b$   
         send (prepared,  $b$ ,  $a\_bal$ ,  $a\_val$ ) to  $p$   
     **else**  
         send (nack,  $b$ ) to  $p$

(4) **upon** (accept,  $b$ ,  $v$ ) from  $p$  **do**:  
     **if**  $b \geq bal$  **then**  
          $bal := b$   
          $(a\_bal, a\_val) := (b, v)$   
         send (accepted,  $b$ ) to  $p$   
     **else**  
         send (nack,  $b$ ) to  $p$

# Complete Leader Protocol

---

**Initially:**

$leader := \perp$ ;  $myBallot := b_0$ ;  $decided := \text{FALSE}$ ;  $bal = b_0$ ;  $a\_bal := b_0$ ;  $a\_val := \text{null}$ ;  $v_0 := \text{value of the Propose request}$

**upon event**  $\langle \Omega, Trust \mid p \rangle$  **do**

$leader := p$

**upon**  $self = leader \wedge decided = \text{FALSE}$  **do**

- ▶  $myBallot := b$  such that  $b$  is unique and  $b > myBallot$
- ▶ Start **Abortable Consensus** for ballot  $myBallot$
- ▶ If returned  $V \neq \text{ABORT}$ : **trigger**  $\langle beb, Broadcast \mid [DECIDE, V] \rangle$

**upon event**  $\langle beb, Deliver \mid [DECIDE, V] \rangle$  such that  $decided = \text{FALSE}$  **do**

$decided := \text{TRUE}$

**trigger**  $\langle Decide \mid V \rangle$

# Correctness: Termination

---

- Eventually some correct process  $p$  is permanently trusted as a leader by all correct processes
- $p$  will be the only process executing the code triggered by the condition  $self = leader \wedge decided = FALSE$
- Since *myBallot* keeps increasing, eventually,  $p$  calls Abortable Consensus with a ballot which is  $>$  than the value of *bal* of any process
- Once this happens, Abortable Consensus is guaranteed to return  $V \neq ABORT$  causing  $p$  to decide  $V$  and broadcast  $V$  to all processes
- Since  $p$  is correct  $V$  will eventually reach all correct processes causing them to decide  $V$  as well

# Further Reading

---

- “Paxos Made Simple” (Lamport, 2001)
  - ▶ A copy is available on the module’s web site
- Adapted to the **standard** setting of the N-process Consensus problem
  - ▶ No separate proposer, acceptor, and learner roles
  - ▶ Every process can be viewed as simultaneously playing all these three roles



# Conclusions

---

- Paxos is a widely used protocol for solving Uniform Consensus in a partially synchronous system
  - Fail-noisy model with Eventual Leader Detector
  - Tolerates up to a minority  $f < N/2$  of process crashes
- Abortable Consensus is the core of Paxos
  - Never violates Agreement, but may fail to reach a decision (abort)
- Terminates once Abortable Consensus is called with a sufficiently high ballot number by a permanently trusted correct leader
  - This will happen once the system becomes synchronous

# Next

---

- Total-order broadcast