COMP 170 HW 7

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Write a CFG for the following langauge over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$:

$$L_1 = \{\mathtt{a}^n\mathtt{b}^n \mid n > 0 \text{ and } n \text{ is not a multiple of } 5\}$$

$$\begin{array}{lcl}
S_0 & \longrightarrow & \mathtt{a}\ S_1\ \mathtt{b} \\
S_1 & \longrightarrow & \mathtt{a}\ S_2\ \mathtt{b} \mid \mathtt{aa}\ S_3\ \mathtt{bb} \mid \mathtt{aaa}\ S_4\ \mathtt{bbb} \mid \varepsilon \\
S_2 & \longrightarrow & \mathtt{a}\ S_3\ \mathtt{b} \mid \mathtt{aa}\ S_4\ \mathtt{bb} \mid \varepsilon \\
S_3 & \longrightarrow & \mathtt{a}\ S_4\ \mathtt{b} \mid \varepsilon \\
S_4 & \longrightarrow & \mathtt{aa}\ S_1\ \mathtt{bb} \mid \varepsilon
\end{array}$$

Prove that the following language is not context-free where $\Sigma = \{0, 1\}$:

$$L_2 = \{w \mid w \text{ is a palindrome that contains an equal number of 1s and 0s}\}$$

Proof by Contradiction: via the Pumping Lemma

Initial Assumption: L_2 is context-free and thus satisfies the Pumping Lemma let p be the pumping length given by the Pumping Lemma.

Test Input: $w = 1^p 0^{2p} 1^p$ This input is in L_2.

By the Pumping Lemma, w can be broken down into five components $uvxyz$ such that:

- $|vxy| \leq p$
- $|vy| > 0$

Given that $|vxy| \leq p$, vxy cannot span across both the boarder between 1s and 0s and the boarder between 0s and 1s. Thus, when we pump vxy, we *either* pump 0s, 1s, or a string of the form $\ldots 0011 \ldots$ or a string of the form $\ldots 1100 \ldots$

Cases:

1. vxy contains only 0s. By pumping $uvxyz$ up to uv^2xy^2z, we produce a string with more 0s than 1s, and thus is not in L_2.
2. vxy contains only 1s. By pumping $uvxyz$ up to uv^2xy^2z, we produce a string with more 1s than 0s, and thus is not in L_2. (it's also not a palindrome but let's not get into it)
3. vxy spans a $\ldots 0011 \ldots$ or $\ldots 1100 \ldots$ transition. By pumping $uvxyz$ up to uv^2xy^2z, we produce a string that is no longer a palindrome, since vxy cannot also contain the other transition.

In all of these cases, by pumping $uvxyz$ to uv^2xy^2z, we produce a string that is no longer in L_2. Thus L_2 does not satisfy the Pumping Lemma.

$$\Longrightarrow\Longleftarrow$$

Thus L_2 is not context-free. $\square$

Consider the following context-free grammar:

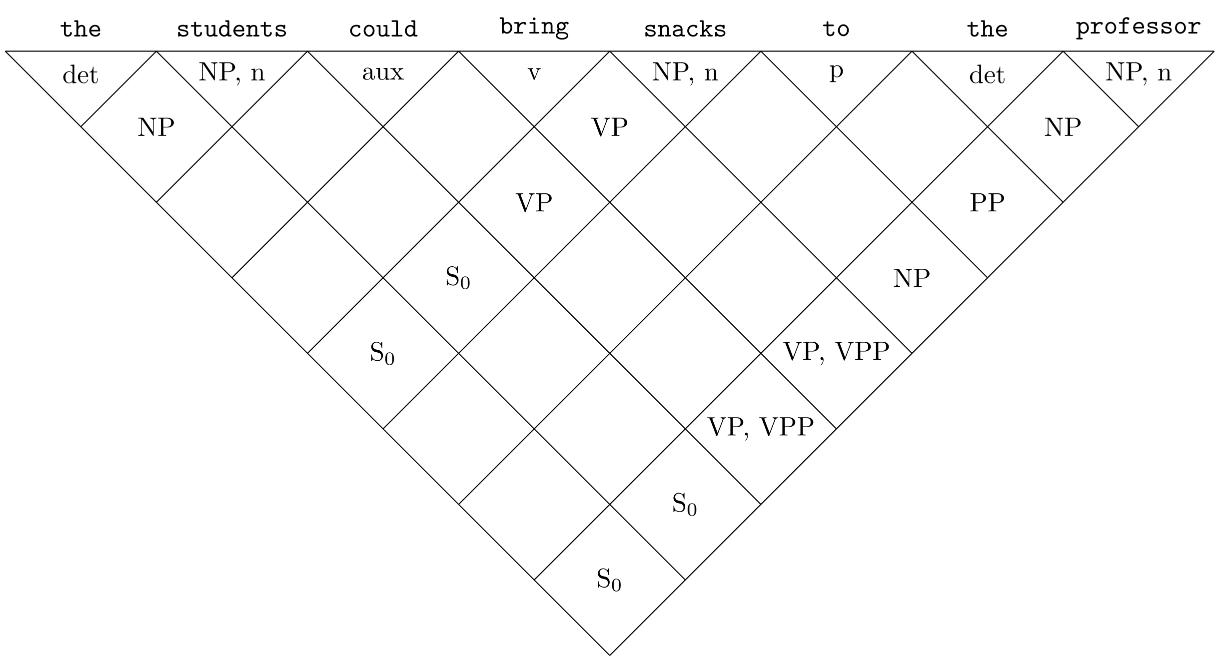
S	$\longrightarrow$	NP VP
S	$\longrightarrow$	NP VP PP
NP	$\longrightarrow$	det n
NP	$\longrightarrow$	n
NP	$\longrightarrow$	NP PP
VP	$\longrightarrow$	aux VP
VP	$\longrightarrow$	v NP
PP	$\longrightarrow$	p NP
det	$\longrightarrow$	`the` | `a` | `an`
n	$\longrightarrow$	`students` | `professor` | `snacks`
aux	$\longrightarrow$	`could` | `should` | `must`
v	$\longrightarrow$	`bring` | `feed`
p	$\longrightarrow$	`to` | `for`

List every derevation of the following input: `the students could bring snacks to the professor`. Your solution should be formatted according to the Sipser derevation on page 104.

In Chomsky Normal Form, the CFG becomes:

S_0	$\longrightarrow$	NP VP | NP VPP
VPP	$\longrightarrow$	VP PP
NP	$\longrightarrow$	det n | NP PP | `students` | `professors` | `snacks`
VP	$\longrightarrow$	aux VP | v NP
PP	$\longrightarrow$	p NP
det	$\longrightarrow$	`the` | `a` | `an`
n	$\longrightarrow$	`students` | `professor` | `snacks`
aux	$\longrightarrow$	`could` | `should` | `must`
v	$\longrightarrow$	`bring` | `feed`
p	$\longrightarrow$	`to` | `for`

Then we can apply the Ice Cream Cone algorithm. . .



$$
\begin{aligned}
\mathrm{S}_0 &\Longrightarrow \langle \mathrm{NP} \rangle \; \langle \mathrm{VP} \rangle \\
&\Longrightarrow \langle \mathrm{det} \rangle \; \langle \mathrm{n} \rangle \; \langle \mathrm{VP} \rangle \\
&\Longrightarrow \texttt{the } \langle \mathrm{n} \rangle \; \langle \mathrm{VP} \rangle \\
&\Longrightarrow \texttt{the students } \langle \mathrm{VP} \rangle \\
&\Longrightarrow \texttt{the students } \langle \mathrm{aux} \rangle \; \langle \mathrm{VP} \rangle \\
&\Longrightarrow \texttt{the students could } \langle \mathrm{VP} \rangle \\
&\Longrightarrow \texttt{the students could } \langle \mathrm{v} \rangle \; \langle \mathrm{NP} \rangle \\
&\Longrightarrow \texttt{the students could bring } \langle \mathrm{NP} \rangle \\
&\Longrightarrow \texttt{the students could bring } \langle \mathrm{NP} \rangle \; \langle \mathrm{PP} \rangle \\
&\Longrightarrow \texttt{the students could bring snacks } \langle \mathrm{PP} \rangle \\
&\Longrightarrow \texttt{the students could bring snacks } \langle \mathrm{p} \rangle \; \langle \mathrm{NP} \rangle \\
&\Longrightarrow \texttt{the students could bring snacks to } \langle \mathrm{NP} \rangle \\
&\Longrightarrow \texttt{the students could bring snacks to } \langle \mathrm{det} \rangle \; \langle \mathrm{n} \rangle \\
&\Longrightarrow \texttt{the students could bring snacks to the } \langle \mathrm{n} \rangle \\
&\Longrightarrow \texttt{the students could bring snacks to the professor}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{S}_0 &\Longrightarrow \langle \mathrm{NP} \rangle \; \langle \mathrm{VPP} \rangle \\
&\Longrightarrow \langle \mathrm{det} \rangle \; \langle \mathrm{n} \rangle \; \langle \mathrm{VPP} \rangle \\
&\Longrightarrow \texttt{the } \langle \mathrm{n} \rangle \; \langle \mathrm{VPP} \rangle \\
&\Longrightarrow \texttt{the students } \langle \mathrm{VPP} \rangle \\
&\Longrightarrow \texttt{the students } \langle \mathrm{VP} \rangle \; \langle \mathrm{PP} \rangle \\
&\Longrightarrow \texttt{the students } \langle \mathrm{aux} \rangle \; \langle \mathrm{VP} \rangle \; \langle \mathrm{PP} \rangle \\
&\Longrightarrow \texttt{the students could } \langle \mathrm{VP} \rangle \; \langle \mathrm{PP} \rangle \\
&\Longrightarrow \texttt{the students could } \langle \mathrm{v} \rangle \; \langle \mathrm{NP} \rangle \; \langle \mathrm{PP} \rangle \\
&\Longrightarrow \texttt{the students could bring } \langle \mathrm{NP} \rangle \; \langle \mathrm{PP} \rangle \\
&\Longrightarrow \texttt{the students could bring snacks } \langle \mathrm{PP} \rangle \\
&\Longrightarrow \texttt{the students could bring snacks } \langle \mathrm{p} \rangle \; \langle \mathrm{NP} \rangle \\
&\Longrightarrow \texttt{the students could bring snacks to } \langle \mathrm{NP} \rangle \\
&\Longrightarrow \texttt{the students could bring snacks to } \langle \mathrm{det} \rangle \; \langle \mathrm{n} \rangle \\
&\Longrightarrow \texttt{the students could bring snacks to the } \langle \mathrm{n} \rangle \\
&\Longrightarrow \texttt{the students could bring snacks to the professor}
\end{aligned}
$$