1) Assume,

1. $P(A \wedge B) = P(A) \times P(B)$

2. $P(A \wedge B) = P(A|B)\,P(B)$ — Product Rule

Divide by $P(B)$

3. $P(A|B) = \dfrac{P(A \wedge B)}{P(B)}$ — using 1. substitute $P(A \wedge B) = P(A) \times P(B)$

4. $P(A|B) = \dfrac{P(A) \times P(B)}{P(B)}$ — $P(B)/P(B)$ cancel out

5. $P(A|B) = P(A)$ — this is (ii)

$\therefore$ (i) and (ii) are equivalent.

2) a. $P(aw \mid ts) = 140/150 \overset{(140+10)}{} = 14/15 = 0.9\overline{33}$

ignore all $\neg aw$ values

b. $P(ts \mid aw) = 140/840 \overset{(140+700)}{} = 1/6 = 0.1\overline{66}$

ignore all $\neg ts$ values

3) a) $P(yng) = 0.25$ can conclude $p(\neg yng) = 0.75$

$p(fomo \mid yng) = 0.95$

$P(fomo \mid \neg yng) = 0.01$

First calc P(fomo): $P(fomo) = \underbrace{[P(fomo|yng) \times P(yng)]}_{P(fomo \wedge yng)} + \underbrace{[P(fomo|\neg yng) \times p(\neg yng)]}_{P(fomo \neg yng)}$

$= (0.95 \times 0.25) + (0.01 \times 0.75)$

$= 0.245$

Next calc $P(yng \mid fomo)$ and $P(\neg yng \mid fomo)$ to determine which is more likely

$P(yng|fomo) = \dfrac{P(yng \wedge fomo)}{P(fomo)} = \dfrac{0.95 \times 0.25}{0.245} = 0.969$ — P(yng) is greater given 'fomo'

$P(\neg yng|fomo) = \dfrac{P(\neg yng \wedge fomo)}{P(fomo)} = \dfrac{0.01 \times 0.75}{0.245} = 0.03$

b) $p(yng) = 0.01$ $\quad P(\neg yng) = 0.99$

$p(fomo|yng) = 0.96$ $\quad P(fomo|\neg yng) = 0.01$

$P(fomo) = [P(fomo|yng) \times P(yng)] + [P(fomo|\neg yng) \times P(\neg yng)]$

$(0.95 \times 0.01) + (0.01 \times 0.99)$

$= 0.0194$

$P(yng|fomo) = \dfrac{P(yng \wedge fomo)}{P(fomo)} = \dfrac{0.95 \times 0.01}{0.0194} = 0.4897$

$P(\neg yng|fomo) = \dfrac{P(\neg yng \wedge fomo)}{P(fomo)} = \dfrac{0.01 \times 0.99}{0.0194} = 0.5103$ — $P(\neg yng)$ greater given fomo.

c) $P(yng) = 0.01$ $P(\neg yng) = 0.99$
$P(fomo \mid yng) = 0.95$ $P(fomo \mid \neg yng) = 0.0001$

$$P(fomo) = [P(fomo \mid yng) \cdot P(yng)] + [P(fomo \mid \neg yng) \cdot P(yng)]$$
$$(0.95 \cdot 0.01) + (0.0001 \cdot 0.99)$$
$$= 0.009599$$

$$P(yng \mid fomo) = \frac{P(yng \wedge fomo)}{P(fomo)} = \frac{0.95 \cdot 0.01}{0.009599} = 0.98968643$$

P(yng) greater given fomo

$$P(\neg yng \mid fomo) = \frac{P(\neg yng \wedge fomo)}{P(fomo)} = \frac{(0.0001 \cdot 0.99)}{0.009599} = 0.0103157$$

4) $P(cool:+) = (62 + 108)/500 = 170/500 = 0.34.$

$P(cool:+ \mid noisy:+) = 62/(62 + 38) = 62/100 = 0.62$

Because $P(cool:+ \mid noisy:+) \neq P(cool:+)$ we can conclude cool:+ and noisy:+ are not independant.

5) $P(cool:+ \mid open:+) = (54+36)/100 = 9/10 = 0.9$

Check $P(cool:+ \mid open:+, noisy:+) = P(cool:+ \mid noisy)$
$P(cool:+ \mid open:+, noisy:+) =$
$54/(54+6) = 54/60 = 0.9$

$P(cool:+ \mid open:+) = P(cool:+ \mid open:+, noisy:+)$ so we can conclude the two events are independant.

6) For $\Theta_h = 0.9$, $0.1 \times 0.9 \times 0.1 \times 0.1 = 0.0009$
$\Theta_h = 0.5$, $0.5^4 = 0.0625$
$\Theta_h = 0.25$, $0.75 \times 0.25 \times 0.75 \times 0.75 = 0.105$
$\Theta_h = 0.1$, $0.9 \times 0.1 \times 0.9 \times 0.9 = 0.0729$