Prob Exercises

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The HAND-IN details:

- deadline is **before 4.00 Fri Sep 29th**
- I am perfectly happy for you to *write answers by hand on paper* – I know I would. I am going to set up a Blackboard submission and if you have made a handwritten answer, you should make **a digital image of it (photo,scan)** and submit that.

  You can also instead directly *create a digital document*, using whatever document creation software you like to work with. You should force it to finally generate a **pdf**, and you should submit that, and not the source document.

the questions:

1. Consider the following equations

   (i) $P(A \wedge B) = P(A) \times P(B)$
   (ii) $P(A|B) = P(A)$

   Give a proof that they are equivalent ie. show that (i) implies (ii) and that (ii) implies (i)

2. Suppose a player called *Talisman* plays for the *AnyOldTown* football team, and suppose Talisman 'often' scores a goal, and also that 'often' AnyOldTown win. Suppose the numbers actually are as follows

	aw	$\neg aw$
ts	140	10
$\neg ts$	700	150

   ts means 'Talisman scored a goal'
   aw means 'AnyOldTown won'

   (a) Calculate $P(aw \mid ts)$, the conditional prob that AnyOldTown won given thatTalisman scored a goal and also indicate which counts are irrelevant to this calculation

   (b) Calculate $P(ts \mid aw)$, the conditional prob that Talisman scored a goal given that AnyOldTown won and also indicate which counts are irrelevant to this calculation

   NB: try to work out the conditional probs as economically as possible

3. A sound clip from YouTube may or may not have been produced by a person below the age of 30. A clip may or may not contain the expression '*FOMO*'.

   You hear a clip coming from YouTube containing the expression '*FOMO*' and want to make the best guess as to whether or not the speaker was a person below the age of 30.

   Suppose formalized with 2 discrete variables

   - Age, values in {'*smaller than 30*', '*not smaller than 30*'}
   - FM, values in $\{true, false\}$, for whether a sound clip contains the expression '*FOMO*'

   and yng stands for $Age =$ '*smaller than 30*', $\neg yng$ stands for $Age =$ '*not smaller than 30*', and $fomo$ stands for $PB = true$

   Suppose you know $fomo$ and that you don't know the value of Age. Work out (showing reasoning) which of yng or $\neg yng$ is the best guess using the probabilities give in the cases below (NB: work out this best guess economically as possible)

   (a) supposing $p(yng) = 0.25, p(fomo|yng) = 0.95, p(fomo|\neg yng) = 0.01$
   (b) supposing $p(yng) = 0.01, p(fomo|yng) = 0.95, p(fomo|\neg yng) = 0.01$
   (c) supposing $p(yng) = 0.01, p(fomo|yng) = 0.95, p(fomo|\neg yng) = 0.0001$

4. Consider someone who lives in a basement flat. Sometimes it is quite noisy in the flat, and sometimes not. Sometimes it is rather cool in the flat, and sometimes not. Let $noisy$ be a variable indicating whether it is rather noisy or not, on a given day, and let $cool$ be a variable indicating whether it is rather cool or not.

   Consider the frequency table

	$noisy:+$	$noisy:-$
$cool:+$	62	108
$cool:-$	38	292

   (1)

   find $p(cool:+)$ and $p(cool:+|noisy:+)$

   and conclude from this whether or not $cool:+$ is independent of $noisy:+$

5. Unknown to the occupant of the flat there is ventilator fixture in the wall which can be opened and shut to let air from the street outside in, or keep it out. Unknown to the occupant a pet cat plays about with this at night-time, sometimes leaving it open and sometimes leaving it shut. The table (1) concerns 500 days. The two tables below split these into a group of 100 days where the cat has left the ventilator open (2), and 400 days where the cat has left it shut (3)

$open:+$	$noisy:+$	$noisy:-$
$cool:+$	54	36
$cool:-$	6	4

   (2)

$open:-$	$noisy:+$	$noisy:-$
$cool:+$	8	72
$cool:-$	32	288

   (3)

With reference to the table (2), find $p(cool : +|open : +)$ and $p(cool : +|open : +, noisy : +)$ and conclude from this whether or not $cool : +$ is conditionally independent of $noisy : +$ given $open : +$.

6. Suppose the following represents 4 tosses of a coin

   $T\ H\ T\ T$

   Suppose θ_h is the probability of a toss of the coin giving H. Work out the probability of the sequence for

   $\theta_h = 0.9$
   $\theta_h = 0.5$
   $\theta_h = 0.25$
   $\theta_h = 0.1$