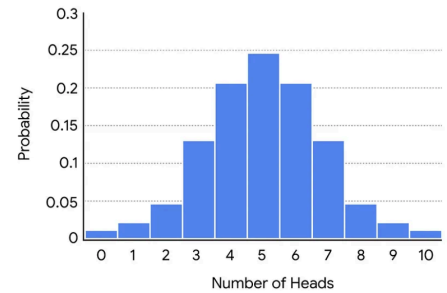# 6.1 Probability Distributions

In this unit we will look at the range of probabilities for all the possible outcomes of an experiment. Generally, a **probability distribution** tells you:
- what outcomes are possible
- the probability of each outcome.

For example, the histogram to the right shows the probability distribution for the number of heads when a fair coin is flipped 10 times.

Random Variables

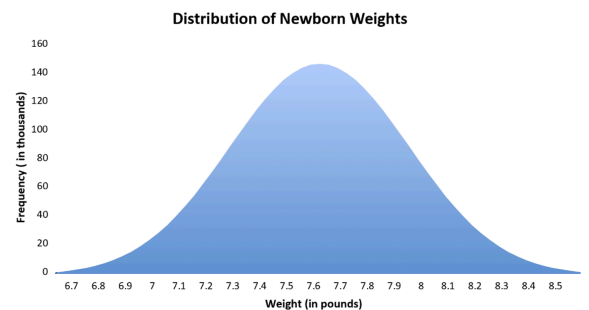A random variable is a quantity that can have a range of values.

Note: we use X to represent the random variable and x to represent the different values X can take on.

For the coin flipped 10 times: X is the number of heads, $x = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, and $P(X = 4) = 0.205$

A **discrete** random variable can have only _____ (E.g., the sum of two dice).

A **continuous** random variable can have infinitely many values

_(generally)_____.

(E.g., a student's reaction time or the mass of a newborn baby).

Distribution of Newborn Weights

Expected Value

The Expected Value is the **average** of a random variable. Specifically, it is the weighted mean of the outcomes (with respect to probability).

Note that this is a theoretical average and can be calculated without taking any measurements or observing any outcomes. To find the expected value, multiply each outcome by the probability of it occurring… and add them all up.
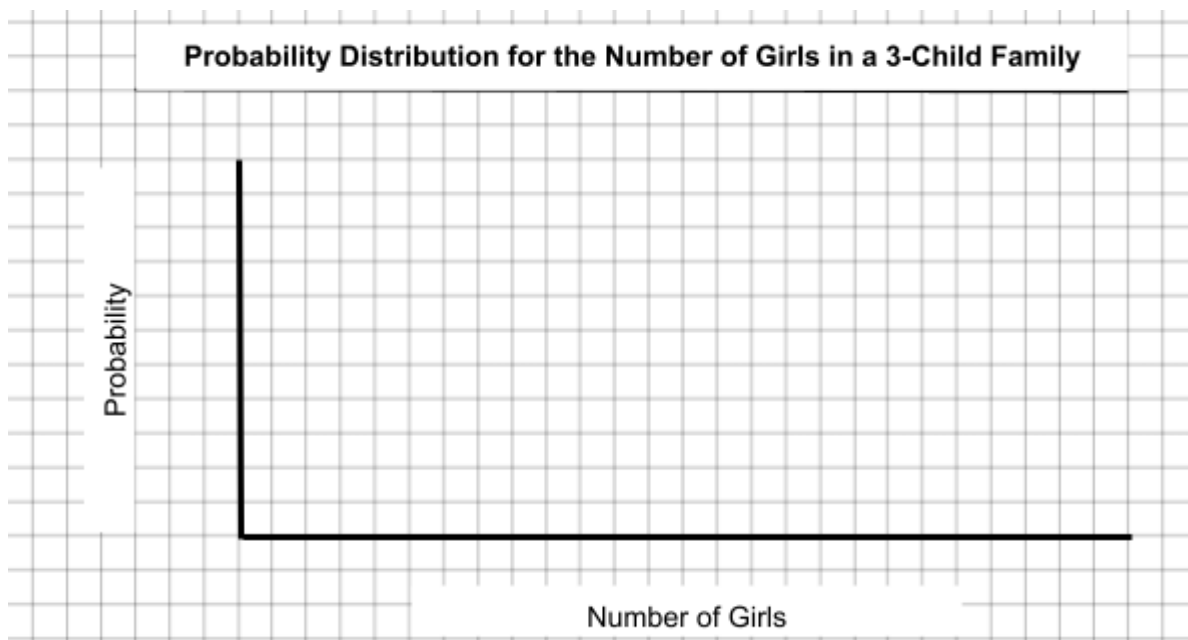
$$E(x) = \sum xP(x)$$

Try it!

1. What are the two key features of a probability distribution?

2. Classify the following random variables as discrete or continuous.

    a) Length of time you stay in class   _____

    b) Number of courses you take in a semester   _____

3. What is the expected value when rolling a fair die?

Like any function, a probability distribution can be represented by an equation or a graph. The graph of a probability distribution is called a probability histogram.

E.g., a) Make a probability distribution in a table for the number of girls in a 3 child family.
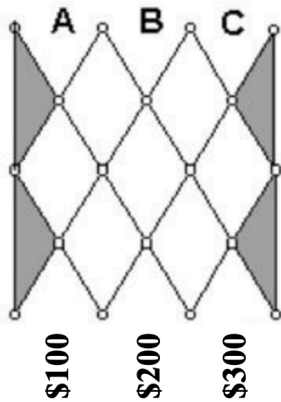
Number of Girls	Distribution	Frequency	Probability
0			
1			
2			
3			

Probability Distribution for the Number of Girls in a 3-Child Family

(Probability on vertical axis, Number of Girls on horizontal axis)

b) Calculate the Expected Value for the number of girls in a 3 child family. What does this answer mean?

Practice

1. You are playing Plinko on the board shown. What is the expected value of your winnings if you drop your disk in slot B?
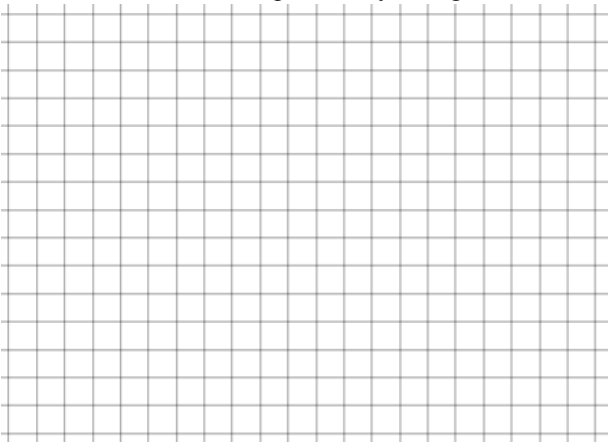

2. A hospital is having a fundraising lottery. A ticket costs $10. There are 2,000,000 tickets available with one $5,000,000 prize, three $100,000 prizes, ten $1,000 prizes and 2000 tickets for a free entry into next year's lottery (which is scheduled to have the same cost to enter).

   a) What is the expected value of a ticket?

   b) What does this answer mean?

   c) How much does the hospital stand to profit from this fundraiser?

3. For the situation involving the rolling of two number cubes and determining the sum:
    a. Identify the discrete random variable and create a probability distribution table

Sum	Frequency	Probability

b. Generate the related probability histogram

c. Determine the expected value when you roll 2 dice.

d. Determine the total area of the bars in the histogram and explain your result.

5. Calculate the expectation for each of the distributions.

a)

x	P(x)
1	0.3
2	0.2
3	0.1
4	0.4

b)

x	P(x)
0	$\frac{1}{5}$
2	$\frac{3}{10}$
4	$\frac{1}{5}$
6	$\frac{1}{10}$
8	$\frac{1}{10}$
10	$\frac{1}{10}$

9. Thinking A school is holding a fundraising raffle. The first prize is $500, the three second prizes are $100 each, and the five third prizes are $50 each. A total of 2000 tickets were sold at $5 each.

a) What is the probability of winning a prize?

b) What is the expected payout per ticket?

c) What is the expected profit per ticket?

d) What price should have been charged to have a 90% profit per ticket?

10. A card is chosen from a standard deck, replaced, then another is chosen. This process is repeated three times.

a) Show the probability distribution for the number of face cards in three trials.

b) Sketch a graph of this distribution.

c) Is the number of face cards a discrete random variable? Justify your response.

d) Calculate the expected value. Explain its meaning.

14. Communication The graph shows the percent of numbers that start with each digit when applied to many different data sets, such as hydro bills, addresses, stock prices, population sizes, death rates, and lengths of rivers.

Distribution of First Digits

a) Why would the distribution look this way?

b) Calculate the expectation. Explain what it means.

15. When continuously cutting a card from a deck with replacement, what is the probability that the first ace will be cut

a) on the first try?

b) on the second try?

c) on the third try?

d) on the nth try?

17. What is the expected sum of two weighted dice on which the number 5 occurs twice as often as the other numbers?
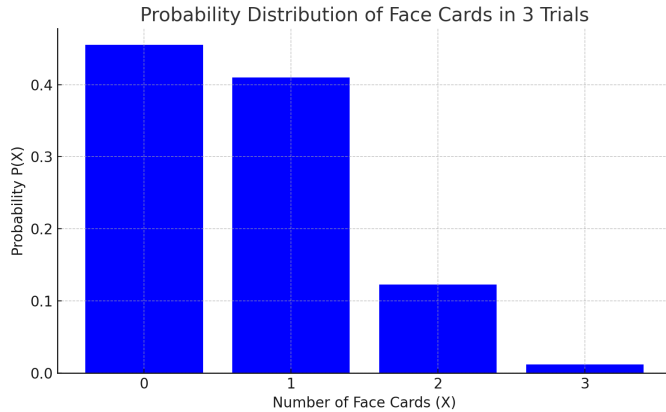
ANSWERS

5. a) 2.6   **b)** 3.8

9. a) $\frac{9}{2000}$ **b)** $0.525   **c)** $4.475   **d)** $5.03

10. a) $P(X = 0) = \frac{1000}{2197}$; $P(X = 1) = \frac{900}{2197}$; $P(X = 2) = \frac{270}{2197}$; $P(X = 3) = \frac{27}{2197}$

b)

Probability Distribution of Face Cards in 3 Trials

c) Yes. The number of face cards in three trials is a discrete random variable because:
It has countable outcomes (0, 1, 2, 3) and each value has a defined probability.

d) $E(X) = \frac{9}{13} \approx 0.692$

Meaning: On average, you can expect to draw 0.692 face cards in 3 random draws with replacement from a standard deck.

14. a) Answers may vary. There are many more values in these data sets that start with one compared to other digits.

b) 3.441

15. a) $\frac{4}{52}$ or $\frac{1}{13}$ **b)** $\frac{12}{169}$

c) $\frac{144}{2197}$ **d)** $\left(\frac{48}{52}\right)^{n-1} \times \frac{4}{52}$

17. a) approximately 7.02