7.1 Logarithmic Functions

Review

1. Rewrite an equivalent term for each of the following:

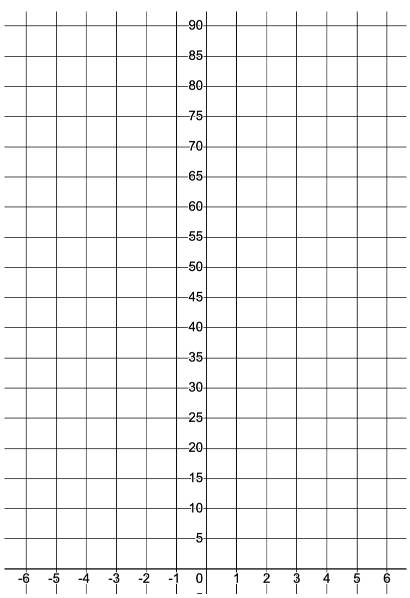
a) $(-\ 18)^{0}$ b) $125^{\frac{1}{3}}$ c) $\left(\frac{8}{27}\right)^{\frac{2}{3}}$

2. Simplify:

a) $(3m)^{4}$ b) $\left(16x^{6}\right)^{\frac{1}{2}}$ c) $\left(\frac{x^{5}y^{3}}{x^{2}y^{7}}\right)^{-2}$

3. Sketch graphs of the following:

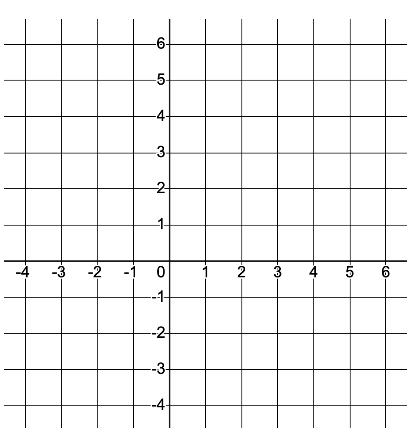
a) $f(x) = 3^{x}$



b) Complete the table of values for: $g(x) = \left(\frac{1}{2}\right)^{x}$

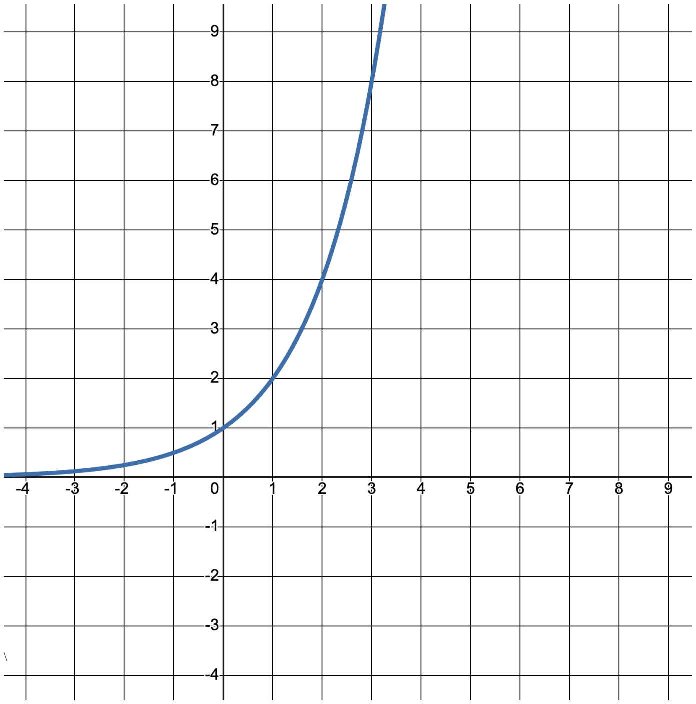
x	y
–2	
–1	
0	
1	
2	
3	

4. Sketch $m(x) = \frac{1}{3}x + 2$ and its inverse. Also determine an equation for its inverse.



Logarithms: Inverses of Exponentials (and key features)

The graph of $y = 2^x$ is shown below. Graph its inverse: $x = 2^y$



Another way to write $x = 2^y$ is:

Logarithm of *x* to the base *a*

What pattern do you notice as we fill in the logarithm values?

Exponential	Logarithm
$2^{-2} = \frac{1}{4}$	$\log_2\left(\frac{1}{4}\right) =$
$2^{-1} = \frac{1}{2}$	$\log_2\left(\frac{1}{2}\right) =$
$2^{0} = 1$	$\log_2(1) =$
$2^{1} = 2$	$\log_2(2) =$
$2^{2} = 4$	$\log_2(4) =$
$2^{3} = 8$	$\log_2(8) =$
$2^{4} = 16$	$\log_2(16) =$
$2^{5} = 32$	$\log_2(32) =$
$2^{6} = 64$	$\log_2(64) =$

Summary

A **logarithm** is an **inverse** of an **exponential** function. Thus (for any "**base**" number $a > 0$) $\log_a x = y$ and $x = a^y$ are equivalent expressions. So when evaluating $y = \log_a x$, remember that y is what the exponent needs to be, on the base a, so that the answer is x.

E.g., $\log_5 125 =$

E.g., $\log_4 32 =$

E.g., $\log 200 =$

E.g., $\log_6 50 =$

Practice

1. Evaluate the following (without a calculator):

a) $\log_3 9 =$

b) $\log_{10}(10,000) =$

c) $\log_2 512 =$

d) $\log_4 256 =$

e) $\log_3\left(\frac{1}{9}\right) =$

f) $\log_{10}\left(\frac{1}{1000}\right) =$

g) $\log_9 27 =$

h) $\log_{25} 3125 =$

2. Evaluate the following using a calculator and explain the results:

a) $\log 4000 =$

b) $\log_{10} 0.5 =$

c) $\log_2 5 =$

d) $\log_5 0.65 =$

3. Sketch $y = 10^x$,

and its inverse: $y = \log x$

4. Write the equation of the inverse, in **both exponential and logarithmic form**:

a) $f(x) = (5)^x$

b) $g(x) = \left(\frac{1}{4}\right)^x$

c) $c(x) = (a)^x$

d) $d(x) = \left(\frac{8}{7}\right)^x$

5. Complete the following table of values:

x	$f(x) = \log_3 x$
0	
$\frac{1}{27}$	
$\frac{1}{3}$	
1	
9	
27	