Math 240 - Tutorial 7 **July 1, 2023**

1) [Quiz Review] Let $T\colon V \to W$ be a linear transformation, let Z be a subspace of W, and let

$$U = \{x \in V\colon\ T(x) \in Z\}.$$

Show that U is a subspace of V.

2) [4.6 Q16] In $\mathbb{P}_2$, find the change-of-coordinates matrix from the basis $\mathcal{B} = \{1-3t^2, 2+t-5t^2, l+2t\}$ to the standard basis. Then write t^2 as a linear combination of the polynomials in $\mathcal{B}$.

3) [4.5 Q13] Determine the dimensions of Null(A), Col(A), and Row(A) of

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4) [4.5 Q42] Show that the space $\mathbb{P}$ of all polynomial functions defined on the real line is an infinite-dimensional vector space.

Let V be a 4-dimensional vector space with basis $\{\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{w}\}$, and S_i be the subspace of vectors in V whose ith coordinate is 0. Then,

$$\forall v \in V, v = a\mathbf{x} + b\mathbf{y} + c\mathbf{z} + d\mathbf{w},$$

and therefore if $v \in S_i$, then

$$v_i = ax_i + by_i + cz_i + dw_i = 0.$$

Writing $v = (a, b, c, d)$, $v \in S_i$ if and only if $(a, b, c, d) \cdot (x_i, y_i, z_i, w_i) = 0$, so S_i is the orthogonal complement of (x_i, y_i, z_i, w_i). Therefore, if $(x_i, y_i, z_i, w_i) \neq (0, 0, 0, 0)$, then S_i is 3-dimensional.