0) [Exam/Course Review?]

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1) [6.1 Q34, 39] Let $u = (5, -6, 7)^T$. Find all vectors v such that $u \cdot v = 0$. Show that $x \in \text{span}\{u\}$ and $x(\in \text{span}\{u\})^{\perp}$ if and only if $x = 0$.

$(5,-6,7)\cdot(a,b,c)=0$

$5a-6b+7c=0$

$\Rightarrow 5a = 6b-7c$

$a = 6/5\, b - 7/5\, c$

Basis $\left\{ \begin{bmatrix} 6/5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7/5 \\ 0 \\ 1 \end{bmatrix} \right\}$

if $\underline{x=0}$, then $0 \in \text{span}\{u\}$ & $(\text{span}\{u\})^{\perp}$, as they're both subspaces

$x \in \text{span}\{u\} \Rightarrow \underline{x} = k\underline{u}$, for some $k \in \mathbb{R}$

$x \in (\text{span}\{u\})^{\perp} \Rightarrow \underline{x}\cdot\underline{u} = 0$

$\Rightarrow k\,\underline{u}\cdot\underline{u} = 0$

$\|$

$k(\underline{u}\cdot\underline{u}) = 0$

$\Downarrow$

$\underline{u} = 0$

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2) [6.3 Q3] Verify $\{u_1, u_2\}$ are orthogonal, then find the orthogonal projection of y onto $\text{span}\{u_1, u_2\}$.

$u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad y = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$

$u_1 \cdot u_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = 1(-1) + 1(1) = 0$ ✓

$\text{proj}_{\{u_1,u_2\}} y = \text{proj}_{u_1} y + \text{proj}_{u_2} y$

$= \dfrac{u_1 \cdot y}{\|u_1\|^2} u_1 + \dfrac{u_2\, y}{\|u_2\|^2}$

$= \dfrac{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}}{2} u_1 + \dfrac{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}}{2} u_2$

$= \frac{3}{2}\underline{u_1} + \frac{5}{2}\underline{u_2}$

$\frac{3}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \frac{5}{2}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix} \leftarrow$ Makes sense

as ← span$\{u_1, u_2\}$

deletes 3rd coordinate.

3) [6.3 Q31] Let A be an $m \times n$ matrix. Prove that every vector in $\mathbb{R}^n$ can be written as $x = p + u$, where $p \in \text{Row}(A)$ and $u \in \text{Null}(A)$.

$A = m \overset{n}{[\quad]}$ Let $x \in \mathbb{R}^n$

$\text{Null}(A) = \text{Row}(A)^\perp$

$\boxed{\text{Theorem 8 orthog. decomp. theorem.}} \Rightarrow$ each $x \in \mathbb{R}^n$ can be written as $x = y + z$ where $y \in \text{Row}(A)$, $z \in \text{Null}(A)$

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4) [6.4 Q2] Let $\{(0,4,2)^T, (5,0,-7)^T\}$ be a basis for $W \subset \mathbb{R}^3$. Use Graham–Schmidt to find orthonormal basis for W.

Graham Schmidt

$v_1 = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$

$v_2 = \begin{bmatrix} 5 \\ 0 \\ -7 \end{bmatrix} - \text{proj}_{\begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}} \begin{bmatrix} 5 \\ 0 \\ -7 \end{bmatrix}$

$= \begin{bmatrix} 5 \\ 0 \\ -7 \end{bmatrix} - \dfrac{\begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 0 \\ -7 \end{bmatrix}}{20} \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 5 \\ 0 \\ -7 \end{bmatrix} + \frac{14}{20} \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -7 \end{bmatrix} + \begin{bmatrix} 0 \\ 14/5 \\ 14/10 \end{bmatrix} = \begin{bmatrix} 5 \\ 14/5 \\ -56/10 \end{bmatrix}$

$v_1 \cdot v_2 = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 14/5 \\ -56/10 \end{bmatrix}$

$= 4(14/5) - 56/5$

$= 56/5 - 56/5$

$= 0$

$v_1 / \|v_1\| = \frac{1}{\sqrt{20}} \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$

$v_2 / \|v_2\| = \underbrace{\dfrac{1}{\sqrt{25 + (14/5)^2 + (56/10)^2}}}_{} \begin{bmatrix} 5 \\ 14/5 \\ -56/10 \end{bmatrix}$

↑ Doesn't really Simplify :(