1) [1.7 Q13] Find the value(s) of h for which the vectors are linearly *dependent*.

$$\begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ -9 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ h \\ -9 \end{bmatrix}$$

$$\left.\begin{aligned} a - 2b + 3c &= 0 \\ 5a - 9b + hc &= 0 \\ -3a + 6b - 9c &= 0 \end{aligned}\right\} \begin{bmatrix} 1 & -2 & 3 \\ 5 & -9 & h \\ -3 & 6 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 3 \\ 5 & -9 & h \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & h-15 \\ 0 & 0 & 0 \end{bmatrix}$$

Depend

for all h

$$\sim \begin{bmatrix} 1 & 0 & 2h-27 \\ 0 & 1 & h-15 \\ 0 & 0 & 0 \end{bmatrix}$$

$c = t, \ t \in \mathbb{R}$

$b = -t(h-15)$

$a = -t(2h-27)$

$t = -1$

$a = 2h-27$

$b = h-15$

$c = -1$

$\boxed{h \in \mathbb{R}}$

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2) [1.8 Q39] Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let $\{v_1, v_2, v_3\}$ be a linearly dependent set in $\mathbb{R}^n$. Explain why the set $\{T(v_1), T(v_2), T(v_3)\}$ is linearly dependent.

$T : \mathbb{R}^n \to \mathbb{R}^m$ L.T. v_1, v_2, v_3 L.D

$\Longleftrightarrow \exists \underbrace{a, b, c}_{\text{at least one non-zero}} \in \mathbb{R}$ s.t. $a\underline{v_1} + b\underline{v_2} + c\underline{v_3} = 0$

then $aT(\underline{v_1}) + bT(\underline{v_2}) + cT(\underline{v_3})$

Theorem 5

$= T(a\underline{v_1}) + T(b\underline{v_2}) + T(c\underline{v_3})$

$= T(a\underline{v_1} + b\underline{v_2} + c\underline{v_3})$

$= T(\underline{0})$

$= 0$

$\therefore \exists a, b, c \in \mathbb{R}$ s.t. $aT(v_1) + bT(v_2) + cT(v_3) = 0$

$\Longleftrightarrow$ L.D.

3) [1.9 Q6,8] Assume that T is a linear transformation. Find the standard matrix of T, where **(1)** $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a horizontal shear transformation that leaves e_1 unchanged and maps e_2 into $e_1 + 3e_2$, **(2)** $T : \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the horizontal x_1-axis and then reflects points through the line $x_2 = x_1$.

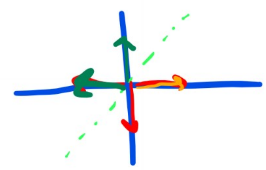
(1) $T(e_1) = e_1 \sim \begin{bmatrix}1\\0\end{bmatrix}$

$T(e_2) = e_1 + 3e_2 = \begin{bmatrix}1\\0\end{bmatrix} + 3\begin{bmatrix}0\\1\end{bmatrix}$ $\therefore$ $\boxed{[T] = \begin{bmatrix}1 & 0\\0 & 3\end{bmatrix}}$

Theorem 10

(2) $T(e_1) = e_2 = \begin{bmatrix}0\\1\end{bmatrix}$

$T(e_2) = -e_1 = -\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-1\\0\end{bmatrix}$



$\boxed{[T] = \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}}$

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4) [1.8 Q41] Show that the transformation T defined by $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$ is not linear.

Counter- Example

$2\,T(x_1, x_2) = 2\,(2x_1 - 3x_2, x_1 + 4, 5x_2)$
$= (4x_1 - 6x_2, 2x_1 + 8, 10x_2)$

$T(2x_1, 2x_2) = (2(2x_1) - 3(2x_2), 2x_1 + 4, 5(2x_2))$
$= (4x_1 - 6x_2, 2x_1 + 4, 10x_2)$

if $x_1 \neq 0$, then $T(2\underline{x}) \neq 2\,T(\underline{x}) \Rightarrow$ T not linear $\square$