0) Midterm Questions?

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1) [4.1 Q6,7] Determine if the given set is a subspace of $\mathbb{P}_n$ for an appropriate value of n. Justify your answers. (i.) All polynomials of the form $p(t) = a + t^2$, where $a \in \mathbb{R}$. (ii.) All polynomials with degree at most 3 with integers as coefficients.

i. $p(t) = a + t^2,\ a \in \mathbb{R}$

For $n \geq 2$

1) Closed under addition? Nope ✗

$(3+t^2)+(1+t^2) = 4 + \underline{2}t^2$

also fails 2) Scalar mult.

$2(1+t^2) = 2+2t^2$

For $n = 0, 1$

not subspace

because has quadratic term

ii. $n \geq 2$

+) $a_1 + b_1 t + c_1 t^2 + d_1 t^3 \in \mathbb{P}_n$

$a_2 + b_2 t + c_2 t^2 + d_2 t^3$

$(a_1+a_2) + (b_1+b_2)t + (c_1+c_2)t^2 + (d_1+d_2)t^3$

still int

✓

·) $\pi(t^3+4t)$

$= \underline{\pi} t^3 + \underline{4\pi} t$

$\notin \mathbb{Z}$ ✗

2) [4.1 Q10] Let H be the set of all vectors of the form $\begin{bmatrix} 2t \\ 0 \\ -t \end{bmatrix}$. Show that H is a subspace of $\mathbb{R}^3$.

$$+)\quad \begin{bmatrix} 2t_1 \\ 0 \\ -t_1 \end{bmatrix} + \begin{bmatrix} 2t_2 \\ 0 \\ -t_2 \end{bmatrix} = \begin{bmatrix} 2(t_1+t_2) \\ 0 \\ -(t_1+t_2) \end{bmatrix} \in H \quad \checkmark \qquad \underline{H \subseteq \mathbb{R}^3} \checkmark$$

$$\cdot)\quad a\begin{bmatrix} 2t_1 \\ 0 \\ -t_1 \end{bmatrix} = \begin{bmatrix} 2at_1 \\ 0 \\ -t_1 a \end{bmatrix} \in H \checkmark$$

$$H = \text{Span}\left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \right\} \text{ subspace of } \mathbb{R}^3$$

if $v_1, v_2, \dots, v_n \in \mathbb{R}^m$, Span$\{v_1, v_2, \dots, v_n\}$ is always a subspace of $\mathbb{R}^m$

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3) [4.2 Q1] Determine if $w = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is in Null(A), where

$$A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$$

$\underline{w} \in \text{Null}(A)$ iff $A\underline{w} = \underline{0}$

$$A\underline{w} = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} = \begin{bmatrix} 3\cdot 1 + (-5)\cdot 3 + (-3)(-4) \\ 6\cdot 1 + (-2)\cdot 3 + 0\cdot(-4) \\ -8\cdot 1 + 4\cdot 3 + 1\cdot(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 3 - 15 + 12 \\ 6 - 6 + 0 \\ -8 + 12 - 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore \underline{w} \in \text{Null}(A)$