Math 240 - Tutorial 12 **August 4, 2023**

0) [Exam/Course Review?]

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1) [6.1 Q34, 39] Let $u = (5, -6, 7)^T$. Find all vectors v such that $u \cdot v = 0$. Show that $x \in \text{span}\{u\}$ and $x \in (\text{span}\{u\})^{\perp}$ if and only if $x = 0$.

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2) [6.3 Q3] Verify $\{u_1, u_2\}$ are orthogonal, then find the orthogonal projection of y onto $\text{span}\{u_1, u_2\}$, where:

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \; u_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \; y = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$$

3) [6.3 Q31] Let A be an $m \times n$ matrix. Prove that every vector in $\mathbb{R}^n$ can be written as $x = p + u$, where $p \in \text{Row}(A)$ and $u \in \text{Null}(A)$.

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4) [6.4 Q2] Let $\{(0, 4, 2)^T, (5, 0, -7)^T\}$ be a basis for $W \subset \mathbb{R}^3$. Use Graham–Schmidt to find orthonormal basis for W.

$$\begin{aligned} &(0, 1, 2, 0, 2, 1) \\ -&(0, 0, 1, 1, 2, 2) \\ =&(0, 1, 1, 2, 0, 2) \end{aligned}$$