CSCE-420-HW2

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1 Question 1

1.1 Prove that "Implication Introduction" (the opposite of Implication Elimination) is a sound rule of inference (ROI) using a truth table.

X	Y	Z	$\neg X \vee Z \vee \neg Y$	$(X \wedge Y) \to Z$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	F
T	F	F	T	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	T	F

1.2 Prove that $(A \wedge B \to C \wedge D) \vdash (A \wedge B \to C)$ ("conjunctive rule splitting") is a sound rule of inference using a truth table.

A	B	C	D	$A \wedge B$	$C \wedge D$	$A \wedge B \to C \wedge D$	$A \wedge B \to C$
T	T	T	T	T	T	T	T
T	T	T	F	T	F	F	T
T	T	F	T	T	F	F	F
T	T	F	F	T	F	F	F
T	F	T	T	F	T	T	T
T	F	T	F	F	F	T	T
T	F	F	T	F	F	T	T
T	F	F	F	F	F	T	T
F	T	T	T	F	T	T	T
F	T	T	F	F	F	T	T
F	T	F	T	F	F	T	T
F	T	F	F	F	F	T	T
F	F	T	T	F	T	T	T
F	F	T	F	F	F	T	T
F	F	F	T	F	F	T	T
F	F	F	F	F	F	T	T

1.3 Also prove $(A \wedge B \to C \wedge D) \vdash (A \wedge B \to C)$ using Natural Deduction. (Hint: use 1a above)

1. $A \wedge B \to C \wedge D$ (Premise)
2. $A \wedge B$ (Assumption for Conditional Proof)
3. A (Elimination from 2)
4. B (Elimination from 2)
5. $C \wedge D$ (Modus Ponens from 1 and 2)
6. C (Elimination from 5)
7. D (Elimination from 5)
8. $A \wedge B \to C$ (Conditional Proof, discharging 2)

1.4 Also prove $(A \wedge B \to C \wedge D) \vdash (A \wedge B \to C)$ using Resolution.

1. $\neg(A \wedge B) \vee (C \wedge D)$ (Assumption)
2. $\neg(A \wedge B) \vee C$ (To be proven)
3. $\neg C \vee (A \wedge B)$ (Negate 2)
4. $(A \wedge B) \vee (C \wedge D)$ (Resolve 1 and 3)
5. $(A \wedge B) \vee C$ (Simplify 4)

2 Question 2

2.1 Using these propositional symbols, write a propositional knowledge base (sammy.kb) that captures the knowledge in this domain (i.e. implications of what different observations or labels mean, as well as constraints inherent in this problem, such as that all boxes have different contents).

n is a specific box

1. $O_{nY} \rightarrow (C_{iW} \vee C_{iB})$
2. $O_{nW} \rightarrow (C_{iY} \vee C_{iB})$
3. $L_{nY} \rightarrow (C_{iY} \vee C_{iB})$
4. $L_{iW} \rightarrow (C_{iW} \vee C_{iB})$
5. $C_{1Y} \vee C_{1W} \vee C_{1B}$
6. $C_{2Y} \vee C_{2W} \vee C_{2B}$
7. $C_{3Y} \vee C_{3W} \vee C_{3B}$
8. $L_{1Y} \equiv \neg C_{1Y}$
9. $L_{2W} \equiv \neg C_{2W}$
10. $L_{3B} \equiv \neg C_{3B}$
11. C_{2W}

2.2 Prove that box 2 must contain white balls (C2W) using Natural Deduction.

1.	$C_{3Y} \vee C_{3B}$	(Observation)
2.	C_{3Y}	(Modus Ponens from L_{3B} and Resolution from 1)
3.	$C_{1Y} \vee C_{1B}$	(Resolution from 1)
4.	$\neg C_{1Y} \vee \neg C_{1B}$	(Modus Ponens from 2)
5.	$\neg C_{1Y}$	(Elimination from 4)
6.	C_{1B}	(Resolution from 3 and 5)
7.	$C_{2W} \vee C_{2B}$	(Modus Ponens from O_{2W})
8.	$\neg C_{2B}$	(Modus Ponens and Elimination from 7)
9.	C_{2W}	(Resolution from 8)

2.3 Convert your KB to CNF.

1. $\neg O_{nY} \vee C_{nW} \vee C_{nB}$
2. $\neg O_{nW} \vee C_{nY} \vee C_{nB}$
3. $\neg L_{nY} \vee C_{nY} \vee C_{nB}$
4. $\neg L_{nW} \vee C_{nW} \vee C_{nB}$
5. $(\neg L_{1Y} \vee \neg C_{1Y}) \wedge (L_{1Y} \vee C_{1Y})$
6. $(\neg L_{2W} \vee \neg C_{2W}) \wedge (L_{2W} \vee C_{2W})$
7. $(\neg L_{3B} \vee \neg C_{3B}) \wedge (L_{3B} \vee C_{3B})$
8. $(\neg C_{1Y} \vee \neg C_{1W}) \wedge (\neg C_{1Y} \vee \neg C_{1B}) \wedge (\neg C_{1W} \vee \neg C_{1B})$
9. $(\neg C_{2Y} \vee \neg C_{2W}) \wedge (\neg C_{2Y} \vee \neg C_{2B}) \wedge (\neg C_{2W} \vee \neg C_{2B})$
10. $(\neg C_{3Y} \vee \neg C_{3W}) \wedge (\neg C_{3Y} \vee \neg C_{3B}) \wedge (\neg C_{3W} \vee \neg C_{3B})$

2.4 Prove C2W using Resolution.

1.	$\neg O_{2W} \vee C_{2Y} \vee C_{2B}$	
2.	$C_{2W} \vee C_{2B}$	
3.	$\neg C_{2B}$	
4.	$\neg O_{2W} \vee C_{2B}$	(Resolution on 1 and 2)
5.	$\neg O_{2W}$	(Resolution on 3 and 4)
6.	C_{2B}	(Resolution on 1 and 5)
7.	C_{2W}	(Resolution on 2 and 6)

3 Do Forward Chaining for the CanGetToWork KB.

note: doesn't need to be sunny, take car rental route, doesn't own car (harder), could bike to work with sunny (Sunny not provable), close to home, and bike

1. Rainy, HaveMoutainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen

2. HaveMoutainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen

3. EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, HaveBike **Rule e**

4. WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, HaveBike

5. WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, HaveBike
6. HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, HaveBike
7. HertzClosed, AvisOpen, McDonaldsOpen, HaveBike
8. AvisOpen, McDonaldsOpen, HaveBike **Rule m**
9. McDonaldsOpen, HaveBike, CarRentalOpen
10. HaveBike, CarRentalOpen
11. CarRentalOpen **Rule o**
12. IsNotAHoliday, CarRentalOpen **Rule k**
13. CanRentCar **Rule j**
14. CanDriveToWork **Rule b**
15. Therefore CanGetToWork

HaveBike,¬IsNotAHoliday,CarRentalOpen,CanDriveToWork,CanRentCar,CanGetToWork, Rainy, HaveMountaiBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen

4 Do Backward Chaining for the CanGetToWork KB.

1. CanGetToWork — pop, **Rule a**
2. CanBikeToWork — pop, **Rule d**
3. Sunny, WorkCloseToHome, HaveBike — backtrack, Sunny not provable
4. CanDriveToWork — pop, **Rule g**
5. OwnCar — backtrack, OwnCar not provable
6. CanDriveToWork — pop, **Rule j**
7. CanRentCar — pop, **Rule k**
8. CarRentalOpen, HaveMoney — pop, **Rule l**
9. HaveMoney, Hertz — backtrack, **HertzOpen not fact**
10. HaveMoney, AvisOpen — pop, **Avis is fact**
11. HaveMoney — pop, **HaveMoney is fact, done**