CSCE-420-HW3

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1 Question 1

1.1 Translate the following sentences into First-Order Logic. Remember to break things down to simple concepts (with short predicate and function names), and make use of quantifiers. For example, don't say "tasteDelicious(someRedTomatos)", but rather:

$$\exists x\, \mathbf{tomato}(x) \wedge \mathbf{red}(x) \wedge \mathbf{taste}(x, \mathbf{delicious}).$$

1. bowling balls are sporting equipment

$$\forall x\, (\text{BowlingBall}(x) \rightarrow \text{SportingEquipment}(x)).$$

2. all horses have a higher speed than any frog

$$\forall x \forall y\, (\text{Horse}(x) \wedge \text{Frog}(y) \rightarrow \text{Speed}(x) > \text{Speed}(y))$$

3. all domesticated horses have an owner

$$\forall x\, (\text{DomesticatedHorse}(x) \rightarrow \text{Owner}(x))$$

4. the rider of a horse can be different than the owner

$$\exists x, y\, \text{Horse}(x) \wedge \text{Rider}(x, y) \wedge \neg(\text{Owner}(x, y))$$

5. a finger is any digit on a hand other than the thumb

$$\forall x (Finger(x) \wedge PartOfHand(x, hand) \rightarrow \neg Thumb(x)$$

6. an isosceles triangle is defined as a polygon with 3 edges connected at 3 vertices, where 2 (but not 3) edges have the same length

$$\begin{aligned} &\forall t, x, y, z\, (Polygon(t) \wedge edge(x) \wedge edge(y) \wedge edge(z) \wedge \\ &\qquad PartOfTriangle(x, t) \wedge PartOfTriangle(y, t) \wedge PartOfTriangle(z, t) \wedge \\ &\qquad VertexFormed(x, y) \wedge VertexFormed(x, z) \wedge VertexFormed(y, z) \wedge \\ &\qquad SameLength(x, y) \wedge \neg SameLength(x, z)) \rightarrow IsoscelesTriangle(t) \end{aligned}$$

2 Question 2

2.1 Convert the following first-order logic sentence into CNF:

$$\forall x person(x) \wedge [\exists z petOf(x,z) \wedge \forall y petOf(x,y) \rightarrow dog(y)] \rightarrow doglover(x)$$

1. Replace implication with $\vee$

   $\forall x\,(\neg\text{person}(x) \vee \neg(\exists z\,\text{petOf}(x,z) \wedge \forall y\,(\text{petOf}(x,y) \rightarrow \text{dog}(y))) \vee \text{doglover}(x)$

2. Distribute negation

   $\forall x\,(\neg\text{person}(x) \vee \neg[\exists z\,\text{petOf}(x,z) \wedge \forall y\,(\neg\text{petOf}(x,y) \vee \text{dog}(y))]) \vee \text{doglover}(x)$

3. Replace $\exists$ with F(x)

   $\forall x\,(\neg\text{person}(x) \vee \neg[\,\text{petOf}(x,F(x)) \wedge \forall y\,(\neg\text{petOf}(x,y) \vee \text{dog}(y))]) \vee \text{doglover}(x)$

4. Remove universal quanitifiers

   $\neg\text{person}(x) \vee [\text{petOf}(x,f(x)) \wedge \text{petOf}(x,y) \vee \text{dog}(y))] \vee \text{doglover}(x)$

3 Question 3

3.1 Determine whether or not the following pairs of predicates are unifiable. If they are, give the most-general unifier and show the result of applying the substitution to each predicate. If they are not unifiable, indicate why. Capital letters represent variables; constants and function names are lowercase. For example, 'loves(A,hay)' and 'loves(horse,hay)' are unifiable, the unifier is u=A/horse, and the unified expression is 'loves(horse,hay)' for both.

1. `owes(owner(X), citibank, cost(X))` and `owes(owner(ferrari),Z, cost(Y))`

   Unification: $\{X/ferrari, Z/citibank, Y/ferrari\}$

   Unified Expression: `owes(owner(ferrari), citibank, cost(ferrari))`

2. `gives(bill, jerry, book21)` and `gives(X, brother(X), Z)`

   Not Unifiable

   Reason: It is not possible to substitute variables within this expression to match one another.

3. `opened(X, result(open(X), s0))` and `opened(toolbox, Z)`

   Unification: $\{X/toolbox, Z/result(open(toolbox), s0)\}$

   Unified Expression: `opened(toolbox, result(open(toolbox), s0))`

4 Question 4

4.1 Translate these sentences to First-Order Logic.

- Pompeian($Marcus$).
- $\forall x$(Pompeian(x) $\rightarrow$ Roman(x)).
- Ruler($Caesar$).
- $\forall x$(Roman(x) $\rightarrow$ (LoyalTo($x, Caesar$) $\oplus$ Hate($x, Caesar$))).
- $\forall x \exists y$ LoyalTo(x, y).
- $\forall x, y$(Ruler(y) $\wedge$ $\neg$LoyalTo(x, y) $\rightarrow$ TryAssassinate(x, y)).
- TryAssassinate($Marcus, Caesar$).

4.2 Prove that Marcus hates Caesar using Natural Deduction. Label all derived sentences with the ROI and which prior sentences and unifier were used.

1. Roman($Marcus$) From (1, 2), unifier={X/Marcus}
2. LoyalTo($Marcus, Caesar$) $\oplus$ Hate($Marcus, Caesar$) From (3, 4), unifier={X/Marcus, Y/Caesar}
3. $\neg$LoyalTo($Marcus, Caesar$) $\rightarrow$ TryAssassinate($Marcus, Caesar$) From (7, 8), unifier={X/Marcus}
4. $\neg$LoyalTo($Marcus, Caesar$) From (6, 9) Modus Ponens
5. Hate($Marcus, Caesar$) From (5, 10) Disjunctive Syllogism

4.3 Convert all the sentences into CNF

- Pompeian($Marcus$)
- $\forall x$($\neg$Pompeian(x) $\vee$ Roman(x)).
- Ruler($Caesar$)
- $\forall x$(($\neg$Roman(x)$\vee$LoyalTo($x, Caesar$)$\vee$Hate($x, Caesar$))$\wedge$($\neg$LoyalTo($x, Caesar$)$\vee$ $\neg$Hate($x, Caesar$)))
- LoyalTo($x, f(x)$)
- $\forall x, y$($\neg$Ruler(y) $\vee$ LoyalTo(x, y) $\vee$ TryAssassinate(x, y))
- TryAssassinate($Marcus, Caesar$)

4.4 Prove that Marcus hates Caesar using Resolution Refutation.

1. Assume ¬Hate($Marcus, Caesar$)
2. Resolution of (1) and (2): Roman($Marcus$)
3. Resolution of (4) and (8): ¬Roman($Marcus$) ∨ LoyalTo($Marcus, Caesar$)
4. Resolution of (9) and (10): LoyalTo($Marcus, Caesar$)
5. From (3), (11), and (6): TryAssassinate($Marcus, Caesar$)
6. Resolution of (7) and (12) leads to a contradiction
7. Therefore, Hate($Marcus, Caesar$) must be true

5 Question 5

5.1 Map-coloring – every state must be exactly 1 color, and adjacent states must be different colors. Assume possible colors are states are defined using unary predicate like color(red) or state(WA). To say a state has a color, use a binary predicate, e.g. 'color(WA,red)'.

- ∀x state(x) → ∃!y color(x,y)
- ∀x, y, z state(x) ∧ state(y) ∧ Adjacent(x, y) ∧ color(x, z) → ¬color(y,z)
- ∀x, y, z state(x) ∧ color(x, y) ∧ color(x, z) → y = z

5.2 Sammy's Sport Shop – include implications of facts like obs(1,W) or label(2,B), as well as constraints about the boxes and colors. Use predicate 'cont(x,q)' to represent that box x contains tennis balls of color q (where q could be W, Y, or B).

- ∀x box(x) → ∃q cont(x, q)
- ∀x box(x) → ∃q label(x, q)
- $\forall x, y, q$ box(x) ∧ box(y) ∧ $x \neq y$ ∧ cont(x, q) → ¬cont(y, q)
- $\forall x, q_1, q_2$ box(x) ∧ cont(x, q_1) ∧ cont(x, q_2) → $q_1 = q_2$
- $\forall x, q_1, q_2$ box(x) ∧ cont(x, q_1) ∧ label(x, q_2) → $q_1 = q_2$
- $\forall x, q_1, q_2$ box(x) ∧ obs(x, q_1) ∧ label(x, q_2) → $q_1 = q_2$

5.3 Wumpus World - (hint start by defining a helper concept 'adjacent(x,y,p,q)' which defines when a room at coordinates (x,y) is adjacent to another room at (p,q). Don't forget rules for 'stench', 'breezy', and 'safe'.

- $\forall x, y, p, q\ (\text{room}(x, y) \wedge \text{W}(p, q) \wedge \text{adjacent}(x, y, p, q)) \rightarrow \text{stenchy}(x, y)$
- $\forall x, y, p, q\ (\text{room}(x, y) \wedge \text{Pit}(p, q) \wedge \text{adjacent}(x, y, p, q)) \rightarrow \text{breezy}(x, y)$
- $\forall x, y\ \text{room}(x, y) \wedge \text{safe}(x, y) \leftrightarrow \neg\text{W}(x, y) \wedge \neg\text{Pit}(x, y) \wedge \forall p, q\ (\text{adjacent}(x, y, p, q) \rightarrow \neg\text{W}(p, q) \wedge \neg\text{Pit}(p, q))$

5.4 4-Queens – assume row(1)...row(4) and col(1)...col(4) are facts; write rules that describe configurations of 4 queens such that none can attack each other, using 'queen(r,c)' to represent that there is a queen in row r and col c.

- $\forall r, c, x, y\ (\text{queen}(r, c) \wedge \text{queen}(x, y) \wedge (r, c) \neq (x, y)) \rightarrow (r \neq x \wedge c \neq y)$
- $\forall r, c, x, y\ (\text{queen}(r, c) \wedge \text{queen}(x, y) \wedge (r, c) \neq (x, y)) \rightarrow \neg\text{diagonal}(r, c, x, y)$
- $\forall r, c, x, y\ \text{diagonal}(x, y, r, c) \leftrightarrow x - r = y - c$