CSCE-420-HW5

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1 Question 1

1.1 Write out the equation* for calculating joint probabilities, P(Smart,Study,Pass).

$$P(Smart, Study, Pass) = P(Pass|Smart, Study) * P(Smart) * P(Study)$$

1.2 Calculate all the entries in the full joint probability table (JPT) [a 4x2 matrix, like Fig 12.3 in the textbook; [Note: names of variables are capitalized, lower-case indicates truth value, e.g. 'pass' means Pass=T, and '-pass' means Pass=F.]

$$P(\text{Smart}, \text{Study}, \text{Pass}) = P(\text{Smart}) \cdot P(\text{Study}) \cdot P(\text{Pass}|\text{Smart}, \text{Study}) = 0.114$$

$$P(\text{Smart}, \text{Study}, \text{Not Pass}) = P(\text{Smart}) \cdot P(\text{Study}) \cdot (1 - P(\text{Pass}|\text{Smart}, \text{Study})) = 0.006$$

$$P(\text{Smart}, \text{Not Study}, \text{Pass}) = P(\text{Smart}) \cdot (1 - P(\text{Study})) \cdot P(\text{Pass}|\text{Smart}, \text{Not Study}) = 0.126$$

$$P(\text{Smart}, \text{Not Study}, \text{Not Pass}) = P(\text{Smart}) \cdot (1 - P(\text{Study})) \cdot (1 - P(\text{Pass}|\text{Smart}, \text{Not Study})) = 0.054$$

$$P(\text{Not Smart}, \text{Study}, \text{Pass}) = (1 - P(\text{Smart})) \cdot P(\text{Study}) \cdot P(\text{Pass}|\text{Not Smart}, \text{Study}) = 0.168$$

$$P(\text{Not Smart}, \text{Study}, \text{Not Pass}) = (1 - P(\text{Smart})) \cdot P(\text{Study}) \cdot (1 - P(\text{Pass}|\text{Not Smart}, \text{Study})) = 0.112$$

$$P(\text{Not Smart}, \text{Not Study}, \text{Pass}) = (1 - P(\text{Smart})) \cdot (1 - P(\text{Study})) \cdot P(\text{Pass}|\text{Not Smart}, \text{Not Study}) = 0.084$$

$$
\begin{aligned}
P(\text{Not Smart}, \text{Not Study}, \text{Not Pass}) =& (1 - P(\text{Smart})) \\
& \cdot (1 - P(\text{Study})) \\
& \cdot (1 - P(\text{Pass}|\text{Not Smart}, \text{Not Study})) \\
=& \ 0.336
\end{aligned}
$$

	Pass	Not Pass
Smart, Study	0.114	0.006
Smart, Not Study	0.126	0.054
Not Smart, Study	0.168	0.112
Not Smart, Not Study	0.084	0.336

1.3 From the JPT, compute the probability that a student is smart, given that they pass the test but did not study.

$$P(\text{Smart}|\text{Pass}, \text{Not Study}) = \frac{P(\text{Smart}, \text{Pass}|\text{Not Study})}{P(\text{Pass}|\text{Not Study})}$$

$$P(\text{Smart}|\text{Pass}, \text{Not Study}) = \frac{0.126}{0.126 + 0.084} = 0.6$$

1.4 From the JPT, compute the probability that a student did not study, given that they are smart but did not pass the test.

$$P(\text{Not Study}|\text{Smart}, \text{Not Pass}) = \frac{P(\text{Smart}, \text{Not Study}, \text{Not Pass})}{P(\text{Smart}, \text{Not Pass})}$$

$$P(\text{Not Study}|\text{Smart}, \text{Not Pass}) = \frac{0.054}{0.054 + 0.006} = 0.9$$

1.5 Compute the marginal probability that a student will pass the test given that they are smart.

$$P(\text{Pass}|\text{Smart}) = P(\text{Smart}, \text{Study}, \text{Pass}) + P(\text{Smart}, \text{Not Study}, \text{Pass}) \tag{1}$$

$$P(\text{Pass}|\text{Smart}) = 0.114 + 0.126 = 0.24 \tag{2}$$

1.6 Compute the marginal probability that a student will pass the test given that they study.

$$P(\text{Pass}|\text{Study}) = P(\text{Smart}, \text{Study}, \text{Pass}) + P(\text{Not Smart}, \text{Study}, \text{Pass}) \tag{3}$$

$$P(\text{Pass}|\text{Study}) = 0.114 + 0.168 = 0.282 \tag{4}$$

2 Question 2

2.1 Using Equation 13.2 in the textbook (p. 415), write out the expression for the joint probability for any state (i.e. combination of truth values for the 5 variables in this problem). [Note: Use capital letters for names of variables, and lower-case to indicate truth value, e.g. 'cold' means Cold=T, and '-cold' means Cold=F.]

$$
\begin{aligned}
&P(\text{Cold}, \text{Sneeze}, \text{Allergic}, \text{Scratches}, \text{Cat}) \\
&\quad = P(\text{Cold}) \cdot P(\text{Cat}) \\
&\qquad \cdot P(\text{Allergic}|\text{Cat}) \\
&\qquad \cdot P(\text{Sneeze}|\text{Cold}, \text{Allergic}) \\
&\qquad \cdot P(\text{Scratches}|\text{Cat})
\end{aligned}
$$

2.2 Use the equation above to calculate the joint probability that the person sneezes, but does not have a cold, has a cat, is allergic, and there are scratches on the furniture:

$$
\begin{aligned}
&P(\text{Sneeze} = \text{True}, \text{Cold} = \text{False}, \text{Allergic} = \text{True}, \\
&\quad \text{Scratches} = \text{True}, \text{Cat} = \text{True}) \\
&\quad = 0.95 \cdot 0.02 \cdot 0.25 \cdot 0.7 \cdot 0.05 \\
&\quad = 0.00016625
\end{aligned}
$$

2.3 Use normalization to calculate the conditional probability that a person has cat, given that they sneeze and are allergic to cats, but do not have a cold, and there are scratches on the furniture.

$$
\begin{aligned}
&P(\text{Cat} = \text{True}|\text{Cold} = \text{False}, \text{Sneeze} = \text{True}, \text{Allergic} = \text{True}, \text{Scratches} = \text{True}) \\
&\quad = \frac{0.95 \cdot 0.02 \cdot 0.25 \cdot 0.7 \cdot 0.05}{0.95 \cdot 0.02 \cdot 0.25 \cdot 0.7 \cdot 0.05 + 0.95 \cdot 0.98 \cdot 0.01 \cdot 0.01 \cdot 0} \\
&\quad = \frac{0.00016625}{0.00016625 + 0} \\
&\quad = 1.0
\end{aligned} \tag{5}
$$

2.4 Use Bayes' Rule to re-write the expression for P(cat—scratches). Look up the values for the numerator in the table above.

$$P(\text{cat}|\text{scratches}) = \frac{P(\text{scratches}|\text{cat}) \cdot P(\text{cat})}{P(\text{scratches})}$$

$$P(\text{A}|\text{B}) = \frac{P(\text{B}|\text{A}) \cdot P(\text{A})}{P(\text{B})}$$

$$\begin{aligned} P(\text{cat}|\text{scratches}) &= \frac{P(\text{scratches}|\text{cat}) \cdot P(\text{cat})}{P(\text{scratches})} \\ &= \frac{0.5 \cdot 0.98}{0.5 \cdot 0.98 + 0.05 \cdot 0.02} = .998 \end{aligned}$$

2.5 The denominator in the answer for (d) would require marginalization over how many joint probabilities? Write out the expressions for these (i.e. expand the denominator, but you don't have to calculate the actual values).

2 joint probabilities

$$P(\text{scratches}) = P(\text{scratches} \cap \text{cat}) + P(\text{scratches} \cap \text{not cat})$$

$$P(\text{scratches}) = P(\text{scratches}|\text{cat}) \cdot P(\text{cat}) + P(\text{scratches}|\text{not cat}) \cdot P(\text{not cat})$$

3 Question 3

3.1 Write a PDDL operator to describe this action. (note: you can express this ego-centrically – you don't have to refer explicitly to the person starting the car; but the operator should take the car being started as an argument)

Reference: *https://www.cs.toronto.edu/ sheila/2542/s14/A1/introtopddl2.pdf*

```
(:action start-car
    :parameters (?car - car)
    :preconditions (and (at ?car)
        (has-key)
        (charged-battery ?car)
        (has-gas ?car)
    )
    :effect (and (car-running ?car)
```

```
        (not (has-gas ?car))
        (at ?car)
        (has-key)
    )
)
```

3.2 Describe the same operator using Situation Calculus (remember to add a situation argument to your predicates)

Action:startCar(car, s)
Preconditions:
atCar(car, s) $\wedge$ hasKey(s) $\wedge$ chargedBattery(car, s) $\wedge$ hasGas(car, s)
Effects:
carRunning(car, do(startCar(car), s)) $\wedge$ $\neg$hasGas(car, do(startCar(car), s))
atCar(car, do(startCar(car), s)) $\wedge$ hasKey(do(startCar(car), s))

3.3 Add a Frame Axiom that says that starting this car will not changes whether any other car is out of gas (tank empty).

$\forall car2, s.\, car2 \neq car \rightarrow (\neg hasGas(car2, s) \leftrightarrow \neg hasGas(car2, do(startCar(car), s)))$