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FRB repetition and non-Poissonian statistics

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ABSTRACT

We discuss some of the claims that have been made regarding the statistics of fast radio bursts (FRBs). In Connor et al. (2015) we conjectured that flicker noise associated with FRB repetition could show up in non-cataclysmic neutron star emission models, like supergiant pulses. We show how if their repeat rate really were non-Poissonian and had a pink or red spectrum then the current limits of repetition ($\lesssim$ daily) would be weakened. The repeat rate has implications for observing strategy, generally favouring shallow wide-field surveys, including if it is zero. We also discuss the statistics of the apparent latitudinal dependence of FRBs, and offer a simple method for calculating the significance of this effect. It is shown how the evidence for a steep latitudinal gradient is less strong than initially suggested and simple explanations like increased scattering and sky temperature in the plane are sufficient to decrease the low-lat burst rate, given current data. The reported dearth of bursts near the plane is further complicated if FRBs have non-Poissonian repetition, since in that case the effective event rate itself depends on survey depth.

Key words:

1 INTRODUCTION

There is mounting evidence that the new class of transients known as fast radio bursts (FRBs) are of astronomical origin. The most striking features of FRBs are their large dispersion measures (DMs) – too high to be attributed to our own Galaxy's ISM – and their event rate ($10^3 - 10^4$ sky^{-1} day^{-1}). They last for ∼millisecond with peak flux of ∼Jy, and none has been conclusively shown to repeat. This has lead to the interpretation that FRBs are cosmological, since the intergalactic medium (IGM) would naturally provide DMs between ∼300-1600 pc cm^{-3} for sources at $z \sim 0.3 - 1$ (Thornton 2013).

Given their apparent phenomenological richness (polarization, scattering, etc.) and considering how little we know about their location and physical origin, it is likely that FRBs will be of interest to the community for years to come, assuming they are not terrestrial. Though we are in the regime of only ∼dozen published FRBs, at present the conventional wisdom is that they are likely cosmological in origin (Thornton 2013), they seem to not repeat regularly (Petroff et al. 2015), and there is a dearth of bursts at low Galactic latitudes (Petroff et al. 2014; Burke-Spolaor & Bannister 2014; Macquart & Johnston 2015).

Based on this premise there have been a number of models proposed to describe cataclysmic, cosmological FRBs (Mickaliger et al. 2012; Totani 2013; Falcke & Rezzolla 2014; Mingarelli et al. 2015). However since the field is still in its infancy it is important to leave as many conceptual doors open as possible; assumptions about the statistics of their event rate, spatial distribution, and repetition are important for the design and observing strategy of upcoming surveys. In section 2 we explore the consequences of repeating FRBs in the case where their burst rate is non-Possionian and exhibits a $1/f$ power spectrum, or pink noise. We also investigate the claims of Maoz et al. (2015) that FRB 140514 could have been the same source as FRB 100220, and comment on its implications. In section 3 discuss the impact of FRB repetition on survey strategy. In section 4 we discuss the statistical treatment of the apparent latitudinal dependence of event rate and the lack of detections at low Galactic latitude.

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2 REPEAT RATES

Though no source has been shown with certainty to repeat, the limits on the repeatability of FRBs are still weak. Several models generically predict repetition, whether periodic or stochastic. Galactic flaring stars (Maoz et al. 2015), radio bursting magnetars (Popov & Postnov 2007; Pen & Connor 2015), and pulsar planet systems (Mottez & Zarka 2014) all predict repetition with varying rates and burst distributions.

In Connor et al. (2015) we suggest supergiant pulses from very young pulsars in supernova remnants of nearby galaxies could explain the high DMs, Faraday rotation, scintillation, and polarization properties of the observed FRBs. We proposed that if the repetition of supergiant pulses were non-Poissonian (with a pink or red distribution) then one might expect several bursts in a short period of time. It is also worth mentioning that the statistics and repeat rates of FRBs could vary from source to source – even if they come from a single class of progenitors – so a long follow-up on an individual burst may not provide global constraints. In this letter we will refer to stationary Poisson processes ($\mu(t)$ is constant) as "Poissonian". When we discuss non-Poissonian statistics we will be focusing on stochastic processes that are correlated on varying timescales. For example we will not discuss periodic signals, which are not Poissonian but we feel they have already been studied (Petroff et al. 2015).

2.1 Flicker noise

Pink noise is ubiquitous in physical systems, showing up in geology and meteorology, a number of astrophysical sources including quasars and the sun, human biology, nearly all electronic devices, finance, and even music and speech (Press 1978; Voss & Clarke 1975). Though there is no agreed upon mathematical explanation for this phenomenon (Milotti 2002), fluctuations are empirically known to be inversely proportional to frequency for a variety of dynamical systems. This can be written as

$$S(f) = \frac{C}{f^{\gamma}} \quad \text{if} \quad f_{min} \leqslant f \leqslant f_{max} \tag{1}$$

where $S(f)$ is the spectral density (i.e. powerspectrum), γ is between 0.5-2, and f_{min} and f_{max} are cutoffs beyond which the powerlaw does not hold.

In the case of a time-domain astronomical source, this results in uniformity on short timescales, i.e. a burst of clustered events followed by extended periods of quiescence. If FRBs were to exhibit such flicker noise then their repetition would not only be non-periodic, but would also have a time-varying pulse rate. Therefore the number of events seen in a follow-up observation would depend strongly on the time passed since the initial event.

In Petroff et al. (2015) the fields of eight FRBs discovered between 2009 and 2013 were followed up from April to October of 2014, for an average of ∼11.4 hours per field. During this follow-up programme FRB 140514 was found in the same beam as FRB 110220, however the authors argue that it is likely a new source due to its lower DM. After its discovery, the field of 140514 was monitored five more times, starting 41 days later on 2014-05-24, without seeing anything. Under the assumption that 140514 was a new

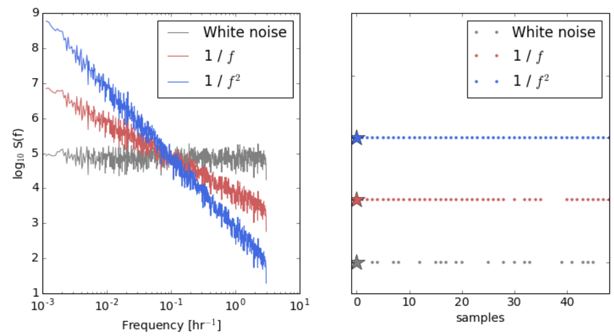

Figure 1. *Left panel*: Power spectrum for pulse arrival times of a single FRB. Grey shows a flat spectrum, corresponding to the often assumed Poissonian repetition rate. The red and blue spectra show flicker noise, with pink ($1/f$) noise and Brownian ($1/f^2$) noise respectively. *Right panel*: We found the first "event" in our Monte Carlo (represented by a star) for the three different spectra and plotted their behaviour in the subsequent 48 hours of followup. Though the average probability over the whole simulation is 0.5 for each distribution, when we zoom in on this short period the strong time-like correlations in the $1/f^{\gamma}$ cases means there are many repetitions: they are in an "on" state at this time.

FRB that only showed up in the same field coincidentally and that the repeat rate is uncorrelated, Petroff et al. (2015) rule out repetition with P $\leqslant$ 8.6 hours and reject 8.6 $<$ P $<$ 21 hours with 90% confidence. However it is possible that one or both of those premises is invalid, so it is useful to explore the possibility of non-stationary repeat rate statistics and repeating FRBs with variable DM.

If the statistics of the FRB's repeat rate were non-Poissonian and initial bursts from FRBs were to have aftershocks similar to earthquakes, then the non-immediate follow-up observations impose far weaker repeat rate limits than has been suggested. We constructed a mock follow-up observation of the eight FRBs whose fields were observed in Petroff et al. (2015). We then asked how many bursts are seen to repeat if we do an immediate follow up vs. a follow-up several years after the initial event at times corresponding to to the actual observations carried out.

We Monte Carlo this simple simulation with one sample per hour and a probability of 0.5 that a given sample has a burst in it. In the stationary Poisson case, the rate of bursts in the immediate follow-up is the same as the multi-year follow-up since all times are uncorrelated. However with flicker noise the variance is strongly time dependent. If we imagine an object that repeated on average once per hour, then if those pulses were Poisson-distributed the probability of seeing zero bursts in 10 hours or longer is ∼0.01. With pink noise one expects this roughly 20% of the time, since the system prefers either to be in "on" or "off" mode. If the average repeat period were more like 5-20 hours, then we would almost certainly expect to see nothing in a multi-day follow-up observation that took place weeks or years after the initial event.

2.2 FRBs 110220 and 140514

Using the event rate of $\sim 10^4$ sky^{-1} day^{-1} from Thornton (2013), it was originally reported that the probability

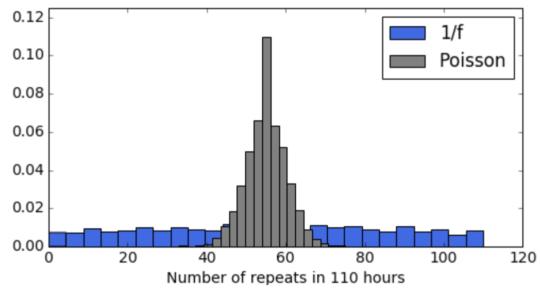

Figure 2. Histogram of a 110 hours mock follow-up observation assuming an average repeat rate of once per hour using two different models for the repetition statistics. A $1/f$ distribution (blue) gives $\sim 1\%$ probability of seeing one or zero events, which is nearly impossible in the Poisson case (grey).

of seeing a new FRB in the field of 110220 during the 85 hours of follow-up was ∼0.32 (Petroff et al. 2015). It was then pointed out by Maoz et al. (2015) that this underestimated the coincidence by an order of magnitude, since they estimated the rate in any one of the 13 beams, while the new event occurred in the identical beam. The probability also dropped due to the updated daily event rate, given the Thornton (2013) estimate is now likely thought to be high. In general we expect the true rate of FRBs to be lower than what is reported due to non-publication bias: If archival data is searched and nothing is found, it is less likely to be published than if something is found. That said, using the rate calculated in Rane et al. (2015) and following the procedure of Maoz et al. (2015), we find the likelihood of $\sim$ 0.25-2.5%.

Given the relatively low probability of finding a new FRB in the same field and since there are models that predict burst repetition with variable DMs (Connor et al. 2015; Maoz et al. 2015) one can ask the question: If one FRB out of eight is found to repeat during 110 hours of follow-up (including extra time spent on 140514), what are the limits on the expected repeat period? Another way of asking this question is what is the probability of a some number of repetitions during the 110 hours, given a repeat rate. The answer to this question depends strongly on the powerspectrum's shape. For the sake of example, if the average repeat rate is once per hour, then the probability of one repeat or fewer in the Poisson cats is effectively zero. With a pink distribution it is closer to 1%.

3 EVENT RATES AND TOTAL NUMBER OF SOURCES

If FRBs were found to repeat, their statistics and the average frequency of their repetition should affect the search strategy of upcoming surveys. For instance, if it were found that FRBs repeated, on average, five times a day, then the number of unique sources would be five times smaller than the per-sky daily event rate. This means the rate $3.3^{+5.0}_{-2.5} \times 10^3$ sky^{-1} estimated by Rane et al. (2015) would be produced by only ∼160-1600 sources. In this scenario there is no FRB in almost every pixel on the sky, which means one could integrate on most patches forever without seeing an event. An example of this strategy is the VLA millisecond search, in which $\sim$ 40% of the time was spent at a single pointing, and almost three quarters of the time was spent at just three locations (Law et al. 2015). It is possible that pointing-to-pointing event rate variance contributed to their not seeing anything.

We therefore warn that deep surveys are at a disadvantage to those that sweep large regions of the sky (CHIME (Bandura 2014), UTMOST[1], HIRAX) because the non-repeating scenario is unaffected; whereas shallow observations should not hurt the detection rate, no matter what their repetition. Ideally, a survey needs only to spend a few dispersion delay times on each beam before moving on.

4 LATITUDINAL DEPENDENCE

There is now evidence that the FRB rate is nonuniform on the sky, with fewer detectible events at low Galactic latitudes (Burke & Graham-Smith 2014). However the statistical significance of this finding may be overstated and some of the initial claims premature. In Petroff et al. (2014) the authors compute the probability of the disparity between the number of bursts seen in the the high and intermediate latitude ($|b| < 15°$) components of the High Time Resolution Universe survey (HTRU). They calculate the probability of seeing N=0 in the low-lat survey and M=4 in the high-lat, despite having searched 88% more data in the latter, and they rule out the uniform sky hypothesis with 99.5% certainty. We would point out that in general P(N|M) describes a very specific outcome, and P($\leqslant$N| $\geqslant$M) would be more appropriate since it includes all outcomes equally or more unlikely. That number might also be multiplied by two, since if the survey found four low-latitude FRBs and zero high-lat ones, we would ask the same question. HTRU has since reported five more bursts in the high Galactic region, but using a dataset that spent ∼2.5 times more time at high-lat. Below we try and quantify the likelihood of this.

If one wants to compare two statistical hypotheses, then the claims of each should be treated as true and their likelihood discrepancy should be computed. In the case of testing the abundance of FRBs at high latitudes, the sky should be partitioned into high and low regions a priori (e.g. the predefined high-lat HTRU and its compliment). The rate in both regions is then taken to be the same, and the likelihood of a given spatial distribution of observed sources can be calculated. This situation is naturally described by a biased binomial distribution with a fixed number of events. Suppose a total of K FRBs are observed in a given survey. We can ask the question, what is the probability of seeing M events in the high region and (K-M) events in the lower region? This probability can be calculated as

$$\mathrm{P(M|\,K}, p) = \binom{\mathrm{K}}{\mathrm{M}} p^{\mathrm{M}} (1-p)^{\mathrm{K-M}} \qquad (2)$$

where p is the probability that an event happens to show up in the northern region. In a survey where more time is spent on one part of the sky than the other, $p = (1+\alpha)/\,\alpha$, where α is the ratio of time spent in the high-lat region vs. the intermediate region. In the case of the HTRU survey,

[1] http://www.caastro.org/news/2014-utmost

K=9 and since none were found in the low-lat region, M=9. Roughly 2500 hours were spent searching the upper region and $\sim$ 1000 hours were spent at $|b| < 15^{\circ}$, giving $\alpha = 0.4$. Using equation 2, this outcome is only $\sim$ 5% unlikely.

The problem is given a quasi-Bayesian treatment in Petroff et al. (2014) which gives the following.

$$\mathrm{P(N|\,M)} = \alpha^{\mathrm{N}}(1+\alpha)^{-(1+\mathrm{M}+\mathrm{N})}\frac{(\mathrm{M}+\mathrm{N})!}{\mathrm{M!N!}} \tag{3}$$

This gives a probability of $\sim$3.5%. This method is Bayesian in the sense that they marginalize over the unknown rate and calculate a likelihood, but they then calculate a confidence and do not look at a posterior.

However as mentioned above, If sticking with the Bayesian approach we should probably calculate

$$\mathrm{P}(\leqslant \mathrm{N}_{obs}|\geqslant \mathrm{M}_{obs}) = \sum_{0}^{N_{obs}}\sum_{M_{obs}}^{\infty}\mathrm{P(N|\,M)} \tag{4}$$

since we are concerned with the total likelihood of anything as or more extreme than the observed event. In the case where we do not see any in the low-lat region and $\mathrm{N}_{obs} = 0$,

$$\mathrm{P}(\leqslant \mathrm{N}_{obs}|\geqslant \mathrm{M}_{obs}) = \frac{(1+\alpha)^{-\mathrm{M}_{obs}}}{\alpha} \tag{5}$$

since equation 4 becomes a geometric series sum. This gives a probability of $\sim$12%, significantly larger than the non-integrated calculation.

The most obvious difference between the approach we have offered (biased coin-flip) and the quasi-Bayesian method is that we take M + N to be fixed. It follows to ask whether or not we *should* regard the total number of FRBs as "given"? We believe the answer is yes, since this is one of the few quantities that we have actually measured, along with M and N. What we are really trying to infer is how much larger μ_{high} is than μ_{low}, so these rates should not be marginalized over. We calculate $P(\mu_{high} \leqslant \alpha\mu_{low}|N, M)$ and find it to be 99.5% if you assume flat priors on μ_{high} and μ_{low}.

Though clearly this is not the final word on the the latitudinal distribution of FRBs, we find the biased binomial approach to be the most transparent and intuitive treatment, while we are in the regime of small number statistics. Like Rane et al. (2015) we argue that the jury is still out on the severity of the latitudinal dependence. With current data the preference for FRBs to be discovered outside of the plane seems consistent with sky-temperature effects and increased scattering. Whether or not more sophisticated explanations (e.g Macquart & Johnston (2015)) are required remains to be seen.

5 CONCLUSIONS

The search for FRBs with multiple surveys that have disparate sensitivities, frequency coverage, and survey strategy has made it difficult even to estimate a daily sky rate (not to mention publication bias). That combined with the relatively low number of total FRBs observed has meant that dealing with their statistics can be non-trivial. In the case of repetition, we remind the reader that several non-cataclysmic models for FRBs are expected to repeat. In the case of supergiant pulses from pulsars, SGR radio flares, or even Galactic flare stars, it is possible that this repetition would by non-stationary and might exhibit strong correlations in time. We have shown that if the repetition had some associated flicker noise and its powerspectrum were $1/f^{\gamma}$, then one should expect the repetition rate to be higher immediately after the initial FRB detection. Therefore follow-up observations to archival discoveries that take place years or months after the first event would not provide strong upper limits. This also means that if no burst is found in a given beam after some integration time, then it is unlikely that one will occur in the following integration, and therefore a new pointing should be searched. In other words, shallow fast surveys are favourable.

In section 4 we offered a simple way of quantifying the latitudinal dependence of FRBs with a binomial distribution. This is akin to a biased coin flip, in which we ask "what is the probability of M bursts being found in one region and N bursts in its compliment, given α times more time was spent in the latter".

6 ACKNOWLEDGEMENTS

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