Long wavelength astrophysics

by

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Abstract

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To my parents and Pop

Acknowledgements

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Beamforming is a signal processing technique that allows for spatial filtering, and has greatly benefited a diverse set of fields from radar and wireless communications to radio astronomy. Historically, this was

By coherently combining the voltages of a multi-element array, sensitivity can be allocated to small regions of the sky and the array's effective forward gain can be increased. The signal from each antenna, x_n, is multiplied by a complex weight whose phases, ϕ_n, are chosen to destructively interfere radio waves in all directions but the desired pointing. The signals from all antennas are then combined to give the formed-beam voltage stream, X_{BF}.

$$X_{\mathrm{BF}} = \sum_{n=1}^{N} a_n e^{i\phi_n} x_n \tag{1.1}$$

Here a_n are real numbers that can be used to as amplitude weightings for the antennas. If we define a more general complex weighting, $w_n \equiv a_n e^{i\phi_n}$, and switch to vector notation, Eq. 1.1 becomes,

$$X_{\mathrm{BF}} = \mathbf{w}\,\mathbf{x}^{\mathrm{T}}. \tag{1.2}$$

In general, X_{BF} and $\mathbf{x}^{\mathrm{T}}$ will be functions of time and frequency. This is also true for $\mathbf{w}$, unless one needs a static, non-tracking beam – which is the case for the CHIME Pathfinder's transient search. We can write this explicitly as follows.

$$\mathbf{w}_{t\nu} = \left(a_1(\nu)e^{i\phi_1(\nu)},\, a_2(\nu)e^{i\phi_2(\nu)}, ...,\, a_N(\nu)e^{i\phi_N(\nu)}\right) \tag{1.3}$$

$$\mathbf{x}_{t\nu} = (x_1(\mathrm{t},\nu),\, \mathrm{x_2}(\mathrm{t},\nu), ...,\, \mathrm{x_N}(\mathrm{t},\nu)) \tag{1.4}$$

1.3.1 Geometric phase

We now need to calculate ϕ_n across the array. Ignoring instrumental phases for now, one can compute the geometric phases for an antenna by projecting its position vector, $\mathbf{d}_n$, onto the pointing vector, $\hat{\mathbf{k}}$. This gives,

$$\phi_n = \frac{2\pi}{\lambda}\,\mathbf{d}_n \cdot \hat{\mathbf{k}} \tag{1.5}$$

where we have taken $\mathbf{d}_n$ to be the baseline vector between feed n and an arbitrary reference point, and ϕ_n is the corresponding phase difference. A sketch for this is shown in Fig. 1.3.1 on page **??**.

To calculate the projection $\mathbf{d}_n \cdot \hat{\mathbf{k}}$, we need to go from celestial coordinates, in this case equatorial, to geographic coordinates. Using the abbreviations in Table 1.3.1, we can get a source's azimuth and altitude if we have its right ascension, declination, and the observer's position and observing time. For the latter we use local sidereal time (LST), which is the RA of the local meridian. This is given by an observer's longitude and a time, e.g. a Coordinated Universal Time (UTC). A sources hour angle is simply the difference between LST and its RA,

$$HA = LST - RA. \tag{1.6}$$

We use the standard interferometric (u, v, w) coordinate system to describe our baseline vector, $\mathbf{d}_n$. This is a right-handed coordinate system where u and v are in the plane whose normal is the zenith, and w measures the vertical direction. They are defined in numbers of wavelengths, with $u = d_{\mathrm{ew}}/\lambda$, $v = d_{\mathrm{ns}}/\lambda$, and $w = d_{\mathrm{vert}}/\lambda$. Eqn 1.5 can we expanded as,

$$\begin{aligned} \phi_n &= 2\pi\,(u, v, w)\cdot\hat{\mathbf{k}} && (1.7)\\ &= 2\pi\left(u\,\hat{\mathbf{u}}\cdot\hat{\mathbf{k}} + v\,\hat{\mathbf{v}}\cdot\hat{\mathbf{k}} + w\,\hat{\mathbf{w}}\cdot\hat{\mathbf{k}}\right), && (1.8)\end{aligned}$$

where each projection component can be obtained using spherical trigonometry. Though we do not go through the derivation here, it is given by the following product,

$$\frac{1}{\lambda}\mathbf{d}_n\cdot\hat{\mathbf{k}} = \begin{pmatrix} u, & v, & w \end{pmatrix}\cdot\begin{pmatrix} -\cos\delta\,\sin HA \\ \cos(lat)\,\sin\delta - \sin(lat)\,\cos\delta\,\cos HA \\ \sin(lat)\,\sin\delta + \cos(lat)\,\cos\delta\,\cos HA \end{pmatrix} \tag{1.9}$$

Variable	Coordinate
δ	Source declination
RA	Source right ascention
LST	Local sidereal time
HA	Source hour angle
alt	Source altitude
az	Source azimuth
lat	Telescope latitude
lon	Telescope longitude

$$\sin(\mathrm{alt}) = \sin(\delta)\sin(\mathrm{lat}) + \cos(\delta)\cos(\mathrm{lat})\cos(\mathrm{HA}) \tag{1.10}$$

$$\cos(\mathrm{az}) = \frac{\sin\delta - \sin(\mathrm{alt})\sin(\mathrm{lat})}{\cos(\mathrm{alt})\cos(\mathrm{lat})} \tag{1.11}$$

1.3.2 Neutrino N-body Particles in CUBEP^{3}M

1.4 Conclusion

Acknowledgements

We thank Ondrej and Nolan and Peterman.

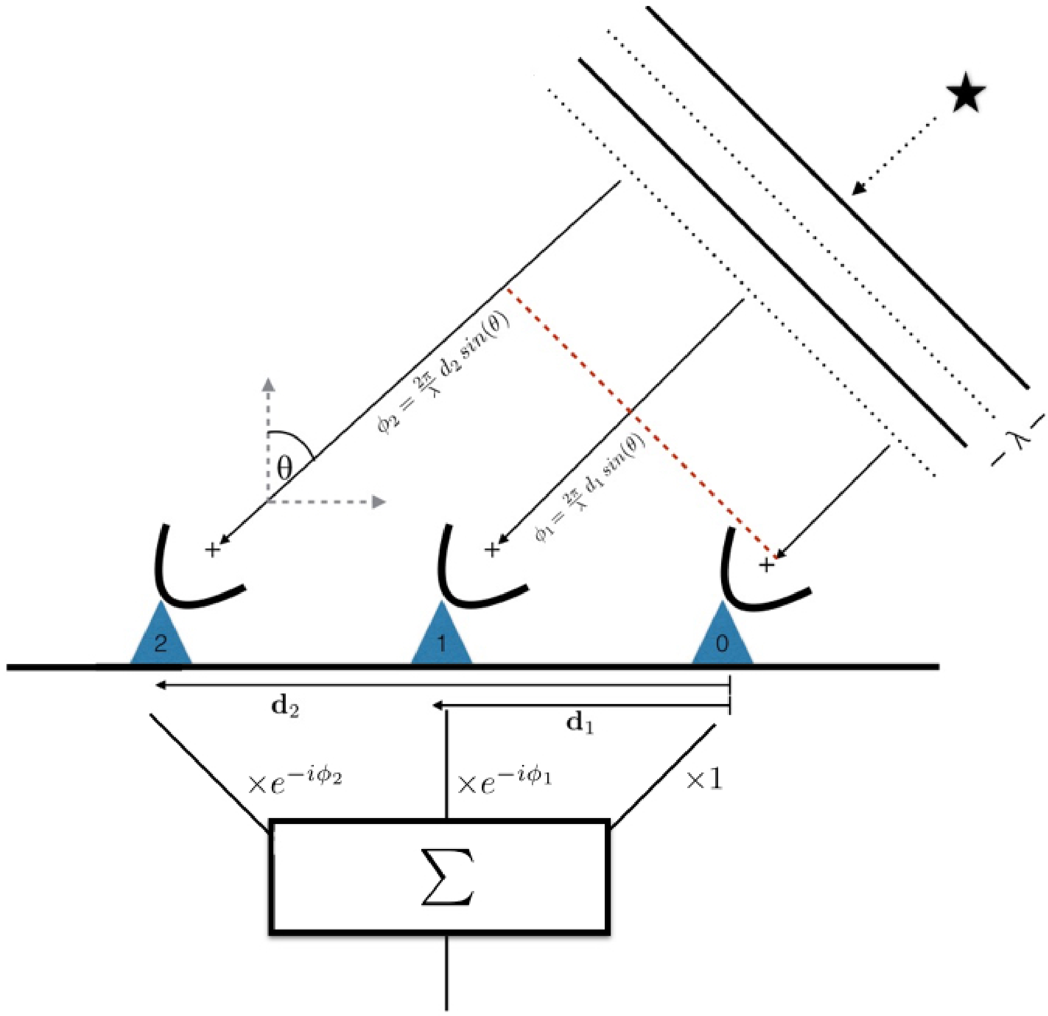

Figure 1.1: Diagrammatic example of a three-element beamformer. The wavefront from a far-field point source arrives at each antenna at different times, but the delay is calculable given an array configuration and a direction to the object. Complex weights can be applied to each antenna's voltage time-stream to account for the geometric delay, allowing for the signals to be summed coherently.