An **orthotropic Johnson Cook** strength model using **eigenspace transforms** for radial-return within material subspaces.

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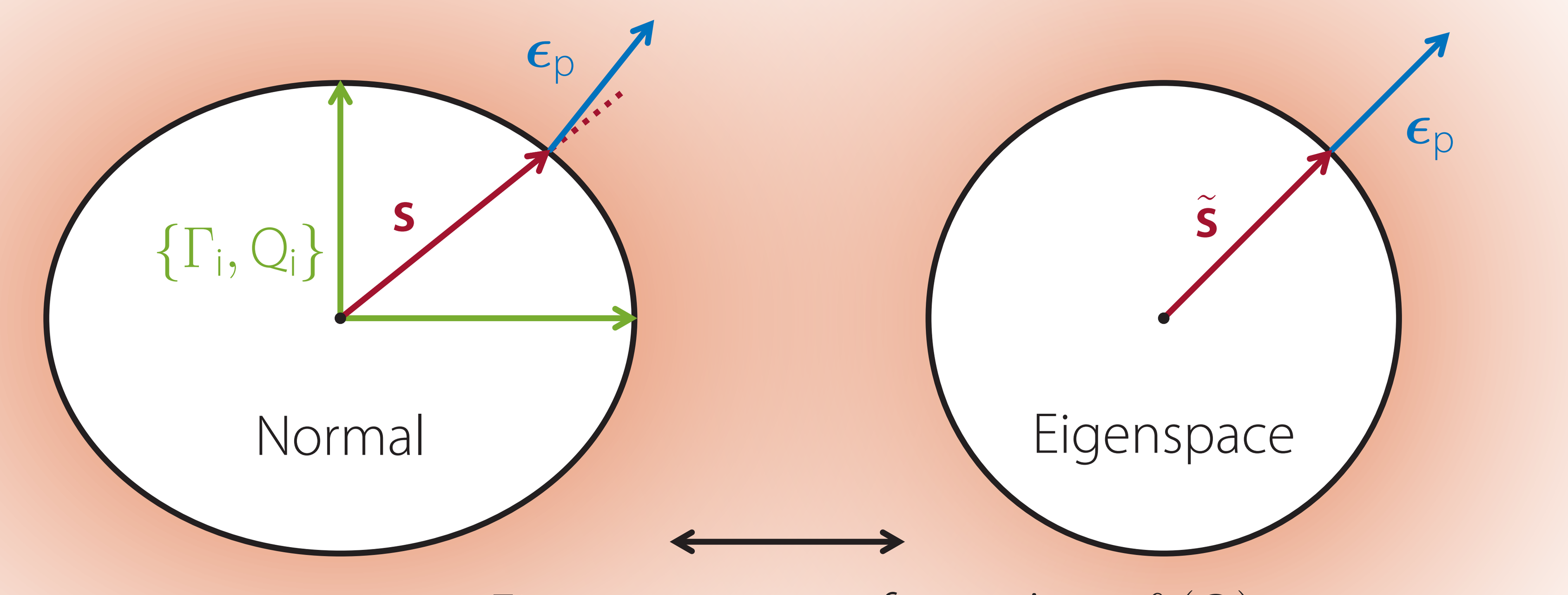
Why: Anisotropic material strength models already exist in finite element codes. However most lack sensitivity to **strain hardening, rate-dependence, or thermal softening**. Those that do are often:

- tabulated, requiring large amounts of data,
- complex, limiting intuitiveness and complicating calibration,
- unable to alter the **degree of anisotropy given varying material, loading or environmental conditions**.

How: A 2018 LANL study[1] detailed a novel implementation of the associated flow rule for orthotropic materials. The method defined **isotropic eigenspaces facilitating conventional radial return**. This paired to a quadratic Hill model, where the aspherical deviatoric yield surface hinders conventional routine efficiency.



Eigenspace transform, $\mathbf{\Lambda} = f(\mathbf{Q})$.

The Hill yield criterion[2] describes a varying threshold dependent on the deviatoric stress direction, interpretable as a **hyper-ellipsoid in deviatoric stress space**.

$$f_{\mathrm{Hill}} = \boldsymbol{\sigma}^{\mathsf{T}}\mathbf{A}\boldsymbol{\sigma} = \mathbf{s}^{\mathsf{T}}\mathbf{A}\mathbf{s} \leq \left(\sigma^{\mathrm{Y}}\right)^2$$

$$\mathbf{A} = \begin{bmatrix} G+H & -H & -G & 0 & 0 & 0 \\ -H & F+H & -F & 0 & 0 & 0 \\ -G & -F & F+G & 0 & 0 & 0 \\ 0 & 0 & 0 & 2L & 0 & 0 \\ 0 & 0 & 0 & 0 & 2M & 0 \\ 0 & 0 & 0 & 0 & 0 & 2N \end{bmatrix} \qquad \tilde{\boldsymbol{\sigma}} = \mathbf{\Lambda}^{-1}\boldsymbol{\sigma} \qquad \mathbf{\Gamma} = \mathbf{\Lambda}^{\mathsf{T}}\mathbf{A}\mathbf{\Lambda}$$

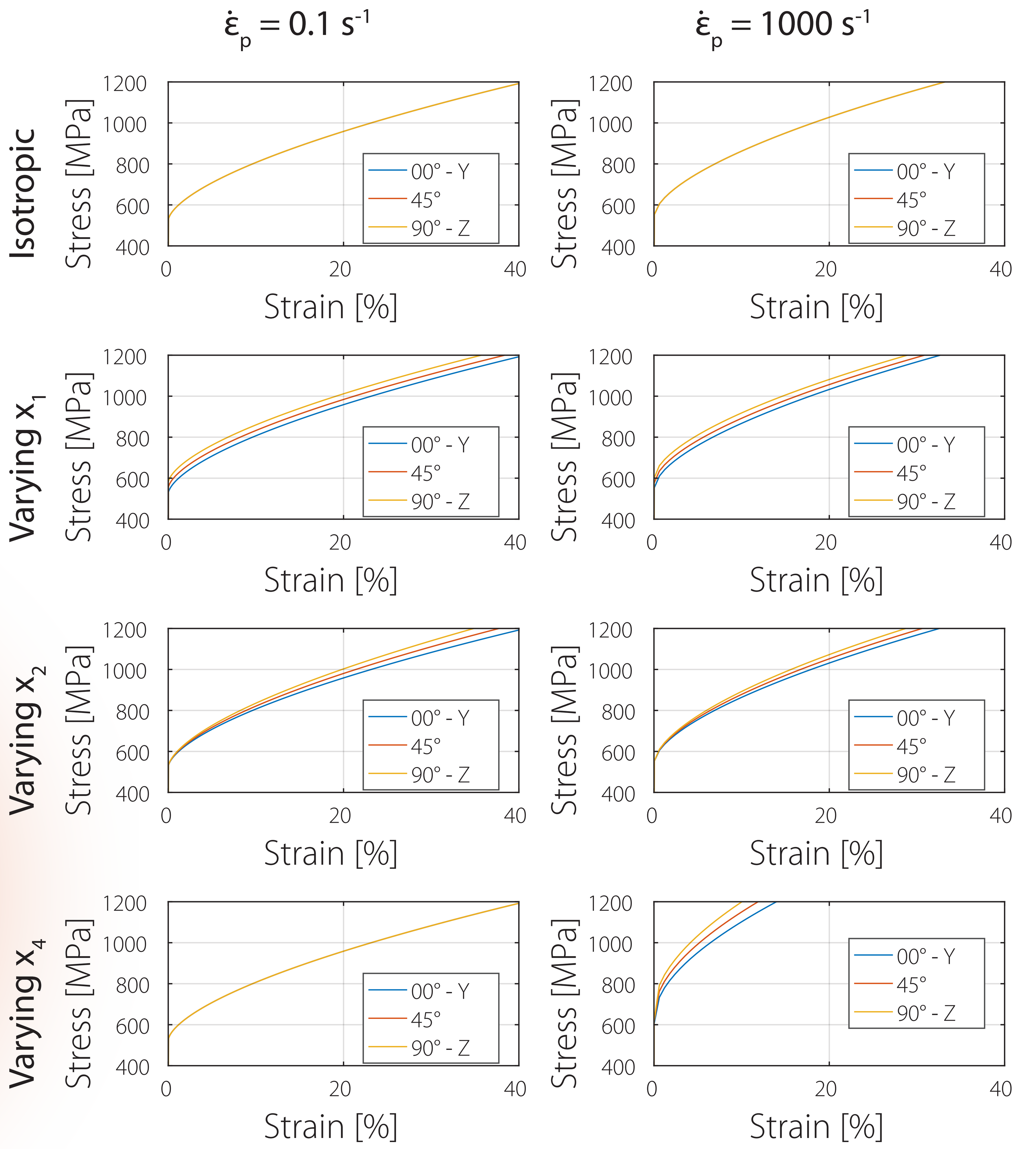
$$\boldsymbol{\kappa} = \left(\mathbf{I} + \Delta\gamma\mathbf{\Gamma}\right)^{-1}\mathbf{\Gamma}\left(\mathbf{I} + \Delta\gamma\mathbf{\Gamma}\right)^{-1}$$

$$f_{\mathrm{Hill}} = \tilde{\boldsymbol{\sigma}}_{\mathrm{tr}}{}^{\mathsf{T}}\boldsymbol{\kappa}\tilde{\boldsymbol{\sigma}}_{\mathrm{tr}} \leq \left(\sigma^{\mathrm{Y}}\right)^2$$

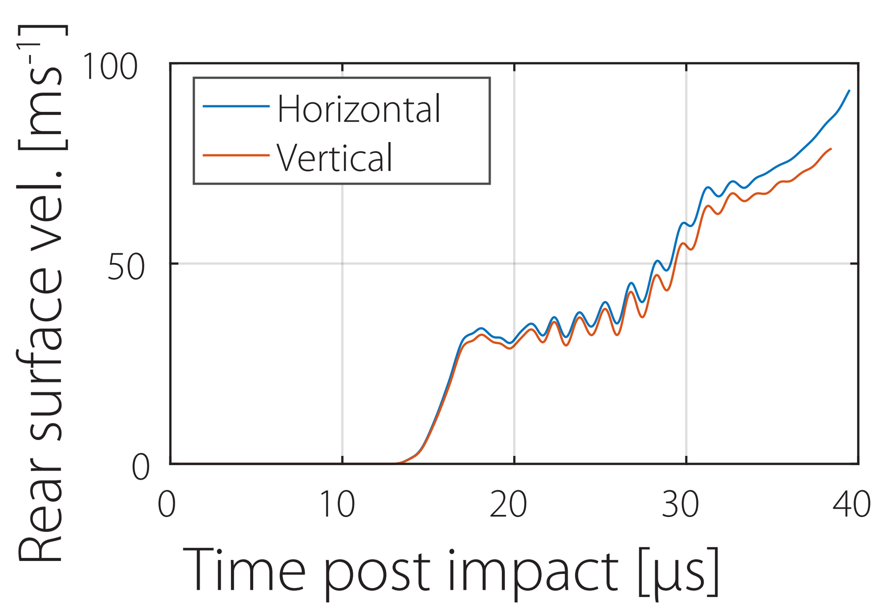
This work develops the concept; **parametrising an effective Johnson Cook[3] potential from several analogous hyper-ellipsoids** defined using on-axis values within deviatoric strain space. The model is then implemented into a vectorial subroutine within LS-DYNA.

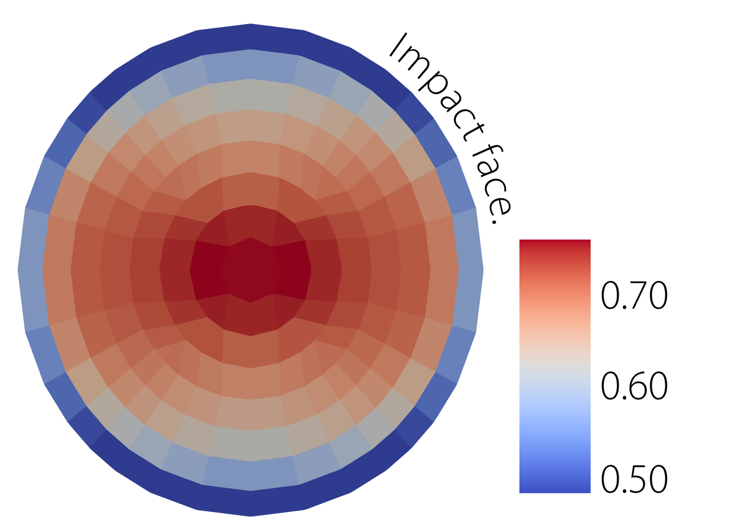
$$\sigma^{\mathrm{Y}} = \left(\mathrm{x}_1 + \mathrm{x}_2\left(\epsilon_{\mathrm{p}}^{\mathrm{eff}}\right)^{\mathrm{x}_3}\right)\left(1 + \mathrm{x}_4\ln\left(\dot{\epsilon}_{\mathrm{p}}^{\mathrm{eff}}\right)\right)\left(1 + \left(\mathrm{T}^*\right)^{\mathrm{x}_5}\right),\ \mathrm{x}_{\mathrm{i}} = \frac{\boldsymbol{\epsilon}_{\mathrm{p}}{}^{\mathsf{T}}\left(\mathbf{A}_{\mathrm{x}_{\mathrm{i}}}\right)\boldsymbol{\epsilon}_{\mathrm{p}}}{\left|\boldsymbol{\epsilon}_{\mathrm{p}}\right|^2}$$

Validation: Responses were simulated covering isotropic degenerate and orthotropic cases across a range of conditions.



Application: Initial results imply the model reproduces the anisotropic, strain hardening, rate sensitive responses of reverse Taylor impacts[4]. Efforts now concern **multi-platform regressions**, creating frameworks to model **anisotropic additively manufactured microstructures.**





Resultant plastic strain, ϵ_p.

Code: A copy of this poster is available now on GitHub at the QR code in the footer below. A version of the source code will also follow shortly.

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AWE

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[4] Liam C. Smith, David J. Chapman, Paul A. Hooper, Glenn Whiteman, and Daniel E. Eakins , "On the dynamic response of additively manufactured 316L", AIP Conference Proceedings 1979, 060007 (2018), doi.org/10.1063/1.5044804

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