Facilitating the calibration of complex quantum photonic circuits with machine learning assisted gate set tomography

By

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Declaration

This dissertation is submitted to the University of Bristol in accordance with the requirements of the degree of MEng in the Faculty of Engineering. It has not been submitted for any other degree or diploma of any examining body. Except where specifically acknowledged, it is all the work of the Author.

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Abstract

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1.1 Tomography

1.1.1 State Tomography

Notes taken from [1], only reworded and trimmed down for my benefit.

In the classical world, characterising the dynamics of a system is trivial and known as *system identification*. The general idea is that we wish to know how the system behaves with respect to any input, thus uniquely identifying it. In the quantum world, the analogue of this is called *quantum process tomography*. To understand process tomography, we must first understand *quantum state tomography*.

State tomography is the procedure of determining an unknown quantum state. This is harder than it sounds: if we're given an unknown state ρ, we can't just measure the state and recover it immediately way since measurement will *disturb* the original state. In fact, *there is no quantum measurement which can distinguish non-orthogonal states with certainty*. However, if we have an *ensemble* of the same quantum state ρ, then it's possible to estimate ρ.

If we represent the state of the system using its density matrix ρ, we may expand ρ as

$$\rho = \frac{\operatorname{tr}(\rho)\, I + \operatorname{tr}(X\rho)\, X + \operatorname{tr}(Y\rho)\, Y + \operatorname{tr}(Z\rho)\, Z}{2} \tag{1.1}$$

Note that $\operatorname{tr}(Z\rho)$ can be interpreted as the *expectation* of the observable Z. Therefore, to estimate $\operatorname{tr}(Z\rho)$, we measure the observable Z m-times to obtain outcomes $z_1, \ldots, z_m$ and calculate

$$\operatorname{tr}(Z\rho) \approx \frac{1}{m}\sum_{i}^{m} z_i \tag{1.2}$$

In general, this estimate is approximately a Gaussian with mean $\operatorname{tr}(Z\rho)$ and standard deviation $\Delta(Z)/\sqrt{m}$, where $\Delta(Z)$ is the standard deviation of a single measurement. We can apply this same method to estimate $\operatorname{tr}(X\rho)\, X$ and $\operatorname{tr}(Y\rho)\, Y$; with a large enough sample size we obtain a good estimate for ρ. Additionally, since density matrices have unit trace, we know that

$$\operatorname{tr}(\rho)\, I = I \tag{1.3}$$

This process can be generalised to a density matrix on n qubits as

$$\rho = \sum_{\vec{v}} \frac{\operatorname{tr}(\sigma_{v_1} \otimes \sigma_{v_2} \otimes \cdots \otimes \sigma_{v_n})\, \sigma_{v_1} \otimes \sigma_{v_2} \otimes \cdots \otimes \sigma_{v_n}}{2^n} \tag{1.4}$$

where $\vec{v} = (v_1, \ldots, v_n)$ with entries v_i chosen from the set $0, 1, 2, 3$, i.e. each σ_{v_i} is a particular Pauli matrix.

1.1.2 Process Tomography

To extend this notion to quantum process tomography is actually quite easy from a theoretical point of view. If the state space of the system has d dimensions ($d = 2$ for a single qubit), then we choose d^2 pure quantum states $|\psi_d\rangle, \ldots, |\psi_{d^2}\rangle$. The corresponding density matrices $|\psi_1\rangle\langle\psi_1|, \ldots, |\psi_{d^2}\rangle\langle\psi_{d^2}|$ for these states should form a *basis set* for the space of possible density matrices.

Now, for each state $|\psi_j\rangle$, we prepare the system in that state and then subject it to the process $\mathcal{E}(|\psi_j\rangle\langle\psi_j|)$. Afterwards, we use state tomography to determine the output state $\mathcal{E}(|\psi_j\rangle\langle\psi_j|$. Theoretically, this is all that we need to do, since the matrices $|\psi_j\rangle\langle\psi_j|$ form a basis set, so any other possible density matrices can be represented as a linear combination of the basis set. e.g.,

$$\mathcal{E}(|\Phi\rangle\langle\Phi| + |\Psi\rangle\langle\Psi|) = \mathcal{E}(|\Phi\rangle\langle\Phi|) + \mathcal{E}(|\Psi\rangle\langle\Psi|) \tag{1.5}$$

However, in reality we need to determine $\mathcal{E}$ from experimental data: operators are just a theoretical tool, whereas experiments involve real numbers.

Bibliography

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition.* USA: Cambridge University Press, tenth ed., 2011.