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1. **(Encapsulation and Layering)** Suppose you transmit packets on a 10 Mbps Ethernet link. You have created packets that belong to application/presentation/session layers, and their size is 1000 bytes. The packets undergo encapsulation by the following layers and their protocols: transport layer protocol is UDP, network layer protocol is IP, link layer protocol is Ethernet. The header sizes of UDP, IP and Ethernet are 8, 20 and 22 bytes respectively. Calculate the maximum number of packets per second that can be transmitted on the Ethernet link (Note: 1. The answer may not be necessarily an integer. 2. Ignore CSMA/CD or any details of the real Ethernet for now! In this problem, you can think of Ethernet as a link providing a pure bandwidth of 10 Mbps.).

   **Sol:** Due to encapsulation, the header size at each layer is added to the packet before transmission. The final packet size at Ethernet layer is 8+20+22+1000=1050 bytes. This corresponds to 8400 bits and since the link has bandwidth of 10 Mbps, approximately 1190 packets are transmitted per second.

2. **(Poisson random variables):**

   (a) Let $X$ and $Y$ be *independent* random variables which take values from $\{0, 1, 2, \dots\}$. Let $Z = X + Y$. Let us define $\mathbb{P}(X = k) := p_X(k)$, $\mathbb{P}(Y = k) := p_Y(k)$ and $\mathbb{P}(Z = k) := p_Z(k)$ for $k = 0, 1, 2, \dots$. Prove that $p_Z(k) = p_X(k) * p_Y(k)$ holds where $*$ denotes the *convolution* operation: in other words,

   $$p_Z(k) = \sum_{l=0}^{k} p_X(l) p_Y(k - l)$$

   (b) Let $X_1$ and $X_2$ be independent Poisson random variables with parameter $\lambda_1$ and $\lambda_2$ respectively. Let $Y = X_1 + X_2$. Prove that, $Y$ is a Poisson random variable with parameter $\lambda_1 + \lambda_2$.

   (c) We can further generalize the result from (b), namely let $X_i$ be independent Poisson random variables with parameter $\lambda_i$, $i = 1, \dots, n$. Then $Y = \sum_{i=1}^{n} X_i$ is also a Poisson random variable with parameter $\sum_{i=1}^{n} \lambda_i$. Suppose there are $n$ users in a slotted ALOHA network. The packet arrival per unit time for *each* user is distributed as an independent Poisson random variable with parameter $\lambda = 0.01$. Find the number of users $n$ which maximizes the throughput.

   **Sol:**

   (a) The probability that $Z = k$ can be derived by, first considering the joint probability of $X = l$ and $Y = k - l$ for some $l$, and second summing up such probabilities for all possible $l$. In other words

   $$\mathbb{P}(Z = k) = \sum_{l=0}^{k} \mathbb{P}(X = l, Y = k - l) = \sum_{l=0}^{k} \mathbb{P}(X = l)\mathbb{P}(Y = k - l)$$

   holds, since $X$ and $Y$ are independent. This is exactly the convolution operation.

   (b) Using the result from (a), we have that

   $$\begin{aligned}
   \mathbb{P}(Y = k) &= \sum_{l=0}^{k} \mathbb{P}(X_1 = l)\mathbb{P}(X_2 = k - l) \\
   &= \sum_{l=0}^{k} \frac{(\lambda_1)^l}{l!} \frac{(\lambda_1)^{k-l}}{(k-l)!} e^{-\lambda_1 - \lambda_2} \\
   &= \frac{e^{-\lambda_1 - \lambda_2}}{k!} \sum_{l=0}^{k} \binom{k}{l} \lambda_1^k \lambda_2^{l-k} = \frac{(\lambda_1 + \lambda_2)^k}{k!} e^{-\lambda_1 - \lambda_2}
   \end{aligned}$$

(c) From (b), the packet arrival times to the overall network per unit time can be modelled as Poisson random variable with parameter $G = n\lambda$. For slotted ALOHA, the network capacity (maximum throughput) is achieved when $G = 1$, meaning that the optimal number of users is $n = \lambda^{-1} = 100$.

3. **(Slotted ALOHA):** Suppose there are $N$ nodes in a slotted ALOHA network. Every node is trying to transmit a packet at every time slot with probability $p$, independently of each other. It takes one time slot to transmit a packet.

   (a) Express the offered load (the average number of packets per time slot that are arriving to the network) in terms of $N$ and $p$.

   (b) Show the throughput (the average number of successfully transmitted packets per time slot) is equal to $Np(1-p)^{N-1}$.

   (c) Suppose the nodes can somehow change the probability of transmission $p$ in their favor. Find value of $p$ that maximizes the throughput.

   (d) Let us denote the maximum throughput by $T(N)$, using the value of $p$ from part (c). We call $T(N)$ as capacity of the network. Calculate the capacity if there are infinite number of nodes, in other words, find $\lim_{N\to\infty} T(N)$. (I hope the answer looks familiar to you!)

   **Sol:**

   (a) $Np$

   (b) Note that one packet can be transmitted per time slot, but only if only one of the users transmits. So the throughput is equivalent to the probability of only one user transmits at a time slot, which is $Np(1-p)^{N-1}$.

   Alternatively, suppose I am the first one to transmit at a time slot. In order for my transmission to be successful, no one else should transmit except me from my perspective. So we have that

$$\text{(Throughput)} = \text{(Offered load)} \times \text{(probability that no one else will send except me)} = Np \cdot (1-p)^{N-1}.$$

   (c) If we differentiate the throughput with respect to $p$, we have that

$$\frac{\partial (Np(1-p)^{N-1})}{\partial p} = N(1-pN)(1-p)^{N-2}.$$

   We see that the derivative changes sign from positive to negative as $p$ increases past $1/N$. So $p = 1/N$ is the maximizer of the throughput.

   (d) By substituting $p$ with $1/N$ in the expression for throughput, we have that

$$T(N) = \left(1 - \frac{1}{N}\right)^{N-1}.$$

   And if we calculate the limit

$$\lim_{N\to\infty} T(N) = \lim_{N\to\infty} \left(1 - \frac{1}{N}\right)^{N-1} = \lim_{N\to\infty} \left(\frac{N-1}{N}\right)^{N-1} = \lim_{N\to\infty} \left\{\left(1 + \frac{1}{N-1}\right)^{N-1}\right\}^{-1} = \frac{1}{e}.$$

   This is exactly the same as the throughput of slotted ALOHA derived in the class. In fact, Poisson distribution, a random model for traffic arrival used in class, is the limiting case of Binomial distribution, one used in this problem. So we should see the same result.

4. **(CSMA/CD:)** Consider building a CSMA/CD network running at 1 Gbps over a 1 km cable with no repeaters. The signal speed in the cable is $2 \times 10^5$ km/sec. What is the minimum frame size?

   **Sol:** The round-trip delay is

$$2 \times \frac{1 \text{ km}}{2 \times 10^5 \text{ km/sec}} = 10^{-5} \text{ sec.}$$

   In order for CSMA/CD to work, the transmitter should be able to detect collision while transmitting, so the transmission has to last longer than this time. The corresponding frame size is 1Gbps $\times 10^{-5}$ sec = $10^4$ bits.

5. **(Partitioning Collision Domains on Ethernet Networks):** Figure 1 shows how network managers can partition an Ethernet network to accommodate more nodes. The figure assumes that each node transmits $R$ bps, on average, during a representative period of time. There are $N$ nodes to be connected. If the total transmission rate $N \times R$ is larger that the rate that can be handled by one Ethernet, then the network manager can try to partition the network. For instance, if the efficiency of one Ethernet connecting the $N$ nodes is $80\%$ and if $N \times R > 8$ Mbps, then one Ethernet cannot handle all the nodes. Let us assume that the $N$ nodes can be divided into two groups that do not exchange messages frequently. For simplicity, say that each group sends a fraction $p$ of its messages to the other group. let us connect the computers in each group with a dedicated Ethernet, as shown in the figure. The two Ethernets are connected by a *bridge*.

   (a) Find an expression for the traffic on each Ethernet.

   (b) The arrangement can handle the $N$ nodes if this rate is less than the efficiency of each Ethernet times 10 Mbps. Of course, the bridge must be able to handle the throughput. Assuming an efficiency of $80\%$, $R = 1$ Mbps and $p = 20\%$ compute how many stations can be supported.

   (c) Under the same efficiency and load assumptions compare the number of hosts that could be supported with a single collision domain versus that with the two collision domains computed above.

   (d) If $p$ were smaller, i.e., the traffic exhibits more *locality*, would the advantages of partitioning the network increase or decrease?

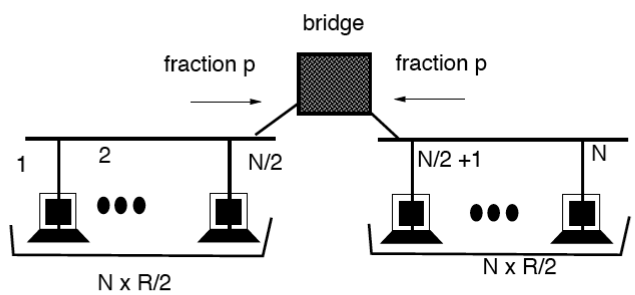



Figure 1: Ethernet load and design.

**Sol:**

(a) The traffic on each Ethernet is

$$(N/2)R + p(N/2)R = (N/2)R(1 + p)$$

(b) Following condition has to hold so that the Ethernet can handle the traffic.

$$(N/2)R(1 + p) < \eta \times 10Mbps$$

where $\eta$ is the efficiency of the Ethernet. Which gives $N \approx 13$.

(c) With a single network we could only support $8$ stations, so by partitioning we increase our capacity by $5$.

(d) A smaller $p$ would allow us to better isolate traffic, and increase the number of hosts that could supported. In the extreme case where $p = 1$, the bridge is useless because the network behaves as if $N$ nodes are in a single collision domain. By contrast if $p = 0$, the traffic in each network should not affect each other, so the bridge is most useful in that case. Therefore, for SMALLER $p$, the advantages of partitioning INCREASES.

6. **(CSMA/CD in wireless?)** Compared to wired networks, state two reasons why CSMA/CD is not appropriate in the context of wireless networks.

**Sol:**

(a) An efficient operation of CSMA/CD requires that a node be able to detect collision during transmission. However it is expensive to make wireless transceivers full-duplex (being able to receive and transmit at the same time) in terms of cost and power.

(b) Hidden node problem.

7. **(Problems with solving exposed node problem)** Consider the wireless network in Figure 1. Node A performs CSMA/CA procedure proposed by MACAW, in other words RTS-CTS-DATA-ACK sequence for transmission. Suppose node B has just received RTS from node A and found out that A tries to transmit to node C, but did not hear CTS from node C. Node B concluded that he became an exposed node and decides to transmit data to node D.

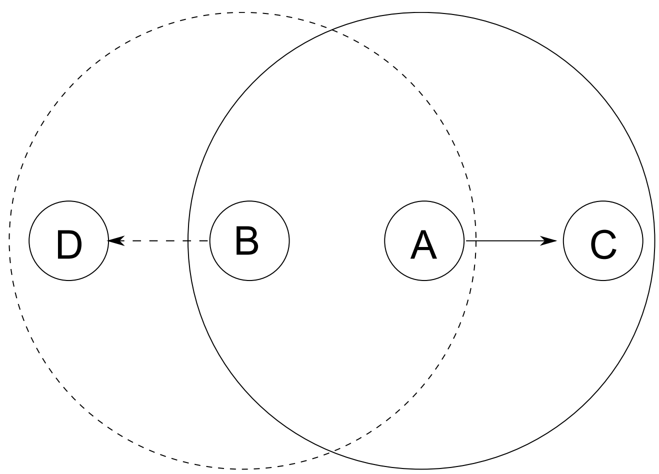



Figure 2: B is an exposed node.

(a) Node B initiates RTS-CTS-DATA-ACK sequence with node D, at the same moment as A starts to transmit data packet. What is the potential problem with node Bs attempt? (Hint: Suppose node A is transmitting a large packet.)

(b) Instead of MACAW procedure, node B starts DATA-ACK sequence (CSMA-CA in weaker sense) for transmission to node D, at the same moment as A finishes transmitting data packet. What is the potential problem with node Bs attempt? (Hint: Suppose node B is transmitting a large packet.)

**Sol:**

(a) RTS from B can be successfully received by D. However CTS from D will not be correctly received at B due to collision if A is still transmitting while D attempts to send CTS to B. This will cause unnecessary retransmissions of RTS packets from B.

(b) A will not receive ACK from C due to collision if B is still transmitting while C attempts to send ACK to A. This will cause unnecessary retransmissions of data packets from A.

These examples show that by trying to solve exposed node problem one may cause side-effect problems that may complicate the protocol even further.

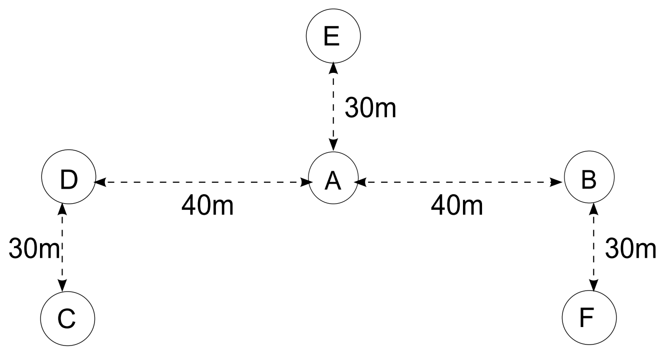



Figure 3: A Wireless Network.

8. **(Capacity of wireless networks and transmission range)**: Consider a wireless network with six nodes and their physical distances as in Figure 2 (All the angles in the geometry in Figure 2 are 90 degrees.). The transmission range of a node is modelled by a circle of radius $R$, in other words, if two nodes are within distance $R$ of each other, one can successfully transmit data to the other. Suppose every node can transmit packets at a rate up to $P$ packets/sec. A node is allowed to transmit to another node as long as its intended receiver is not interfered by other transmissions. For a given $R$, we define the capacity of the network as the maximum of the sum of successfully transmitted packets per second in the network. Assume that the nodes cannot transmit and receive at the same time (half-duplex). Figure 2: A wireless network.

   (a) Suppose you can control the value of $R$ (for example, by adjusting radio transmission power). Obviously, the choice of a large $R$ will be poor: consider $R$ to be 200 meters for example, then at any instant, only one node will be able to transmit to another node, in which case the capacity will be only $P$ packets/sec. You can achieve higher capacity by appropriate setting the value of $R$, but $R$ has to be greater than some minimum radius $R_{\text{min}}$. Find the maximum capacity of this network and find the value of $R_{\text{min}}$. (Hint: Think in terms of the number of concurrent transmissions that are allowed in the network.)

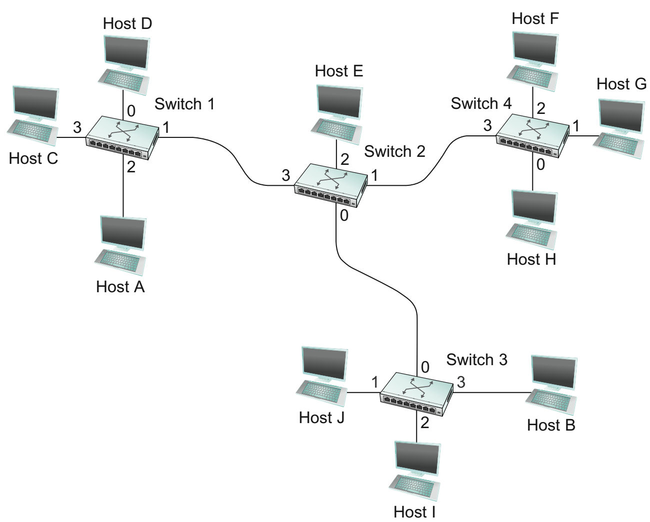
   (b) If we increase the value of R starting from $R_{\text{min}}$, you will see that we cannot achieve the maximum capacity derived in part (a) if $R$ reaches some value $R_{\text{max}}$. Find $R_{\text{max}}$.

**Sol:**

   (a) We can make the maximum capacity 3P packets/sec by allowing three concurrent transmissions, provided that the transmission range is at least 30m ($R_{\text{min}}$=30m). One of the possible (transmitter)→(receiver) pairs are D→C, A→E, B→F. Note this configuration makes receivers as further away as possible from each other, and makes transmitters more or less spatially clustered.

(b) In order to achieve 3P, every node must become either transmitter or receiver, since there must be three concurrent transmissions and nodes are half-duplex. We will show that there exists at least one node that cannot become either transmitter or receiver AND the network has three concurrent transmissions if the transmission range is greater than 50m. We pick node A. Suppose node A is a transmitter and R > 50m. This makes every other node lie within the transmission range of A. Then clearly any other node (except the receiver of A) cannot become a receiver of other transmitter than A. This prohibits the network from having two or more concurrent transmissions. Now suppose that node A is a receiver. If R > 50m, node A is within the transmission range of any other two transmitters in the network: in other words, if we pick any two nodes other than A, node A is within the transmission range of those nodes. This makes any two or more concurrent transmissions impossible in the network. Thus if R > 50m = Rmax and if we would like to have concurrent transmissions in the network, node A cannot be either transmitter or receiver, which implies that capacity must be less than 3P packets/sec. Note we still can achieve 3P packets/sec with the transmitter-receiver configuration given in part (a) as long as the transmission range is less than 50m.

9. **(Virtual Circuit Networks)** Using the example network given in Figure, give the virtual circuit tables for all the switches after each of the following connections is established. Assume that the sequence of connections is cumulative; that is, the first connection is still up when the second connection is established, and so on. Also assume that the VCI assignment always picks the lowest unused VCI on each link, starting with 0, and that a VCI is consumed for both directions of a virtual circuit.




(a) Host A connects to host C.

(b) Host D connects to host B.

(c) Host D connects to host I.

(d) Host A connects to host B.

(e) Host F connects to host J.

(f) Host H connects to host A.

**Sol:**

| | SW1 | | | | SW2 | | | | SW3 | | | | SW4 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | In | In VCI | Out | outVCI | In | In VCI | Out | outVCI | In | In VCI | Out | outVCI | In | In VCI | Out | outVCI |
| (a) | 2 | 0 | 3 | 0 | | | | | | | | | | | | |
| (b) | 0 | 0 | 1 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 3 | 0 | | | | |
| (c) | 0 | 1 | 1 | 1 | 3 | 1 | 0 | 1 | 0 | 1 | 2 | 0 | | | | |
| (d) | 2 | 1 | 1 | 2 | 3 | 2 | 0 | 2 | 0 | 2 | 3 | 1 | | | | |
| (e) | | | | | 1 | 0 | 0 | 3 | 0 | 3 | 1 | 0 | 2 | 0 | 3 | 0 |
| (f) | 1 | 3 | 2 | 2 | 1 | 1 | 3 | 3 | | | | | 0 | 0 | 3 | 1 |

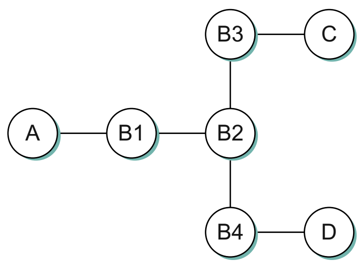
10. **(Learning bridges)** Consider the arrangement of learning bridges shown in Figure 3.49. Assuming all are initially empty, give the forwarding tables for each of the bridges B1 to B4 after the following transmissions:

    (a) A sends to C.

    (b) C sends to A.

    (c) D sends to C.

    Identify ports with the unique neighbor reached directly from that port; that is, the ports for B1 are to be labeled A and B2.




**Sol:**

| B1 | | B2 | | B3 | | B4 | |
|---|---|---|---|---|---|---|---|
| A-interface | A | B1-interface | A | B2-interface | A | B2-interface | A |
| B2-interface | C | B3-interface | C | C-interface | C | D-interface | D |
| | | B4-interface | D | B2-interface | D | | |