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1. **(Encapsulation and Layering)** Suppose you transmit packets on a 10 Mbps Ethernet link. You have created packets that belong to application/presentation/session layers, and their size is 1000 bytes. The packets undergo encapsulation by the following layers and their protocols: transport layer protocol is UDP, network layer protocol is IP, link layer protocol is Ethernet. The header sizes of UDP, IP and Ethernet are 8, 20 and 22 bytes respectively. Calculate the maximum number of packets per second that can be transmitted on the Ethernet link (Note: 1. The answer may not be necessarily an integer. 2. Ignore CSMA/CD or any details of the real Ethernet for now! In this problem, you can think of Ethernet as a link providing a pure bandwidth of 10 Mbps.).

Answer: The total packet is like Figure 1. Thus, the maximum number of packets per second is $\frac{10^6}{8400} \approx 119$ packets .

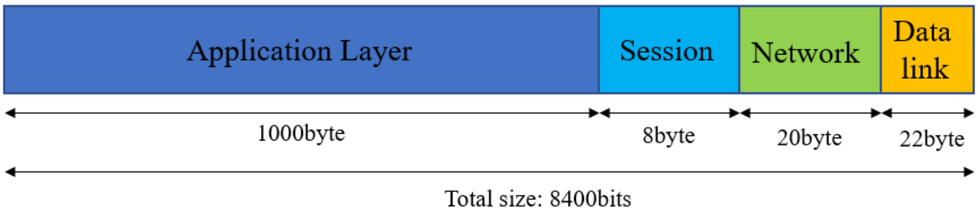


Figure 1. packet

2. **(Poisson random variables):**

(a) Let X and Y be *independent* random variables which take values from $\{0, 1, 2, \ldots\}$. Let $Z = X + Y$. Let us define $P(X = k) := p_X(k)$, $P(Y = k) := p_Y(k)$ and $P(Z = k) := p_Z(k)$ for $k = 0, 1, 2, \ldots$. Prove that $p_Z(k) = p_X(k) * p_Y(k)$ holds where $*$ denotes the *convolution* operation: in other words,

$$p_Z(k) = \sum_{l=0}^{k} p_X(l)p_Y(k-l)$$

Answer(Proof): If X and Y are independent integer-valued random variables with probability mass function P_X and P_Y, then Z=X+Y is also an integer-valued random variable with probability mass function P_Z.

$$\begin{aligned}
P_Z(k) &= P\{X+Y=k\} \\
&= \textstyle\sum_{l=0}^{k} P\{X=l, Y=k-l\} \\
&= \textstyle\sum_{l=0}^{k} P\{X=l\}P\{Y=k-l\} \quad //(\text{X and Y are independent random variables}) \\
&= \textstyle\sum_{l=0}^{k} P_X(l)P_Y(k-l)
\end{aligned}$$

Thus, $P_Z(k) = P_X(k) * P_Y(k)$ holds where $*$ denotes the *convolution* operation.

(b) Let X_1 and X_2 be independent Poisson random variables with parameter λ_1 and λ_2 respectively. Let $Y = X_1 + X_2$. Prove that, Y is a Poisson random variable with parameter $\lambda_1 + \lambda_2$.

Answer(Proof):

$$\begin{aligned}
P_Y(k) &= P\{X_1+X_2=k\} \\
&= \textstyle\sum_{l=0}^{k} P\{X_1=l, X_2=k-l\} \\
&= \textstyle\sum_{l=0}^{k} P\{X=l\}P\{Y=k-l\} \quad //(\text{X and Y are independent random variables}) \\
&= \textstyle\sum_{l=0}^{k} \frac{e^{-\lambda_1}\lambda_1^l}{l!}\frac{e^{-\lambda_2}\lambda_2^{k-l}}{(k-l)!} \quad //(\text{X and Y are independent Poisson random variables}) \\
&= \textstyle\sum_{l=0}^{k} \frac{e^{-(\lambda_1+\lambda_2)}\lambda_1^l\lambda_2^{k-l}}{l!(k-l)!}\frac{k!}{k!} \\
&= e^{-(\lambda_1+\lambda_2)}\frac{(\lambda_1+\lambda_2)^k}{k!} \quad //\left((\lambda_1+\lambda_2)^k = \textstyle\sum_{l=0}^{k}\binom{k}{l}\lambda_1^l\lambda_2^{k-l}\right)
\end{aligned}$$

In above derivation, we can see that $P_Y(k)$ is a probability which follow Poisson distribution. Thus Y is a Poisson random variable with parameter $\lambda_1 + \lambda_2$.

(c) We can further generalize the result from (b), namely let X_i be independent Poisson random variables with parameter λ_i, $i = 1, \ldots, n$. Then $Y = \sum_{i=1}^{n} X_i$ is also a Poisson random variable with parameter $\sum_{i=1}^{n} \lambda_i$.

Suppose there are n users in a slotted ALOHA network. The packet arrival per unit time for *each* user is distributed as an independent Poisson random variable with parameter $\lambda = 0.01$. Find the number of users n which maximizes the throughput.

Answer: The number of Tx attempts during unit time follows $\text{Poisson}(0.01n)$ (n is the number of users). Throughput is defined as the number of packets successfully Txed per unit time. Thus, in this case, Throughput is $S = 0.01n \times P(\{\text{no packets generated during unit time}\}) = 0.01n \times e^{-0.01n}$ (a window of vulnerability is unit time). The number of users which maximizes the throughput satisfies the statement $S' = e^{-0.01n}(0.01 - 0.0001n) = 0$. Thus, the answer is n=100.

3. **(Slotted ALOHA):** Suppose there are N nodes in a slotted ALOHA network. Every node is trying to transmit a packet at every time slot with probability p, independently of each other. It takes one time slot to transmit a packet.

(a) Express the offered load (the average number of packets per time slot that are arriving to the network) in terms of N and p.

Answer: the number of packets per time slot that are arriving to the network follows $\text{Binomial}(N, p)$. Thus, The average of that is Np.

(b) Show the throughput (the average number of successfully transmitted packets per time slot) is equal to $Np(1 - p)^{N-1}$.

Answer: the average number of packets per time slot is Np. Beside, the probability that a given node succeed in transmitting a packet is equal to $(1 - p)^{N-1}$. Thus the average number of successfully transmitted packets per time slot is $Np(1 - p)^{N-1}$.

(c) Suppose the nodes can somehow change the probability of transmission p in their favor. Find value of p that maximizes the throughput.

Answer: If we differentiate $Np(1-p)^{N-1}$, $(Np(1-p)^{N-1})' = N(1-p)^{N-2}\big((1-p) - p(N-1)\big)$ is obtained. The probability p that maximizes the throughput satisfies the equality $N(1-p)^{N-2}\big((1-p) - p(N-1)\big) = 0$. Thus, the answer is $p = \frac{1}{N}$ (p>0).

(d) Let us denote the maximum throughput by $T(N)$, using the value of p from part (c). We call $T(N)$ as capacity of the network. Calculate the capacity if there are infinite number of nodes, in other words, find $\lim_{N\to\infty} T(N)$. (I hope the answer looks familiar to you!)

Answer: Given$T(N) = N\frac{1}{N}\left(1 - \frac{1}{N}\right) = \left(1 - \frac{1}{N}\right)^{N-1}$, *The solution is* $\lim_{N\to\infty}\left(1 - \frac{1}{N}\right)^{N-1} = \frac{1}{e}$.

4. **(CSMA/CD:)** Consider building a CSMA/CD network running at 1 Gbps over a 1 km cable with no repeaters. The signal speed in the cable is 2×10^5 km/sec. What is the minimum frame size?

Answer: the minimum frame size in CSMA/CD has $2t$ slot where t is propagation time between two stations to receive error message(jam signal) before the entire frame is transmitted. The propagation time t is equal to $\frac{1\times10^6\text{m}}{2\times10^5\times10^3 m/sec} = 5$ μs. Since the time for round-trip is 10 μs, the minimum frame size is $10^9\text{bps} \times 10 \times 10^{-6}\text{sec} = 10\text{kb}$

5. **(Partitioning Collision Domains on Ethernet Networks):** Figure 1 shows how network managers can partition an Ethernet network to accommodate more nodes. The figure assumes that each node transmits R bps, on average, during a representative period of time. There are N nodes to be connected. If the total transmission rate $N \times R$ is larger that the rate that can be handled by one Ethernet, then the network manager can try to partition the network. For instance, if the efficiency of one Ethernet connecting the N nodes is 80% and if $N \times R > 8$ Mbps, then one Ethernet cannot handle all the nodes. Let us assume that the N nodes can be divided into two groups that do not exchange messages frequently. For simplicity, say that each group sends a fraction p of its messages to the other group. let us connect the computers in each group with a dedicated Ethernet, as shown in the figure. The two Ethernets are connected by a *bridge*.

(a) Find an expression for the traffic on each Ethernet.

Answer: Each Ethernet has as the $\left(\frac{N\times R}{2} + \frac{N\times R}{2}p\right)$ bps as the traffic. Thus the answer is $\frac{N\times R}{2}(1+p)$bps.

(b) The arrangement can handle the N nodes if this rate is less than the efficiency of each Ethernet times 10 Mbps. Of course, the bridge must be able to handle the throughput. Assuming an efficiency of 80%, $R = 1$ Mbps and $p = 20\%$ compute how many stations can be supported.

Answer: The traffic that can be handled in each group is equal to $8\text{M}bps$. Beside, each group has $1.2 \times \frac{N}{2}$Mbps about the number of nodes. So, it can accommodate $13(\frac{8Mbps}{0.6Mbps} \approx 13.33)$ nodes.

(c) Under the same efficiency and load assumptions compare the number of hosts that could be supported with a single collision domain versus that with the two collision domains computed above.

Answer: In single collision domain, a group can accommodate $\frac{8\text{Mbps}}{1\text{Mbps}} = 8$ nodes. Thus, the number of hosts that the two collision domain can have is bigger than that of the single collision domain.

(d) If p were smaller, i.e., the traffic exhibits more *locality*, would the advantages of partitioning the network increase or decrease?

Answer: the purpose of partitioning the network that I thought was to accommodate more node. if p is smaller, the traffic which happens by receiving message of other group is smaller. That means each group can accommodate more nodes. Consequently, it is the increasing of advantages.

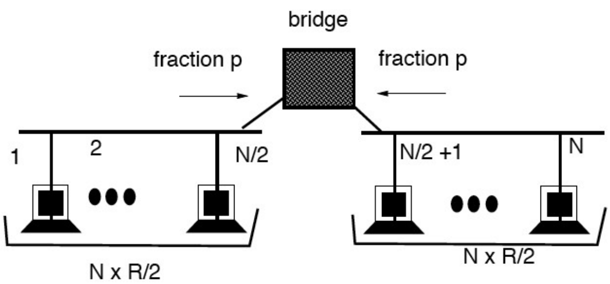


Figure 1: Ethernet load and design.

6. **(CSMA/CD in wireless?)** Compared to wired networks, state two reasons why CSMA/CD is not appropriate in the context of wireless networks.

   Answer: First of all, in the wired networks, transmitter immediately detects collision because he can compare what is sent and what is received using full-duplex data flow. However, in wireless networks, transmitter cannot do it because of self-interference which lets transmitter use half-duplex like data flow. Second problem is that the transmitter in wireless networks can not feel other collision out of his transmission range. It results in late feedback about that collision. However, transmitters wire networks don't need to care about that because of connections having nothing to do with range.

7. **(Problems with solving exposed node problem)** Consider the wireless network in Figure 2. Node A performs CSMA/CA procedure proposed by MACAW, in other words RTS-CTS-DATA-ACK sequence for transmission. Suppose node B has just received RTS from node A and found out that A tries to transmit to node C, but did not hear CTS from node C. Node B concluded that he became an exposed node and decides to transmit data to node D.

   (a) Node B initiates RTS-CTS-DATA-ACK sequence with node D, at the same moment as A starts to transmit data packet. What is the potential problem with node Bs attempt? (Hint: Suppose node A is transmitting a large packet.)

   Answer: the larger is node A' packet, the longer the transmission time is. while the node A is transmitting packet to node C, the CTS packet of D can be transmitted to B. Like figure 2, B receives simultaneously the packets which have different destination. Thus there is a possibility that the collision happen and node B cannot hear CTS.

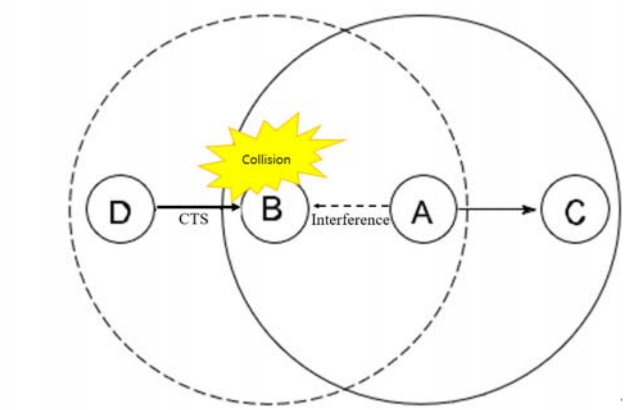


Figure 3. collision in node B

   (b) Instead of MACAW procedure, node B starts DATA-ACK sequence (CSMA-CA in weaker sense) for transmission to node D, at the same moment as A finishes transmitting data packet. What is the potential problem with node Bs attempt? (Hint: Suppose node B is transmitting a large packet.)

   Answer: Like the previous subproblem, the situation that node D cannot receive CTS packet may not happens. However, the collision at node A can happen because the transmission from C to A can be interfered with node B' DATA phase.

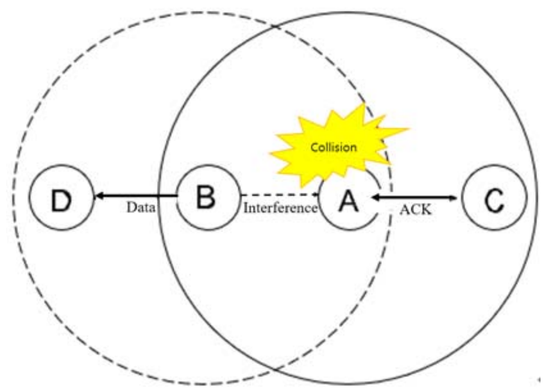


Figure 4. collision in node A

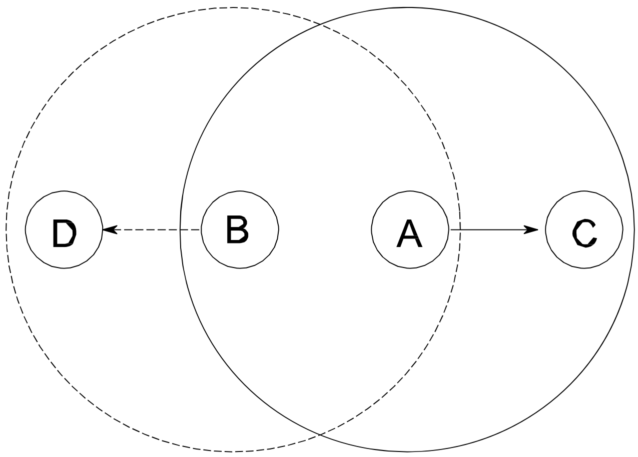


Figure 2: B is an exposed node.

8. **(Capacity of wireless networks and transmission range)**: Consider a wireless network with six nodes and their physical distances as in Figure 2 (All the angles in the geometry in Figure 2 are 90 degrees.). The transmission range of a node is modelled by a circle of radius R, in other words, if two nodes are within distance R of each other, one can successfully transmit data to the other. Suppose every node can transmit packets at a rate up to P packets/sec. A node is allowed to transmit to another node as long as its intended receiver is not interfered by other transmissions. For a given R, we define the capacity of the network as the maximum of the sum of successfully transmitted packets per second in the network. Assume that the nodes cannot transmit and receive at the same time (half-duplex). Figure 2: A wireless network.

   (a) Suppose you can control the value of R (for example, by adjusting radio transmission power). Obviously, the choice of a large R will be poor: consider R to be 200 meters for example, then at any instant, only one node will be able to transmit to another node, in which case the capacity will be only P packets/sec. You can achieve higher capacity by appropriate setting the value of R, but R has to be greater than some minimum radius R_{min}. Find the maximum capacity of this network and find the value of R_{min}. (Hint: Think in terms of the number of concurrent transmissions that are allowed in the network.)

   Answer: the number of node is six, So we can make maximum three pair of nodes doing communication with each other. Thus, the maximum capacity of this network is 3P packets/sec. Beside, if R is less than 30m, no one can transmit data to other because all of nodes are out of transmission range. Thus R_{min} should be 30m.

   (b) If we increase the value of R starting from R_{min}, you will see that we cannot achieve the maximum capacity derived in part (a) if R reaches some value R_{max}. Find R_{max}.

   Answer: if we increase R to 40m, the collision domains will be generate. For example, when C transmit packets to D, A cannot transmit packets to anyone because of collision in node D. Thus R should be less than 40m for every node to transmit packets at maximum rate. The answer R_{max} is 40m

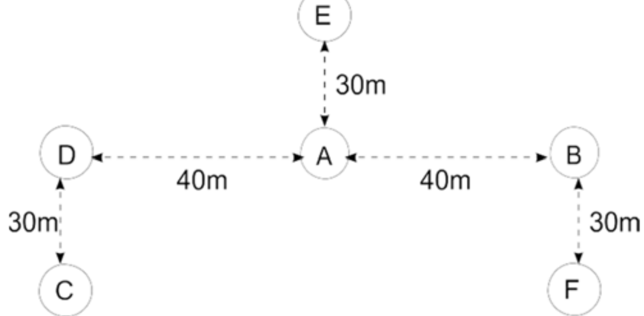
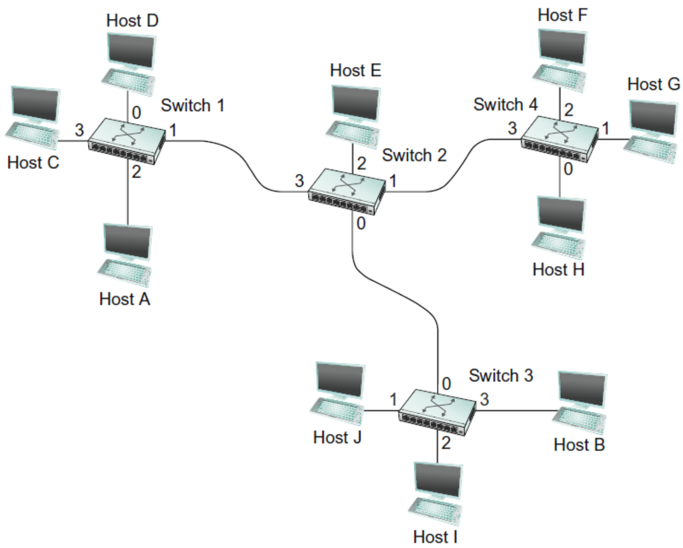


Figure 5. A Wireless Network

9. (Virtual Circuit Networks) Using the example network given in Figure, give the virtual circuit tables for all the switches after each of the following connections is established. Assume that the sequence of connections is cumulative; that is, the first connection is still up when the second connection is established, and so on. Also assume that the VCI assignment always picks the lowest unused VCI on each link, starting with 0, and that a VCI is consumed for both directions of a virtual



(a) Host A connects to host C.
(b) Host D connects to host B.
(c) Host D connects to host I.
(d) Host A connects to host B.
(e) Host F connects to host J.
(f) Host H connects to host A.

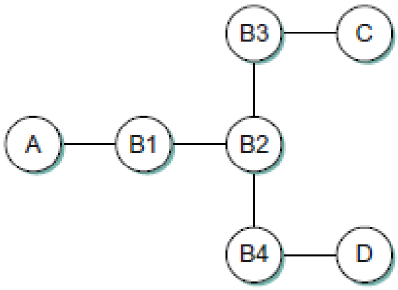
Answer: The answer is a below table.

Switch 1				Switch 2				Switch 3				Switch 4			
Incoming		**Outgoing**		**Incoming**		**Outgoing**		**Incoming**		**Outgoing**		**Incoming**		**Outgoing**	
Port	VCI	Port	VCI	Port	VCI	Port	VCI	Port	VCI	Port	VCI	Port	VCI	Port	VCI
2	0	3	0	3	0	0	0	0	0	3	0	2	0	3	0
0	0	1	0	3	1	0	1	0	1	2	0	0	1	3	1
0	1	1	1	3	2	0	2	0	2	3	1				
2	1	1	2	1	0	0	3	0	3	1	0				
1	3	2	2	1	1	3	3								

(a) (b) (c) (d) (e) (f)

Figure 6. table after establishing all connections

10. (Learning bridges) Consider the arrangement of learning bridges shown in Figure 3.49. Assuming all are initially empty, give the forwarding tables for each of the bridges B1 to B4 after the following transmissions:
(a) A sends to C.
(b) C sends to A.
(c) D sends to C.
Identify ports with the unique neighbor reached directly from that port; that is, the ports for B1 are to be labeled A and B2.



Answer: The answer is a below table

B1		B2	
Destination	Neighbor	Destination	Neighbor
A	A	A	B1
C	B2	C	B2
		D	B4

B3		B4	
Destination	Neighbor	Destination	Neighbor
A	B2	A	D2
C	C	D	D
D	B2		

Figure 7. forwarding table for problem 10