Continued Fractions 1

1. Expand the fractions $\frac{17}{3}$ and $\frac{3}{17}$ into continued fractions.

2. Convert into rational numbers: $[2; 1, 4]$, $[-3; 2, 12]$, $[0; 1, 1, 100]$.

3. Given positive integers b, c, d with $c > d$, prove that $[a; c] < [a; d]$ but $[a; b, c] > [a, b, d]$ for any integer a.

4. Let $a_1, a_2, \ldots, a_n$ and c be positive real numbers. Prove that
$$[a_0; a_1, \ldots, a_n] > [a_0; a_1, \ldots, a_n + c]$$
holds if n is odd, but is false if n is even.

5. Verify that $[2; 3, 7]$ and $[2; 3, 6, 1]$ are two continued fraction expansions of $\frac{51}{52}$. In general, any rational number $\frac{u_0}{u_1}$ has at least two continued fraction expansions:
$$\frac{u_0}{u_1} = [a_0; a_1, \ldots, a_{j-1}, a_j] = [a_0; a_1, \ldots, a_{j-1}, a_j - 1, 1]. \tag{1}$$

6. Prove that the expansions in (1) are the only continued fraction expansions of $\frac{u_0}{u_1}$. In other words, show that if $[a_0; a_1, \ldots, a_j] = [b_0; b_1, \ldots, b_n]$ with $a_j > 1$ and $b_n > 1$, then $j = n$ and $a_i = b_i$ for $i = 0, 1, \ldots, n$.