Matrix of a Linear Transformation

1. Assume that T is a linear transformation. Find the standard matrix of T.

   (a) $T : \mathbb{R}^3 \to \mathbb{R}^2$. $T(e_1) = (1, 4)$, $T(e_2) = (-2, 9)$, and $T(e_3) = (3, -8)$, where e_1, e_2, and e_3 are the columns of the 3×3 identity matrix.

   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a vertical shear transformation that maps e_1 into $e_1 - 3e_2$, but leaves e_2 unchanged.

   (c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ rotates points about the origin through $3\pi/4$ radians, clockwise.

   (d) $T : \mathbb{R}^3 \to \mathbb{R}^3$ contracts points by a factor of $\frac{1}{2}$ ($T(x) = \frac{1}{2}x$).

   (e) $T(x_1, x_2, x_3, x_4) = 3x_1 + 4x_3 - 2x_4$

2. Fill in the missing entries of the matrix, assuming that the equation holds for all values of the variables.

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_1 - 4x_2 \\ x_1 - x_3 \\ -x_2 + 3x_3 \end{bmatrix}$$

3. Determine if the specified linear transformation is (a) one-to-one and (b) onto.

   (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects points about the x-axis.

   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ that projects points onto the y-axis.

   (c) $T : \mathbb{R}^3 \to \mathbb{R}$ that sends (x, y, z) to $x + y + z$.

   (d) $T : \mathbb{R} \to \mathbb{R}^3$ that sends t to $(t, 2t, 3t)$.

4. If a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ maps $\mathbb{R}^n$ onto $\mathbb{R}^m$, can you give a relation between m and n? If T is one-to-one, what can you say about m and n?