Subspaces

1. Let $v_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 4 \\ -4 \\ 5 \\ 7 \end{bmatrix}$, $v_3 = \begin{bmatrix} 5 \\ -3 \\ 6 \\ 5 \end{bmatrix}$, and $u = \begin{bmatrix} -1 \\ -7 \\ -1 \\ 2 \end{bmatrix}$. Determine if u is in the subspace of $\mathbb{R}^4$ generated by $\{v_1, v_2, v_3\}$.

2. Let $u = \begin{bmatrix} -5 \\ 5 \\ 3 \end{bmatrix}$ and $A = \begin{bmatrix} -2 & -2 & 0 \\ 0 & 3 & -5 \\ 6 & 3 & 5 \end{bmatrix}$. Is u in Nul A?

3. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ -5 & -1 & 0 \\ 2 & 7 & 11 \\ 3 & 3 & 4 \end{bmatrix}$. Find a nonzero vector in Nul A and a nonzero vector in Col A.

4. Do $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 16 \\ -3 \end{bmatrix}$ form a basis for $\mathbb{R}^2$?

5. Do $\begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -4 \\ 10 \end{bmatrix}$ form a basis for $\mathbb{R}^3$? How about just the first two vectors?

6. True or false?

   (a) A subset H of $\mathbb{R}^n$ is a subspace if the zero vector is in H.

   (b) Let H be a subspace of $\mathbb{R}^n$. If x is in H, and y is in $\mathbb{R}^n$, then $x + y$ is in H.

   (c) The solution set to $Ax = b$, where A is an $m \times n$ matrix, forms a subspace of $\mathbb{R}^n$.

7. Find a basis for the column space and null space of the matrix.

$$A = \begin{bmatrix} 3 & 4 & 0 & 7 \\ 1 & -5 & 2 & -2 \\ -1 & 4 & 0 & 3 \\ 1 & -1 & 2 & 2 \end{bmatrix}.$$

8. Suppose F is a 5×5 matrix whose column space is not equal to $\mathbb{R}^5$. What can be said about F's nullspace?

9. What can be said about the shape of an $m \times n$ matrix A when the columns of A form a basis for $\mathbb{R}^m$?

10. If B is a 6×6 matrix and Nul B is not the zero subspace, what can be said about Col B?