Eigenvalues and Eigenvectors

1. Is $\lambda = -3$ an eigenvalue of $\begin{bmatrix} -1 & 4 \\ 6 & 9 \end{bmatrix}$?

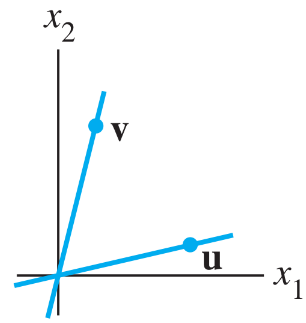
2. Is $\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$ an eigenvector of $\begin{bmatrix} 3 & 6 & 7 \\ 3 & 2 & 7 \\ 5 & 6 & 4 \end{bmatrix}$? If so, find the eigenvalue.

3. Find a basis for the eigenspace corresponding to each listed eigenvalue.

   (a) $A = \begin{bmatrix} 4 & 1 \\ 3 & 6 \end{bmatrix}$, $\lambda = 3, 7$.

   (b) $A = \begin{bmatrix} 4 & 0 & -1 \\ 3 & 0 & 3 \\ 2 & -2 & 5 \end{bmatrix}$, $\lambda = 3$.

4. Let u and v be the vectors shown in the figure and suppose that u and v are eigenvectors of a 2×2 matrix A that correspond to eigenvalues 2 and 3, respectively. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by $T(x) = Ax$, and let $w = u + v$. Make a copy of the figure and on the same set of axes, plot $T(u)$, $T(v)$, and $T(w)$.



5. Find the characteristic polynomial and the eigenvalues of the matrices.

   (a) $\begin{bmatrix} -4 & -1 \\ 6 & 1 \end{bmatrix}$, (b) $\begin{bmatrix} 9 & -2 \\ 2 & 5 \end{bmatrix}$

6. Find the characteristic polynomial

$$\begin{bmatrix} 3 & 1 & 1 \\ 0 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix}$$

7. List the eigenvalues along with their algebraic multiplicities.

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 6 & 2 & 0 & 0 \\ 0 & 3 & 6 & 0 \\ 2 & 3 & 3 & -5 \end{bmatrix}$$