Determinant Properties

1. Compute the determinants by using row reduction shortcuts.

   (a)
$$\begin{vmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{vmatrix}$$

   (b)
$$\begin{vmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{vmatrix}$$

   (c)
$$\begin{vmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 5 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{vmatrix}$$

2. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$, compute the following determinants.

   (a)
$$\begin{vmatrix} a & b & c \\ d & e & f \\ 5g & 5h & 5i \end{vmatrix}$$

   (b)
$$\begin{vmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{vmatrix}$$

   (c)
$$\begin{vmatrix} b & 3a-2c & c \\ e & 3d-2f & f \\ h & 3g-2i & i \end{vmatrix}$$

3. Use determinants to determine if the vectors are linearly independent.

   (a)
$$\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$$

   (b)
$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 8 \\ 9 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix}$$

4. Compute $\det B^5$, where
$$B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}.$$

5. Show that if A is invertible, then $\det A^{-1} = \frac{1}{\det A}$.

6. Find a formula for $\det(rA)$ when A is an $n \times n$ matrix

7. Let U be a square matrix such that $U^T U = I$. Show that $\det U = \pm 1$.

8. Let A and P be square matrices, with P invertible. Show that $\det(PAP^{-1}) = \det A$.