Quiz 6

Student ID Number: Name _____

Math 140B, 5PM

Please justify all your answers February 28, 2019

Please also write your full name on the back

1. Let f be bounded on $[a, b]$ and suppose there exists a partition P with $L(f, P) = U(f, P)$. Show that f is constant. What is the value of $\int_a^b f$?

2. Show that if f is bounded on $[a, b]$ and $|f(x) - f(y)| \leq M|x - y|$ for some constant M and all $x, y \in [a, b]$ then for any partition P of $[a, b]$, we have

$$U(f, P) - L(f, P) \leq M(b - a) \cdot \text{mesh}(P).$$

Recall that the mesh of the partition $P = \{a = x_0 < x_1 < \ldots < x_n = b\}$ is defined to be $\max_{1 \leq i \leq n}(x_i - x_{i-1})$.