Matrix Equations and Solution Sets

1. Compute the matrix product. If the product is undefined, explain why.

   (a)
   $$\begin{bmatrix} -1 & 6 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 4 \\ -7 \end{bmatrix}$$

   (b)
   $$\begin{bmatrix} 5 & 5 & -1 \\ 1 & -7 & 0 \end{bmatrix} \begin{bmatrix} 12 \\ 4 \end{bmatrix}$$

2. Write the system as a vector equation and then as a matrix equation.

   (a)
   $$\begin{array}{rcrcr} 2x & + & 5y & = & -10 \\ -2x & + & 4y & = & 0 \end{array}$$

   (b)
   $$\begin{array}{rcrcrcr} x & - & 7y & & & = & 12 \\ -2x & + & 5y & + & 13z & = & 0 \\ & & 6y & - & z & = & -1 \end{array}$$

3. Let $u = \begin{bmatrix} 4 \\ -1 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 5 & -1 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix}$. Is u in the subset of $\mathbb{R}^3$ spanned by the columns of A? What does this say about the solution set for the equation $Ax = u$?

4. Let $A = \begin{bmatrix} 1 & -2 & -1 \\ -2 & 2 & 0 \\ 4 & -1 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Show that the equation $Ax = b$ does not have a solution for all possible b. Describe the set of all b for which $Ax = b$ *does* have a solution.

5. Write the solution set of the given homogeneous system in parametric vector form.
   $$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & - & 3x_3 & = & 0 \\ 2x_1 & + & x_2 & - & 3x_3 & = & 0 \\ -x_1 & + & x_2 & & & = & 0 \end{array}$$

6. Write the solution set to $Ax = 0$ in parametric vector form where A is given by
   $$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ -2 & -3 & 4 & 5 \\ 2 & 4 & 0 & -2 \end{bmatrix}.$$

7. Describe the solutions of the following system in parametric vector form. Give a geometric description of the solution set.
   $$\begin{array}{rcrcrcrcr} x_1 & + & 3x_2 & - & 4x_3 & + & 4x_4 & = & 4 \\ x_1 & + & 4x_2 & - & 7x_3 & + & 6x_4 & = & 3 \end{array}$$

8. Find a parametric equation of the line ℓ through $p = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$ and $q = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$.

9. Suppose that the equation $Ax = b$ is consistent for some b, and let p be a solution. Prove that the solution set of $Ax = b$ is the set of all vectors of the form $w = p + v_h$, where v_h is any solution of the homogeneous equation $Ax = 0$.

10. (Bonus) Suppose A is a 3×3 matrix and b is a vector in $\mathbb{R}^3$ such that the equation $Ax = b$ does *not* have a solution. Does there exist a vector y in $\mathbb{R}^3$ such that the equation $Ax = y$ has a *unique* solution?