Row Reduction and Echelon Forms

1. Consider each matrix as the augmented matrix of a linear system. What are the next two row operations that should be performed in the process of solving the system? Finish solving the system.

   (a)
   $$\begin{bmatrix} 5 & 1 & 0 & 13 \\ 1 & -4 & 0 & -10 \\ 0 & 3 & -1 & 5 \end{bmatrix}$$

   (b)
   $$\begin{bmatrix} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 3 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 3 & 1 & 6 \end{bmatrix}$$

2. Solve the systems.

   (a)
   $$\begin{aligned} x_2 + 4x_3 &= -5 \\ x_1 + 3x_2 + 5x_3 &= -2 \\ 3x_1 + 7x_2 + 7x_3 &= 6 \end{aligned}$$

   (b)
   $$\begin{aligned} x_1 - 3x_2 &= 5 \\ -x_1 + x_2 + 5x_3 &= 2 \\ x_2 + x_3 &= 0 \end{aligned}$$

3. Do the three planes $x + 2y + z = 4$, $y - z = 1$, and $x + 3y = 0$ have at least one common point of intersection?

4. Suppose a, b, c, and d are constants such that a is nonzero and the system below is consistent for all possible values of f and g. What can you say about the numbers a, b, c, and d?
   $$\begin{aligned} ax_1 + bx_2 &= f \\ cx_1 + dx_2 &= g \end{aligned}$$

5. Row reduce the matrix to reduced echelon form. Circle the pivot positions in the final matrix and in the original matrix and list the pivot columns.
   $$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$$

6. Find the general solutions of the system whose augmented matrices are given here.

   (a)
   $$\begin{bmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{bmatrix}$$

   (b)
   $$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix}$$

7. Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

   (a)
   $$\begin{bmatrix} 1 & 1 & h & 1 \\ 1 & h & 1 & 1 \\ h & 1 & 1 & 1 \end{bmatrix}$$

8. Choose h and k such that the system has (a) no solution, (b) a unique solution, and (c) many solutions.

$$\begin{aligned} x &+ 3y &= 2 \\ 3x &+ hy &= k \end{aligned}$$

9. True or False? Justify.

   (a) If one row in an echelon form of an augmented matrix is [0 0 0 5 0], then the associated linear system is inconsistent.

   (b) The echelon form of a matrix is unique.

   (c) Whenever a (consistent) system has free variables, the solution set contains many solutions.

10. Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.