Linear Equations

1. What does it mean for a linear system to be consistent? Give an example of a consistent system and an example of an inconsistent system.

2. One of the fundamental row operations is scaling: we can multiply all entries in a row by a nonzero constant. Why is it important to us that this constant be nonzero?

3. Come up with a system of two equations in two unknowns that satisfies the following constraint, if possible. If no such system exists, try to explain why.

   (a) The system has exactly one solution.

   (b) The system has no solution.

   (c) The system has infinitely many solutions.

   (d) The system has exactly two solutions.

4. Solve the system using elementary row operations on the equations or on the augmented matrix.

$$\begin{aligned} 3x_1 &+ 6x_2 &&= -3 \\ 5x_1 &+ 7x_2 &&= 10 \end{aligned}$$

5. What are the next few row operations that should be performed in the process of solving the system?

   (a)
$$\begin{bmatrix} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 4 & 1 & 2 \end{bmatrix}$$

   (b)
$$\begin{bmatrix} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

6. Perform a substitution to turn the system of equations into a linear system and then solve it.

   (a)
$$\begin{aligned} -2\sin t &- 2\cos s &&= -1 \\ 2\sin t &+ \cos s &&= 1 \end{aligned}$$

   (b)
$$\begin{aligned} \sqrt{x} &- e^y &&= 1 \\ 2\sqrt{x} &- e^y &&= 5 \end{aligned}$$

7. Determine if the system is consistent.

$$\begin{array}{rcrcrcr} 2x_1 & & & & - & 4x_4 & = & -10 \\ & & 3x_2 & + & 3x_3 & & = & 0 \\ & & & & x_3 & + & 4x_4 & = & -1 \\ -3x_1 & + & 2x_2 & + & 3x_3 & + & x_4 & = & 5 \end{array}$$

8. Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

(a)

$$\begin{bmatrix} 1 & h & -5 \\ 2 & -8 & 6 \end{bmatrix}$$

(b)

$$\begin{bmatrix} -4 & 12 & h \\ 2 & -6 & -3 \end{bmatrix}$$

9. Suppose a, b, c, and d are constants such that a is not zero and the system below is consistent for all possible values of f and g. What can you say about the numbers a, b, c, and d?

$$\begin{array}{rcrcl} ax_1 & + & bx_2 & = & f \\ cx_1 & + & dx_2 & = & g \end{array}$$