MATH 2B - Integration By Parts

1. Evaluate the integrals by parts.

   (a)
   $$\int t^2 \sin(4t)\ dt$$

   (b)
   $$\int y e^{9y} dy$$

   (c)
   $$\int \tan^{-1}(2y)\ dy$$

2. Make a substitution and then integrate by parts.

   (a)
   $$\int e^{\sqrt{x}} dx$$

   (b)
   $$\int e^s \cos(1+s)\ ds$$

   (c)
   $$\int x \sin x \cos x\ dx$$

3. Use integration by parts to prove the reduction formula
   $$\int (\ln x)^n\ dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx.$$
   Find a similar formula for $\int x^n e^x\ dx$.

4. Evaluate the integral (not just integration by parts).

   (a)
   $$\int \frac{1}{\sqrt{e^{2x}-1}} dx$$

   (b)
   $$\int \cos(\log x) dx$$

   (c)
   $$\int \frac{\arctan x}{1+x^2} dx$$

   (d)
   $$\int \log \sqrt{1+x^2}\ dx$$

   (e)
   $$\int \arctan x\ dx$$

   (f)
   $$\int x \arctan x\ dx$$

5. This result (the Riemann-Lebesgue lemma) is really important in a branch of mathematics called harmonic analysis. Assuming f' is continuous on $[a, b]$, use integration by parts to prove that
   $$\lim_{n\to\infty} \int_a^b f(t) \sin(nt)\ dt = 0.$$