Midterm 2 – Math 130B

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Instructions:

- You must show your work and clearly explain your line of reasoning.
- You need to solve all of the problems below. The maximum score is 100
- The exam is 1 hour, but you will automatically get extra half an hour for scanning and uploading your solution (and making sure that the file is readable. If not, then please rewrite).

GOOD LUCK!

1. Every day, Alice comes to the bus stop exactly at 7am. She takes the first bus that arrives. The arrival of the first bus is an exponential random variable with expectation 20 minutes. Also, every working day, and independently, Bob comes to the same bus stop at a random time, uniformly distributed between 7 and 7:30am.

   (a) (10 points) What is the probability that tomorrow Alice will wait for more than 30 minutes?

   (b) (15 points) Assume day-to-day independence. Consider Bob late if he comes after 7:20. What is the probability that Bob will be late on 2 or more working days among the next 10 working days?

   (c) (15 points) What is the probability that Alice and Bob will meet at the station tomorrow?

2. Let X, Y, and Z be independent random variables, each of which is uniformly distributed on the interval $[0, 1]$.

   (a) (15 points) Find the joint density function of XY and Z^2, and compute $\Pr[XY < Z^2]$.

   (b) (15 points) Compute $Var(XY + Z)$.

3. (30 points) Let K_n be the *complete graph* on n vertices; that is, its edge-set consists of all $\binom{n}{2}$ possible unordered pairs of vertices. Suppose that some coloring of the edge-set of K_n is given. A triangle is called *rainbow* if it has at most one edge from each color. Show that there exists a coloring of the edges of K_n using three colors with at least $\binom{n}{3}\frac{2}{9}$ rainbow triangles.