Math 180B - Pell's Equation

1. Let d be a positive integer that is not a perfect square. If k is any positive integer, prove that there are infinitely many solutions in integers of $x^2 - dy^2 = 1$ with $k \mid y$.

2. Find the smallest positive solution of $x^2 - dy^2 = 1$ by successively substituting $y = 1, 2, 3, \ldots$ when d is (a) 7 and (b) 11.

3. Find all positive solutions to $x^2 - 2y^2 = 1$ for which $y < 250$.

4. A Pell's equation has the form $x^2 - dy^2 = 1$ where d is a positive integer that is not a perfect square. Why don't we want d to be a perfect square?

5. Consider a right triangle, the lengths of whose sides are integers. Prove that the area cannot be a perfect square.