Math 130B - Joint Random Variables

1. Let X and Y be independent random variables taking values in the positive integers having the same distribution given by

$$\Pr[X = n] = \Pr[Y = n] = 2^{-n}$$

for all $n \in \mathbb{N}$. Find $\Pr[X \text{ divides } Y]$.

2. Let X and Y be independent continuous random variables with densities f_X and f_Y, respectively. Express the density of XY in terms of the densities of X and Y.

3. Suppose that n points are independently chosen at random on the circumference of a circle, and we want the probability that they all lie in a semicircle. That is, we want the probability that there is a line passing through the center of the circle such that all the points are on one side of that line.

Let $P_1, \ldots, P_n$ denote the n points. Let $A^{(n)}$ denote the event that all the points are contained in some semicircle, and let $A_i^{(n)}$ be the event that all the points lie in the semicircle beginning at the point P_i and going clockwise for $180°$, $i = 1, \ldots, n$.

   (a) Express $A^{(n)}$ in terms of the $A_i^{(n)}$.

   (b) Find $\Pr[A^{(n)}]$ and show that it is $o(1)$.