Math 130B - Continuous Random Variables

1. For what values of C are the following functions probability density functions?

   (a) $f(x) = C[x(1-x)]^{-1/2}$, $x \in (0,1)$.

   (b) $f(x) = C\exp(-x - e^{-x})$, $x \in \mathbb{R}$.

   (c) $f(x) = C/(1+x^2)$, $x \in \mathbb{R}$.

2. For what values of α is $E[|X|^\alpha]$ finite if X has the density f. (This is called the α-th moment of X).

   (a) $f(x) = e^{-x}$, $x \geq 0$.

   (b) $f(x) = 1/[\pi(1+x^2)]$, $x \in \mathbb{R}$.

3. You arrive at a bus stop at 10 AM knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30.

   (a) What is the probability that you will have to wait longer than 10 minutes?

   (b) If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

4. Let X be a random variable that takes on values between 0 and c. Show that

$$\mathrm{Var}[X] \leq \frac{c^2}{4}.$$

   One way to do this is to first show that $E[X^2] \leq cE[X]$ and then show that

$$\mathrm{Var}[X] \leq c^2[\alpha(1-\alpha)],$$

   where $\alpha = \frac{E[X]}{c}$.

5. Say you want to write a computer program that needs to simulate a continuous random variable X whose distribution function is F. You don't know how to simulate X directly, but you can simulate uniform random variables just fine. Assuming the distribution function F is strictly increasing, describe a way to simulate X.