HomeWork 5 Math 271B, Winter 2020.

1. Consider

$$dX_t = \mu(X_t)dt + \sigma(X_t)d\beta_t, \quad X_0 = 1, \tag{1}$$

with $\mu(x) = x + a, \sigma(x) = 4x$. Assuming $X_t > 0$ find dY_t when $Y_t = \sqrt{X_t}$. Can you find Y_t ?

2. For X_t as in Eq.(1), but with $\mu(x) = 2x$ and $\sigma(x) = x^a$ and $Y_t = X_t^b$. Can you find b so that $\langle Y \rangle_t$ is linear in t (constant diffusion coefficient/volatility).

3. Let

$$dX_t = \sqrt{1 + X_t}d\beta_t, \quad X_0 = 0,$$

find $\mathbb{E}[X_t]$ and $\mathbb{E}[X_t^2]$.

4. Do Problem 4.16 page 60-61 in Øksendal.