Quiz 2

Student ID Number: Name _____

Math 175, 12PM

Please justify all your answers January 23, 2020

Please also write your full name on the back

1. True or false?

   (a) If A and B are finite sets, then $|A \cup B| = |A| + |B|$.

   (b) There are $\binom{52}{5}$ ways to draw a hand of five cards from a deck of 52 cards.

2. Briefly explain what is wrong with this inductive proof.

   Theorem. $\frac{d}{dx}x^n = 0$ *for all* $n \geq 0$.

   Proof. For the base case, suppose $n = 0$. Then

   $$\frac{d}{dx}x^0 = \frac{d}{dx}1 = 0.$$

   Suppose the claim holds for all $k \leq n$. By the product rule we have

   $$\frac{d}{dx}x^{n+1} = \frac{d}{dx}(x^n \cdot x^1) = x^n \cdot \frac{d}{dx}x^1 + x^1 \cdot \frac{d}{dx}x^n = x^n \cdot 0 + x^1 \cdot 0 = 0.$$

   The claim follows by induction. □

3. How many positive integers below 100 are divisible by 3, 5, or 7?