Today's Problems on (cnues under Files > Discussion Documents -Make an account on Overleaf. com . 6 - dounland somether like Miktex

3. Prove that the number of positive irreducible fractions ≤ 1 with denominator $\leq n$ is $\phi(1) + \phi(2) + \cdots + \phi(n)$.

+ P(N+1)

A

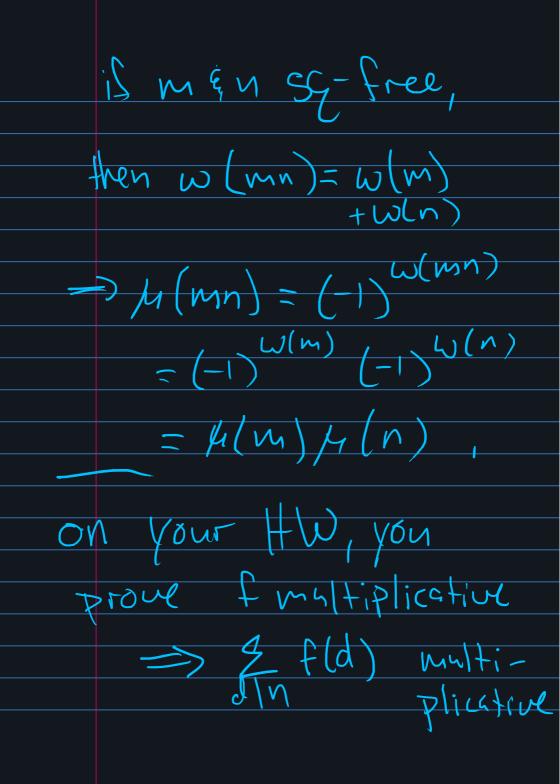
5. Define the function $\mu: \mathbb{N} \to \{-1,0,1\}$ by

$$\mu(n) = \begin{cases} 1, & \text{if } n \text{ is square-free and has an even number of prime divisors} \\ -1, & \text{if } n \text{ is square-free and has an odd number of prime divisors} \\ 0, & \text{if } n \text{ has a squared prime factor} \end{cases}$$

(a) Prove that $\mu(n)$ is multiplicative and

$$\sum_{d|n} \mu(d) = \begin{cases} 1, & \text{if } n = 1 \\ 0, & \text{if } n > 1 \end{cases}.$$

Primes P/MN} = {Primes p | m} 1 { primes P | n } · if m or n divisible by a square, whoy, m is. Then so is mn => 0= 4(mn)=4(m)4(n) = 0 /4(n)



$$\begin{array}{ccc}
50 & 2 & \mu(d) & \text{is multiplicst} \\
d & \text{ive} \\
\end{aligned}$$

$$\begin{array}{ccc}
\text{Case } : & \text{n=1} \\
2 & \mu(d) = \mu(1) \\
4 & \text{in}
\end{array}$$

So
$$2 \mu dd$$

$$= 2 \mu (p^{j}) = \mu(p^{0}) + \mu(p^{1})$$

$$= 1 - 1 = 0$$

$$geners | case,$$

$$N = p^{e_{1}} - p^{e_{k}}$$

$$e_{k} = f(n) = 2 \mu dd$$

$$F(n) = F(p^{e_{1}} - p^{e_{k}})$$

$$= F(p^{e_{1}}) - F(p^{e_{k}})$$

$$= 0 \cdot 0 \cdot \cdot \cdot 0 = 0$$

Mobius inversion (b) Prove that if $F(n) = \sum_{d|n} f(d)$ for every positive integer n, then $f(n) = \sum_{d|n} \mu(d) F(n/d)$. FI & Mld) F (Md) 1symmetric In d&k, 50 we can switch them 2 f(k) 2 M(d) k|n d|1/2

only term that sorvives is
$$k = N$$

$$= f(N)$$

$$= f(N)$$
(c) Prove the converse to (b): if $f(n) = \sum_{d|n} \mu(d) F(n/d)$, then $F(n) = \sum_{d|n} f(d)$.

Pt. $2f(d) = 2f(d) = 2f(d) f(d)$.

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