Math 130B - Conditional Density and Probabilistic Method

1. The joint density of X and Y is

$$f(x,y) = c(x^2 - y^2)e^{-x}, \quad 0 \le x < \infty, \ -x \le y \le x.$$

Find the conditional distribution of Y, given $X = x$.

2. X and Y have joint density function

$$f(x,y) = \frac{1}{x^2 y^2}, \quad x, y \ge 1.$$

(a) Compute the joint density function of $U = XY$, $V = X/Y$.

(b) What are the marginal densities?

3. Suppose X is a random variable taking only nonnegative integer values. Show that if $E[X] < 1$, then $\Pr[X = 0] > 0$.