Math 130B - Conditional Expectation

1. A mouse is placed in a maze with two rooms pictured in Figure 1. Starting from room 1, what is the expected number of steps the mouse takes before it reaches the exit?

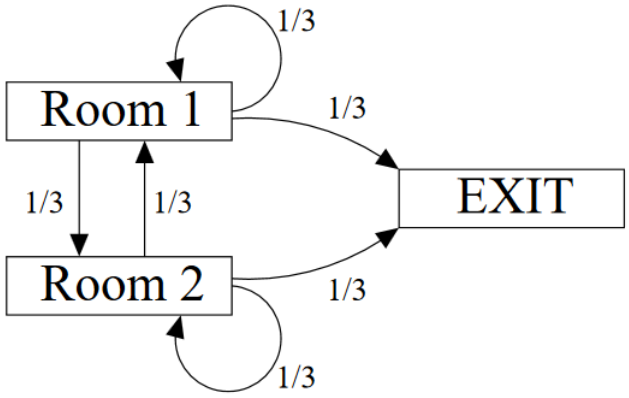

Figure 1: Maze

2. On the day before an exam, Math 130B students go to Liam's office hours to ask questions. Each question asked will appear on the exam with probability p. The number of questions asked is a Poisson distributed random variable with mean λ. What is the probability that Liam does not have to answer an exam question?

3. If X and Y are independent continuous random variables, show that

$$\Pr[Y < X] = \int_{\mathbb{R}} F_Y(x) f_X(x)\, dx,$$

where $F_Y(\cdot)$ is the cdf of Y and $f_X(\cdot)$ is the pdf of X. Use this to compute $\Pr[Y < X]$ where $X \sim Exp(\mu)$ and $Y \sim Exp(\lambda)$ are independent.