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# 271B - Homework 2

**Problem 1.** Let $S$, $T$, and $T_n$, $n = 1, 2, \ldots$ be stopping times (with respect to some filtration $\{\mathcal{F}_t\}_{t\geq 0}$). Show that $T \vee S$, $T \wedge S$, $T + S$, $\sup_n T_n$ are also stopping times.

*Proof.* The pointwise minimum, maximum, sum, and supremum of measurable functions are measurable. For the minimum and maximum we have

$$\{(T \wedge S) \leq t\} = \{T \leq t\} \cup \{S \leq t\}$$

$$\{(T \vee S) \leq t\} = \{T \leq t\} \cap \{S \leq t\}.$$

Unions and intersections of measurable sets are measurable, so both of these sets live in $\mathcal{F}_t$. Thus, $T \wedge S$ and $T \vee S$ are stopping times. For the sum, we can write the set $\{T + S \leq t\}$ as a countable union:

$$\{T + S \leq t\} = \bigcup_{\alpha,\beta\in\mathbb{Q},\ \alpha+\beta\leq t} \{T \leq \alpha\} \cap \{S \leq \beta\}.$$

As $\mathcal{F}_t$-measurability is closed under countable union and intersection, the sum is a stopping time. Finally, we have

$$\{\sup_n T_n \leq t\} = \bigcap_{n=1}^{\infty} \{T_n \leq t\},$$

which is measurable, so the supremum is also a stopping time. □

**Problem 2.** Let $X_t$ be an adapted and continuous stochastic process, and define

$$T_\Gamma = \inf\{t \geq 0 : X_t \in \Gamma\}$$

for $\Gamma$ a closed set. Show that $T_\Gamma$ is a stopping time.

*Proof.* □