Math 180B - Curves mod p

1. Let p be a prime with $p \not\equiv 1 \pmod 5$. Prove that the equation $y^2 = x^5 + 1$ has exactly p solutions modulo p.

2. Let p be a prime greater than 3.

   (a) Show that the elliptic curve $y^2 = x^3 + p$ has p as a bad prime and find a_p. *Hint: Consider (and justify) the substitution $y = xz$.*

   (b) Show that the elliptic curve $y^2 = x^3 + x^2 + p$ has p as a bad prime and find a_p.

   (c) Show that the elliptic curve $y^2 = x^3 - x^2 + p$ has p as a bad prime and find a_p.