

Math 130B - Joint Distributions of Functions of Random Variables

1. If X and Y are independent and identically distributed uniform random variables on $(0, 1)$, compute the joint density of

(a) $U = X + Y, V = X/Y,$

(b) $U = X, V = X/Y,$

(c) $U = X + Y, V = X/(X + Y).$

2. If X_1 and X_2 are independent exponential random variables, each having parameter λ , find the joint density function of $Y_1 = X_1 + X_2$ and $Y_2 = e^{X_1}$.

3. The joint density function of X and Y is given by

$$f(x, y) = xe^{-x(y+1)}, \quad x > 0, y > 0.$$

- (a) Find the conditional density of X , given $Y = y$, and that of Y , given $X = x$.
(b) Find the density function of $Z = XY$.