Math 180B - Gaussian Integers

1. (a) Show that $a + bi$ divides $c + di$ if and only if $a^2 + b^2$ divides both of the integers $ac + bd$ and $-ad + bc$.

   (b) Suppose that $a + bi$ divides $c + di$. Show that $a^2 + b^2$ divides $c^2 + d^2$.

2. Verify that each of the following subsets R_1, R_2, R_3, and R_4 of the complex numbers is a ring. In each case, describe the ring's units.

   (a) $R_1 = \{a + bi\sqrt{2} : a, b \in \mathbb{Z}\}$.

   (b) Let $\omega = -\frac{1}{2} + \frac{1}{2}i\sqrt{3}$. $R_2 = \{a + b\omega : a, b \in \mathbb{Z}\}$.

   (c) Let p be a fixed (integer) prime. $R_3 = \{a/d : a, d \in \mathbb{Z}, \ p \nmid d\}$.

   (d) $R_4 = \{a + b\sqrt{3} : a, b \in \mathbb{Z}\}$.

3. Factor each of the following Gaussian integers into a product of Gaussian primes.

   (a) $91 + 63i$

   (b) 975

   (c) $53 + 62i$