3

3. If X has distribution function F, what is the distribution function of e^{X} ?

By definition, P[X & t] = F(t) YtelR.

We went P[ex \ t].

P[ex = t] = P[X = logt] = F(lyt)

Thany time I write log 4 I don't specify
otherwise, it's loge

4.

4. Let X be a binomial random variable with parameters (n, p). What value of p maximizes $\Pr[X = k]$ for k = 0, 1, ..., n?

Recall that $P[X=k]=\binom{n}{k}p^k(1-p)^{n-k}=:f(p)$ for $X \sim Bin(nik)$.

we went to find p that maximizes this.

Since log() is monotone increasing, maximizing log (f(P)) will maximize f(P) too.

Use coloulus to maximize:

 $\frac{d}{dP}\log(f(P)) = \frac{d}{dP}\left[\log(\frac{n}{k}) + \frac{k}{k}\log P + \frac{(n-k)\log(1-p)}{k}\right]$

Set derivative = 0

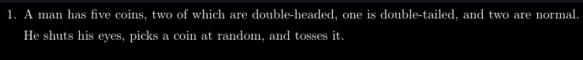
$$K = N-k$$
 $P = 1-p = 0 \iff p = k/n$
 $SE P = k/n$ is a critical point.

 $PE[0:1]$, so have to check endpoints too

 $F(\delta) = \binom{n}{k} \binom{k}{1-0}^{n-k} = 0$
 $F(K) = \binom{n}{k} \binom{k}{n} \binom{n-k}{n}^{n-k} = 0$

2. Consider an experiment whose sample space consists of a countably infinite number of points. Show that not all points can be equally likely. Can all points have a positive probability of occurring?

 $P[\Lambda] = 2 P[\omega] = 2 P = \begin{cases} 0 & \text{if } P = 0 \\ +00 & \text{if } P > 0 \end{cases}$ Since 1 is infinite. Consequence: You can't randomly select an integer from all of Z unitermy et rondon! The probabilities can be positive, just not all equal. Since It is countably infinite, we can enumerate it: $\Lambda = \{ \omega_1, \omega_2, \omega_3, \dots \}$ Let IP[Wn] = /21. Then P[1]= 2 |P[u]= 2 /2"= 1.



a)

(a) What is the probability that the lower face of the coin is a head?

(b) He opens his eyes and see that the coin is showing heads. What is the probability that the lower face is heads as well?

coin 1, so we're interested in

[P[C, | showing heads]

= P[C, , showing heads]

[P[Shaving heads]

P[Showing heads]= P[lower is heads], so

P[C, | showing heads]=
$$\frac{1}{3}$$
, | $\frac{1}{2}$

(c) He shuts his eyes again, and tosses the coin again. What is the probability that the lower face is head?

(d) He opens his eyes and sees that the coin is showing heads. What is the probability that the lower face is a head?

(e) He discards this coin, picks another one at random (but not the same coin), and tosses it. What is the probability that it shows heads?

$$50 (t) = \frac{1}{12} + \frac{1}{24}$$

$$-\frac{5}{12} = \frac{3}{10}$$

$$from before$$