

## 271B - Homework 5

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**Problem 1.** Consider

$$dX_t = \mu(X_t)dt + \sigma(X_t)dB_t, \quad X_0 = 1, \quad (1)$$

with  $\mu(x) = x + a$ ,  $\sigma(x) = 4x$ . Assuming  $X_t > 0$ , find  $dY_t$  when  $Y_t = \sqrt{X_t}$ . Can you find  $Y_t$ ?

*Solution.* Let  $g(t, x) = \sqrt{x}$  so that  $Y_t = g(t, X_t)$ . By Itô's lemma we have

$$\begin{aligned} dY_t &= \frac{1}{2}X_t^{-1/2}dX_t - \frac{1}{8}X_t^{-3/2}(dX_t)^2 \\ &= \frac{1}{2}X_t^{-1/2}[(X_t + a)dt] - \frac{1}{8}X_t^{-3/2}(16X_t^2dt) \\ &= \frac{a - 3Y_t^2}{2Y_t}dt + 2Y_tdB_t. \end{aligned}$$

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