Math 130B - More Continuous Random Variables

1. Let X be a random variable that takes on values between 0 and c. Show that
$$\mathrm{Var}[X] \leq \frac{c^2}{4}.$$
One way to do this is to first show that $E[X^2] \leq cE[X]$ and then show that
$$\mathrm{Var}[X] \leq c^2[\alpha(1-\alpha)],$$
where $\alpha = \frac{E[X]}{c}$.

2. Say you want to write a computer program that needs to simulate a continuous random variable X whose distribution function is F. You don't know how to simulate X directly, but you can simulate uniform random variables just fine. Assuming the distribution function F is strictly increasing, describe a way to simulate X.

3. If X is an exponential random variable with mean $1/\lambda$, show that
$$E[X^k] = \frac{k!}{\lambda^k}, \quad k = 1, 2, \ldots$$

4. If X is an exponential random variable with parameter $\lambda = 1$¡ compute the probability density function of the random variable Y defined by $Y = \log X$.

5. The time in hours required to repair a machine is an exponentially distributed random variable with parameter $\lambda = \frac{1}{2}$. What is
   (a) The probability that a repair time exceeds 2 hours?
   (b) The conditional probability that a repair takes at least 10 hours, given that that its duration exceeds 9 hours?