2. From ten married couples, we want to select a group of six people that is not allowed to contain a married couple. How many choices are there? How many choices are there if the group must also consist of three men and three women? Assume couples are MF

each person we pick must come from a different couple, so choosing six people amounts to choosing six couples, then choosing one person from that couple.

There are $\binom{10}{6}$ ways to choose the couples & two people per couple $\Rightarrow \binom{10}{6} \cdot 2^6$ ways total.

For the second part, we're still choosing six couples to choose a person from. From these six couples, we then choose the woman from three of them. There are then $\binom{10}{6}\binom{6}{3}$ ways to do this.

$\boxtimes$

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4. Determine the number of solutions to the inequality

$$x_1 + x_2 + \cdots + x_n \leq k,$$

where each x_i is a nonnegative integer.

Introduce the new variable $x_{n+1} \geq 0$ & consider the equation

$$x_1 + x_2 + \cdots + x_n + x_{n+1} = k. \quad (*)$$

If this equation is satisfied, then $x_1 + \cdots + x_n \leq k$ since $x_{n+1} \geq 0$.

Conversely, if $x_1 + \cdots + x_n \leq k$, there is a unique value of x_{n+1} that solves

$$x_1 + \cdots + x_n + x_{n+1} = k.$$

Thus, solutions to the given inequality $\iff$ solutions to $(*)$

To solve $(*)$, think of distributing k objects among $n+1$ bins, where bins are allowed to be empty.

Ex: $x_1 + x_2 + x_3 = 5 \rightsquigarrow 5$ objects, 3 bins

$$\underbrace{\bullet\ \bullet}_{2}\ \big|\ \underbrace{\bullet}_{1}\ \big|\ \underbrace{\bullet\ \bullet}_{2} \quad \text{or} \quad \underbrace{\bullet\ \bullet\ \bullet}_{3}\ \big|\underbrace{}_{0}\big|\ \underbrace{\bullet\ \bullet}_{2}$$

To make $n+1$ bins, imagine placing n bars in the spaces between the k objects laid out in a row.

There are then $n+k$ items to configure the objects & the bars. We choose n of these items to be bars, so there are $\binom{n+k}{n}$ solutions to $(*) \Rightarrow \binom{n+k}{n}$ solutions to the inequality. $\boxtimes$