

Math 130B - Homework 4

You must type up your solutions to this assignment in L^AT_EX.

1. Suppose that n points are independently chosen at random on the circumference of a circle, and we want the probability that they all lie in a semicircle. That is, we want the probability that there is a line passing through the center of the circle such that all the points are on one side of that line.

Let P_1, \dots, P_n denote the n points. Let $A^{(n)}$ denote the event that all the points are contained in some semicircle, and let $A_i^{(n)}$ be the event that all the points lie in the semicircle beginning at the point P_i and going clockwise for 180° , $i = 1, \dots, n$.

- (a) Express $A^{(n)}$ in terms of the $A_i^{(n)}$.
 - (b) Find $\Pr[A^{(n)}]$ and show that it is $o(1)$.
2. The joint density function of X and Y is given by

$$f(x, y) = xe^{-x(y+1)}, \quad x > 0, y > 0.$$

- (a) Find the conditional density of X , given $Y = y$, and that of Y , given $X = x$.
 - (b) Find the density function of $Z = XY$.
3. Suppose that A, B, C are independent random variables, each being uniformly distributed over $(0, 1)$.
 - (a) What is the joint cumulative distribution function of A, B, C ?
 - (b) What is the probability that all of the roots of the equation $Ax^2 + Bx + C = 0$ are real?
 4. Show that the jointly continuous (discrete) random variables X_1, \dots, X_n are independent if and only if their joint probability density (mass) function $f(x_1, \dots, x_n)$ can be written as

$$f(x_1, \dots, x_n) = \prod_{i=1}^n g_i(x_i)$$

for nonnegative functions $g_i(x)$, $i = 1, \dots, n$.

5. Suppose that a random real number x has been chosen according to some distribution and you have two sealed envelopes in front of you. One contains the number x and the other contains the number $2x$.
 - (a) If you need to guess which of the envelopes contains the number x , what would be your probability of guessing correctly?
 - (b) Now suppose that you are allowed to pick one envelope and look at the number inside of it before you make a decision. Can you come up with a strategy that gives you a better success rate than the one in part (a)?