Math 180B - Review

1. Show that the n-th Fibonacci number can be written

$$F_n = \binom{n-1}{0} + \binom{n-2}{1} + \binom{n-3}{2} + \cdots + \binom{n-j}{j-1} + \binom{n-j-1}{j},$$

where j is the largest integer less than or equal to $(n-1)/2$. *Hint: you might want to use the recursive formula for the binomial coefficient,* $\binom{m}{k} = \binom{m-1}{k} + \binom{m-1}{k-1}$.

2. Show that the elliptic curve $y^2 = x^3 - x^2 + p$ has p as a bad prime and find a_p.

3. A theorem of Hurwitz says that given any irrational number ξ, there exist infinitely many different rational numbers h/k such that

$$\left|\xi - \frac{h}{k}\right| < \frac{1}{\sqrt{5}k^2}.$$

Show that $\sqrt{5}$ is the best possible constant in the denominator. Do this by setting $\xi = (1+\sqrt{5})/2$ and showing that if there are infinitely many rational numbers h_j/k_j and a positive real number m such that

$$\left|\frac{h_j}{k_j} - \xi\right| < \frac{1}{mk_j^2},$$

then $m \leq \sqrt{5}$.