HomeWork 2 Math 271C, Spring 2020.

1. Show that if a, b are deterministic (and of class I^*) then
   a) if
   $$dX_t = a(t)dt + b(t)d\beta_t,$$
   $X(t)$ is a Gaussian process with independent increments.
   b) if
   $$dX_t = a(t)X_t dt + b(t)X_t d\beta_t,$$
   $X(t)$ is a log-normal process.
2. Solve the SDE
   $$dX_t = \beta_t X_t dt + \beta_t X_t d\beta_t, \quad X_0 = 1.$$
3. Find the stochastic exponential
   $$\mathcal{E}(\beta_t^2 + t).$$
4. Prove Thomas's Lemma:
   Let $X, Y, Z \in \mathcal{M}^{c,loc}$ then
   $$X_t \circ (Y_t \circ dZ_t) = (X_t Y_t) \circ dZ_t.$$
5. Problem 4.10 page 58 Øksendal.
6. Prove that Tanaka's equation:
   $$dX_t = \text{sign}(X_t)d\beta_t, \quad X_0 = 0.$$
   has no strong solution.
7. Problem 5.11 page 76 Øksendal.