

Math 175 - Homework 6

1. (a) Is there a graph on six vertices with degrees 2, 3, 3, 3, 3, 3?
(b) Is there a graph on seven vertices with degrees 6, 6, 5, 5, 4, 2, 1?
(c) Is there a graph on five vertices with degrees 0, 1, 2, 3, 4?

2. (a) We delete an edge from a connected graph G . Show by an example that the remaining graph might not be connected.
(b) Prove that if we assume that the deleted edge belongs to a cycle in G , then the remaining graph is connected.

3. Let G be a graph with 100 vertices and minimal degree 6. Prove that G must contain a C_3 , C_4 , or C_5 .

4. Let G be a connected graph in which ever pair of edges has an endpoint in common. Show that G is either a star or a triangle. A *star* on n vertices is formed by taking the empty graph on n vertices and connecting one vertex to all of the others.

5. (a) Let a_m be the number of cycle graphs on the vertex set $[m]$. Show that $a_0 = a_1 = a_2 = 0$ and for $m \geq 3$, $a_m = \frac{1}{2}(m-1)!$.
(b) Find the function whose power series is given by $\sum_{n=0}^{\infty} \frac{a_n}{n!} x^n$. This is called an *exponential generating function*.