1. Find the continued fraction expansion for the following numbers.

(a)
$$61/14$$
 |= $P(x) = |4x - 6|$

Pecall: He continued Fraction expansion

of XEIR is periodic iff

X's a quadratic irrationality.

ite, X is a zero of a polynomial

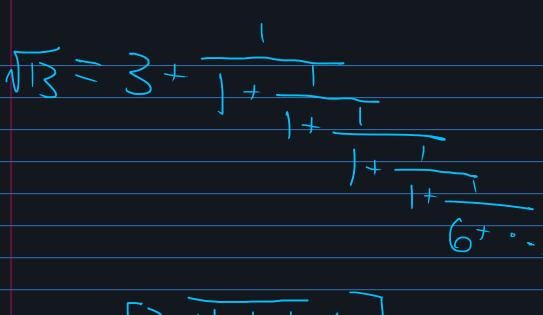
P(x) \(\int \mathred{Z[X]}, \deg(p) \leq 2.

$$\frac{61}{14} = 4 + \left(\frac{61}{14} - 4\right) = 4 + \frac{5}{14}$$

$$= 4 + \frac{1}{14/5}$$

 $\frac{14}{5} = 2 + \frac{4}{5} = 2 + \frac{1}{5/4}$

 $\frac{5}{4} = 1 + \frac{1}{4} \quad \text{done} \quad \text{ferminates}$ $\frac{5}{4} = 1 + \frac{1}{4} \quad \text{done} \quad \text{ferminates}$ $\frac{5}{4} = 1 + \frac{1}{4} \quad \text{done} \quad \text{ferminates}$ $\frac{5}{4} = 1 + \frac{1}{4} \quad \text{done} \quad \text{ferminates}$



You can do this for any XEIR only repeats for guadratic

2. Let c_1, \ldots, c_n be integers such that the continued fraction $[c_1; \ldots, c_n]$ exists. Show that we can describe the continued fraction in terms of matrix multiplication

$$\begin{pmatrix} c_1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} c_2 & 1 \\ 1 & 0 \end{pmatrix} \cdots \begin{pmatrix} c_n & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \implies [c_1; \dots, c_n] = \frac{A}{C}.$$

Pf: Induction n.

suppose it holds for sequences 6 Length N-1. Then $\left[C_{2};C_{3},...,C_{n}\right]=\frac{A}{C}$ where $(Cz)(C_3)$ (Cn)=(AB) $\begin{bmatrix} C_1 & C_2 & C_n \end{bmatrix} = C_1 + \begin{bmatrix} C_2 & C_3 & \dots & C_n \end{bmatrix}$ $=C_{1}+\frac{C}{A}=\frac{A_{C_{1}}+C}{A}$ Also, we have $\binom{C_1}{0}$ $\binom{C_2}{0}$ $\binom{C_n}{0}$ $= \begin{pmatrix} C & A & B \\ + & C & C \end{pmatrix} = \begin{pmatrix} A & C & C \\ A & C & C \end{pmatrix}$ $Ac_{1}+C$ A