MATH 2B - Integrals and the Fundamental Theorem

1. If $\int_0^{10} f(x)\ dx = 17$, and $\int_0^8 f(x)\ dx = 12$, find the value of $\int_8^1 2f(x)\ dx$.

2. Estimate $\int_0^2 e^{-x^2}\ dx$ using upper and lower bounds.

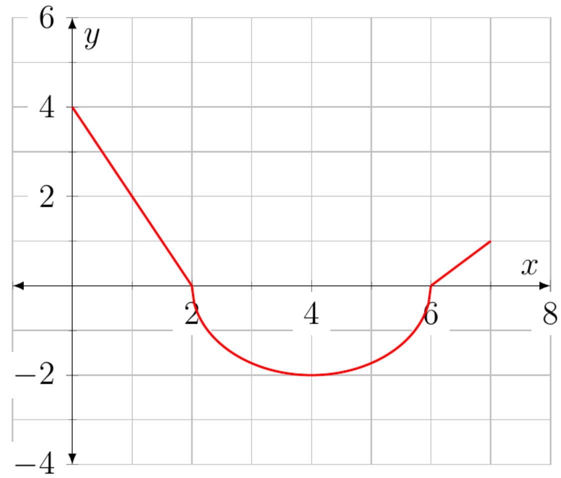
3. Evaluate the integral using right endpoints and the definition of the integral.

$$\int_2^5 (4-2x)\ dx$$

4. Interpret the limit as a definite integral over the given interval. Do not evaluate.

$$\lim_{n\to\infty}\sum_{i=1}^{n} x_i\sqrt{1+x_i^3}\ \Delta x, \quad [2,5]$$

5. Use the graph of $g(x)$ below to evaluate the following integrals by interpreting it in terms of areas. Note that g is composed of two linear functions and a semi-circle.



(a)

$$\int_0^2 g(x)\ dx$$

(b)

$$\int_2^6 g(x)\ dx$$

(c)

$$\int_0^7 g(x)\ dx$$

6. Use the Fundamental Theorem of Calculus to find the derivative of the function.

(a)
$$g(x) = \int_2^x \ln(1+t^2)\ dt$$

(b)
$$R(y) = \int_y^2 t^3 \sin t\ dt$$

(c)
$$h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1} dz$$

(d)
$$y = \int_0^{x^4} \cos^2\theta\ d\theta$$

(e)
$$y = \int_{\sin x}^1 \sqrt{1+t^2}\ dt$$

7. Evaluate the integral.

(a)
$$\int_{-5}^5 e\ dx$$

(b)
$$\int_0^1 \cosh t\ dt$$

(c)
$$\int_1^{18} \sqrt{\frac{3}{z}}\ dz$$

(d)
$$\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}} dx$$

8. Find $\int_{-2}^2 f(x)\ dx$ where

$$f(x) = \begin{cases} 2 & \text{if } -2 \le x \le 0 \\ 4-x^2 & \text{if } 0 < x \le 2 \end{cases}.$$

9. Find the derivative of the function.

(a)
$$F(x) = \int_x^{x^2} e^{t^2} dt$$

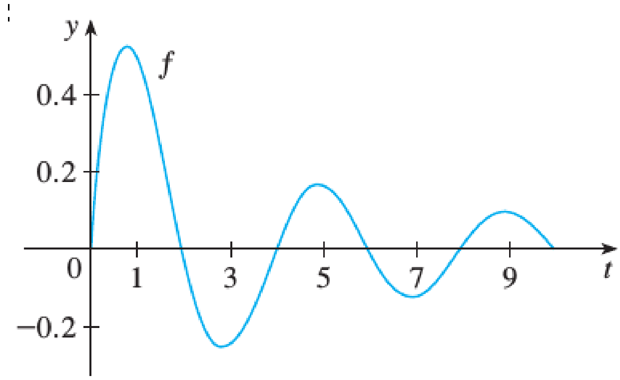
(b)
$$y = \int_{\cos x}^{\sin x} \ln(1+2v)\ dv$$

10. Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown.

(a) At what values of x do the local maximum and minimum values of g occur?

(b) Where does g attain its absolute maximum value?

(c) On which intervals is g concave downward?



11. Show that this expression does not depend on x. (That is, show that its derivative with respect to x is zero).

$$\int_0^x \frac{dt}{1+t^2} + \int_0^{1/x} \frac{dt}{1+t^2}.$$