MATH 2B: Substitution and Area

1. Compare the following two indefinite integrals. What parts of your strategy are similar/different?

$$\int x\sqrt{1+x^2}\ dx$$
$$\int x^7\sqrt{1+x^2}\ dx$$

2. Make a substitution and then integrate.

   (a)
$$\int \cos^3\theta \sin\theta\ d\theta$$

   (b)
$$\int \frac{\cos \ln t}{t} dt$$

   (c)
$$\int_0^1 xe^{-x^2}\ dx$$

   (d)
$$\int \frac{2^t}{1+2^t} dt$$

3. Suppose h is continuous and $\int_1^3 h(s)\ ds = 4$. Find $\int_1^9 \frac{h(\sqrt{t})}{\sqrt{t}} dt$.

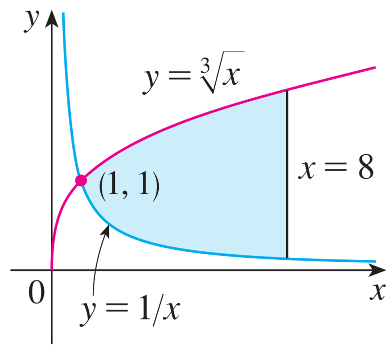
4. Suppose g and f are continuous functions. Suppose further that g is an *odd function* (i.e. $g(-x) = -g(x)$ for all x) and that f is an *even function* (i.e. $f(-x) = f(x)$ for all x). Let $a > 0$ be any positive number.

   (a) Show that $\int_{-a}^a g(x)\ dx = 0$.

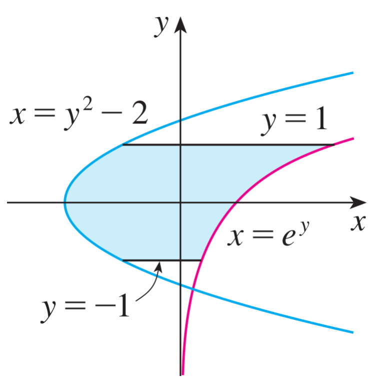
   (b) Show that $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$.

5. Find the area of the shaded region.

(a)



(b)



6. Sketch the region enclosed by the given curves and find its area.

(a) $y=x^2$, $y=4x-x^2$.

(b) $y=x^4$, $y=2-|x|$.

(c) $x=2y^2$, $x=4+y^2$.

7. Find the area between the top half of a circle of radius 1 and $y=\frac{3}{5}\sqrt{1-x^2}$.