HomeWork 4 Math 271A, Fall 2019.

1. One more Brownian construction:
   Let B_t be Brownian motion, show that
$$Y_t = (1+t)B_{(1+t)^{-1}} - B_1$$
   is Brownian motion on $[0, \infty)$.

2. Consider the (in general non-centered) random walk
$$S_n = \sum_{i=1}^{n} \zeta_i$$
   for $\mathbb{P}(\zeta_i = 1) = p$ and $\mathbb{P}(\zeta_i = -1) = 1 - p$ and the ζ_i independent. Given λ find γ so that
$$\exp(\gamma S_n - \lambda n)$$
   is a martingale with respect to the filtration generated by the ζ_is.

3. Assume that $X_n \to X$ in probability and $X_n \to Y$ a.s. Show that $X = Y$ a.s.

4. Here the (ordinary) p-variation below refers to
$$L_t^{(p)}(X) = \lim_{|||\Pi|| \to 0} V_t^{(p)}(\Pi, X), \quad V_t^{(p)}(\Pi, X) = \sum_{i=1}^{n} |X_{t_i} - X_{t_{i-1}}|^p,$$
   limit in proabability and Π the partition $0 = t_0 < t_1 < \cdots < t_n = t$.

   a. Show that a continuous stochastic process with non-zero and finite 2-variation has infinite 1-variation.

   b. Show that a continuous process with finite 1-variation has a 2-variation of 0.

   c. Show that with B Brownian $L_t^{(1)}(B)$ is infinite a.s.

5. Consider the compound Poisson process introduced in class and Homework 3:
$$X_t = \sum_{i=1}^{N_t} Y_i,$$
   with N_t Poisson with parameter λ and independent of the Y_i which are iid mean zero and variance σ^2.

   a. Show that X_s and $X_t - X_s$ are independent

   b. Find the quadratic variation associated with this process, denote it $\langle X \rangle_t$

   c. Compute $\mathbb{E}[X_t^2 - \langle X \rangle_t]$.