130B - Homework 6

1. Suppose we perform a sequence of independent Bernoulli trials, each with success probability p. We fix a positive integer r and keep performing trials until we observe r successes. Let X be the random variable that counts the number of trials needed to observe r successes.

   (a) Compute the probability mass function of X,
   $$f(k; r, p) = \Pr[X = k].$$

   (b) Compute the mean and variance of X. *Hint: An efficient way to do this is to compute the moments, $E[X^k]$ for any positive integer k.*

   (c) In words, how is X related to a binomial random variable?

2. Suppose we have a bin containing N balls, m of which are red and $N - m$ are blue. Draw a sample of n balls (without replacement) and let X be the number of red balls in the sample.

   (a) Compute the probability mass function of X,
   $$f(k; N, m, n) = \Pr[X = k].$$

   (b) Compute the mean and variance of X. *Hint: Same hint as in 1(b).*

   (c) In words, how is X related to a binomial random variable?

3. Suppose a laser pointer sits at a unit distance from the x-axis. We spin the laser about its center and consider the point X at which the beam intersects the x-axis once the beam has stopped spinning. If the beam doesn't point toward the x-axis, just repeat the experiment.

   (a) Find the probability density function, $f(x)$, of X. The laser setup is illustrated in Figure 1, where θ is the angle the beam makes with the y-axis. Assume that θ is uniformly distributed between $-\pi/2$ and $\pi/2$.

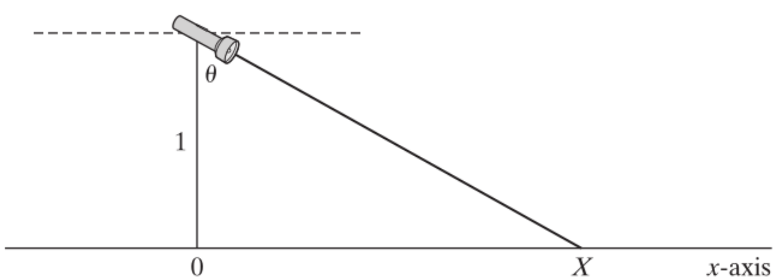

Figure 1: Laser setup

   (b) Find the expectation $E[X]$.

4. Let S be a semicircle of unit radius on a diameter D.

(a) A point P is picked at random on D. If X is the distance from P to S along the perpendicular to D, show that $E[X] = \pi/4$.

(b) A point Q is picked at random on S. If Y is the perpendicular distance from Q to D, show that $E[Y] = 2/\pi$.