REAL ANALYSIS
MATH 205/H140, HW#5

Chapter 4, exercises 41, 46, 49, 59, 60, 69; Chapter 5, exercises 1, 2, and the following problems:

Problem 1.

Let (M, d) be a metric space. Let us denote by ∂A the boundary of A, by $\bar{A}$ the closure of A, and by int A the interior of A. Prove or disprove (i.e. provide a counterexample) each of the following statements:

a) $\partial A = \partial \bar{A}$

b) $\text{int}\,(\partial A) = \emptyset$

c) $\partial(\partial A) = \partial A$

d) $\partial A = \partial(\text{int}\, A)$

e) $\partial(\partial(\partial A)) = \partial(\partial A)$

Problem 2.

Prove or disprove that for any closed subset $F \subset \mathbb{R}$ of the real line, there exists a set $A \subset \mathbb{R}$ such that $\partial A = F$.