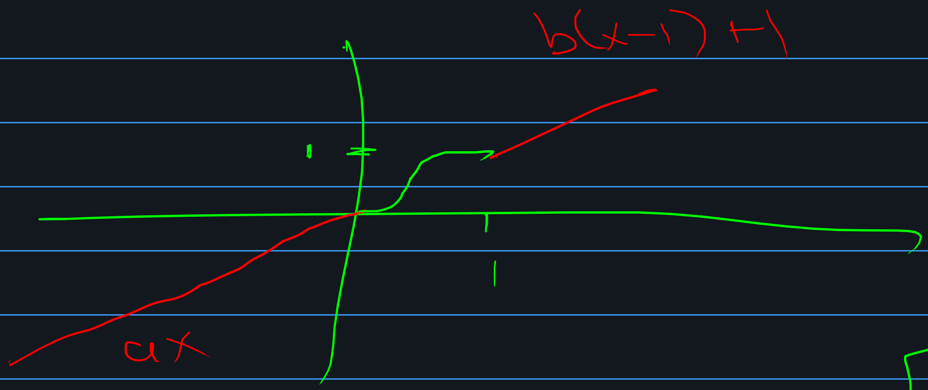
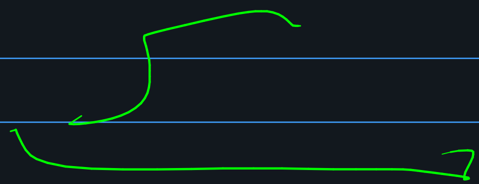
want to smoothly go from

$y = ax$ to $(y-1) = b(x-1)$





$\psi(x) = e^{-1/x}$ smoothly goes to zero

$\rightsquigarrow \psi(x) = \psi(x+1)\,\psi(1-x)$

Adapt Lemma 11.12 

$e^{-1/x} \longrightarrow 0$ at 0 w/ 0 slope

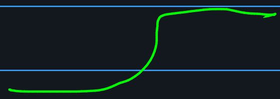
make it go to 0 w/ different slope

$ce^{-1/x} + ax$

$L_x \quad \dfrac{c e^{-1/x}}{x^2}$

$\downarrow$

0



Problem 1.

TRUE or FALSE: If $\{f_n\}$, $f_n : [0,1] \to \mathbb{R}$, is a sequence of functions such that for some $M > 0$, $L > 0$ for all $x, y \in [0,1]$ and $n \in \mathbb{N}$ we have

$$|f_n(x) - f_n(y)| \leq L|x-y|, \quad |f_n(x)| \leq M,$$

then there exists a subsequence that converges in the norm $\|\cdot\|_{BV}$.

equicts

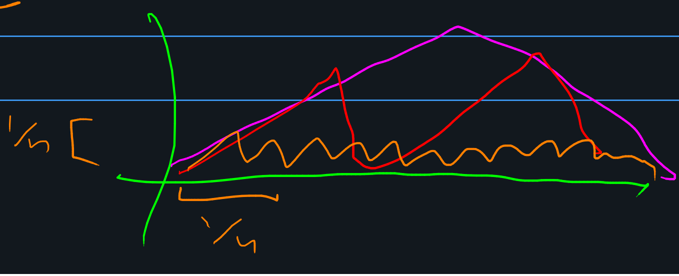
A-A $\Longrightarrow$ wlog $f_n \rightrightarrows f$ (actually, a subseq)

if $f_n \xrightarrow{BV} g$, $g = f$ Lemma 13.2

we know $\forall f_n, g$ $\|f_n - g\|_\infty \leq \|f_n - g\|_{BV}$

can we find wlog $f_n \rightrightarrows 0$, but $f_n \not\to 0$ in BV?

(do something w/ hat functions)



6. We can test several of the inclusions implicit in our discussion up to this point by means of a single family of functions. For $\alpha \in \mathbb{R}$ and $\beta > 0$, set $f(x) = x^{\alpha} \sin(x^{-\beta})$, for $0 < x \le 1$, and $f(0) = 0$. Show that:

(a) f is bounded if and only if $\alpha \ge 0$.

(b) f is continuous if and only if $\alpha > 0$.

(c) $f'(0)$ exists if and only if $\alpha > 1$.

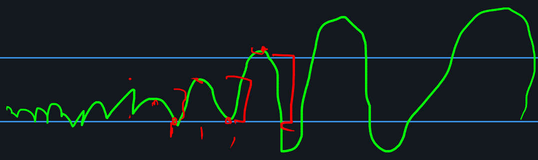
(d) f' is bounded if and only if $\alpha \ge 1 + \beta$.

(e) If $\alpha > 0$, then $f \in BV[0,1]$ for $0 < \beta < \alpha$ and $f \notin BV[0,1]$ for $\beta \ge \alpha$.
[Hint: Try a few easy cases first, say $\alpha = \beta = 2$.]

manually choose a partition

$x^{\alpha} \sin(x^{-\beta})$

Peak when $\sin(x^{-\beta}) = 1$



$\iff x = \left(\frac{\pi}{2} + 2k\pi\right)^{-1/\beta}$

valley $\iff x = \left(\frac{3\pi}{2} + 2\pi k\right)^{-1/\beta}$

choose a partition of these

$$|f(x_n) - f(x_{n-1})| = |x_n^{\alpha} \sin(x_n^{-\beta}) - x_{n-1}^{\alpha} \sin(x_{n-1}^{-\beta})|$$

$$= |x_n^{\alpha} + x_{n-1}^{\alpha}|$$

Series problem

$\sqrt{x}$ is BV(?) but its deriv is unbdd

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even if g unbdd

if $|f'| \le |g|$ that is integrable,

then f is BV?

idea $V_0^1 f \le \int |g|$