Finding and Counting Substructures in Graphs and Hypergraphs

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1. A Finding Problem

Quick Definitions

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- picture here

History and Motivation

Theorem (L. Lovász, T. Gallai - 1979)

Let $G = (V, E)$ be any graph. Then G admits a partitioning of its vertex set into two parts, $V = V_1 \cup V_2$, so that each vertex in $G[V_1]$ and each vertex in $G[V_2]$ has even degree. In particular, any graph on n vertices has an even subgraph of order at least $n/2$.

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Proof sketch:

asdf $\square$

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Every graph $G(V, E)$ on n vertices, none of which are isolated, contains a set $W \subseteq V(G)$ such that $|W| \geq \frac{n}{900 \log n}$ and $G[W]$ has all degrees odd.

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Conjecture (A. Scott - 2001)

There exists some constant $c > 0$ such that every graph $G(V,E)$ on n vertices, none of which are isolated, contains a set $W \subseteq V(G)$ such that $|W| \geq cn$ and $G[W]$ has all degrees odd.