Real Analysis
Math 205B/H140B, HW#5

Chapter 12, exercises 27, 28; Chapter 13, exercises 3, 6, 12, 13, 14, 19, and the following problems:

Problem 1.

TRUE or FALSE: If $\{f_n\}$, $f_n : [0,1] \to \mathbb{R}$, is a sequence of functions such that for some $M > 0$, $L > 0$ for all $x, y \in [0,1]$ and $n \in \mathbb{N}$ we have

$$|f_n(x) - f_n(y)| \leq L|x - y|, \quad |f(x)| \leq M,$$

then there exists a subsequence that converges in the norm $\|\cdot\|_{BV}$.

Problem 2.

The norm $\|\cdot\|_{BV}$ in $BV[0,1]$ was defined as

$$\|f\|_{BV} = |f(0)| + V_0^1(f).$$

Show that a norm $|||\cdot|||$ given by $|||f||| = |f(1)| + V_0^1(f)$ is equivalent to the norm $\|\cdot\|_{BV}$.