Real Analysis
Math 205B/H140B, HW#4

Chapter 12, exercises 4, 6, 7, 10, 14, and the following problems:

Problem 1.

Use the Stone-Weierstrass Theorem to confirm that piecewise-linear continuous functions are dense in $C[0,1]$.

Problem 2.

Prove that any continuous function on $[0,1]$ can be uniformly approximated by a finite linear combination of exponential functions.

Problem 3.

Consider the collection of functions

$$A = \{f : [0,1] \to \mathbb{R} \mid f(x) = e^x p(x),\ p \text{ is a polynomial}\}.$$

Is A a linear subspace in $C[0,1]$? Is A a subalgebra of $C[0,1]$? Is A dense in $C[0,1]$?

Problem 4.

Let $A \subseteq C[0,1]$ be the set

$$A = \{p : [0,1] \to \mathbb{R} \mid p \text{ is a polynomial},\ p'(0) = 0\}.$$

Is A a linear subspace in $C[0,1]$? Is A a subalgebra of $C[0,1]$? Is A dense in $C[0,1]$?

Problem 5.

Let us recall that by $C_{1/3}$ we denoted the standard Cantor set. Let $A \subseteq C(C_{1/3})$ be the set of real valued locally constant functions (a function $f : C_{1/3} \to \mathbb{R}$ is locally constant if for any $x \in C_{1/3}$ there exists an open neighborhood $U \subset C_{1/3}$, $x \in U$, such that $f|_U$ is a constant function). Is A a linear subspace in $C(C_{1/3})$? Is A a subalgebra of $C(C_{1/3})$? Is A dense in $C(C_{1/3})$?