Real Analysis
Math 205/H140, HW#2

Chapter 1, exercises 34, 35, 36, 47; Chapter 2, exercises 9, 17, and the following problems:

Problem 1.

Suppose $\{a_{n,m}\}_{n,m\in\mathbb{N}}$ is a bounded collection of real numbers. Prove or disprove (provide a counterexample) each of the following statements:

a) $\limsup_{n\to\infty}(\liminf_{m\to\infty} a_{n,m}) = \liminf_{m\to\infty}(\limsup_{n\to\infty} a_{n,m})$;

b) $\limsup_{n\to\infty}(\liminf_{m\to\infty} a_{n,m}) \geq \liminf_{m\to\infty}(\limsup_{n\to\infty} a_{n,m})$;

c) $\limsup_{n\to\infty}(\liminf_{m\to\infty} a_{n,m}) \leq \liminf_{m\to\infty}(\limsup_{n\to\infty} a_{n,m})$.

Problem 2.

Prove that for any real numbers $x, y \in \mathbb{R}$ the sequence $\{a_n\}_{n\in\mathbb{N}}$ defined by

$$a_1 = x, a_2 = y, a_{n+2} = \frac{a_n + a_{n+1}}{2}, n \geq 1,$$

converges. Find the limit.

Problem 3.

Prove that any two open subsets of $\mathbb{R}^2$ are equivalent (i.e. have the same cardinality).

Problem 4.

Is it possible for a continuous function $f : \mathbb{R}^2 \to \mathbb{R}$ to have uncountably many strict local minima?