a) $\partial A = \partial(\bar{A})$?

T? : $x \in \partial A \implies x \in \bar{A}$

$$\bar{A} = A^\circ \cup \partial A$$
$$= A \cup A'$$

$A' \rightarrow$ limit points

F: $M = \mathbb{R}$
$A = \mathbb{Q} \cap [0,1]$

$\partial A = [0,1]$
$\partial(\bar{A}) = \partial[0,1] = \{0,1\}$

4.49 ℓ_2 & H^∞ are separable

have countable dense subsets

(ℓ_2, d_2)

$$d_2((a_n),(b_n)) = \left(\sum |a_n - b_n|^2 \right)^{1/2}$$

(H^∞, d_∞)

$$d_\infty((a_n),(b_n)) = \sum 2^{-n} |a_n - b_n|$$

Idea: Show the $M (= l_2 \text{ or } H^\omega)$

$\xi \forall \varepsilon > 0$ that

$B(x, \varepsilon)$ hits an element of

the proposed countable set.

l_2: Let $S_n = \left\{ (r_1, r_2, \ldots, r_n, 0, 0 \ldots) \right\}$
                          s.t. $r_i \in \mathbb{Q}$

$\cdot S_n$ is countable

$\Rightarrow$ So is $\displaystyle\bigcup_{n=1}^{\infty} S_n = D$

Pick $(a_n) \in l_2$ & let $\varepsilon > 0$ be

given. WTS $\exists (q_n) \in D$ s.t.

$d_2 ((a_n), (q_n)) < \varepsilon$

Idea: since $(a_n) \in l_2$ its tail most
      eventually be small (after N)

$\cdot$ Pick $r_i \in \mathbb{Q}$ s.t. $|r_i - a_i|$ small
              $\forall i \leq N$

pick N s.d. $\sum\limits_{N}^{\infty} |a_n|^2 < \varepsilon/2 \quad \leq \min\left(\frac{\varepsilon}{2}, \frac{1}{2}\right)$

" now pick $r_i \in \mathbb{Q}$ s.d.

$$|a_i - r_i|^2 \leq \varepsilon/2N \quad \left(\begin{array}{c}\text{can do since} \\ \overline{\mathbb{Q}} = \mathbb{R}\end{array}\right)$$

$\Rightarrow$ set $(q_n) = (r_1, \ldots, r_N, 0, 0, \ldots)$

$$\in D$$

& $d_2^2\left((a_n), (q_n)\right)$

$$= \sum\limits_{i=1}^{N} |a_i - r_i|^2 + \sum\limits_{i=N+1}^{\infty} |a_i|^2$$

$$< \sum\limits_{i=1}^{N} \varepsilon/2N + \varepsilon/2$$

$$\leq \varepsilon$$

1e) $\partial\partial\partial A \overset{?}{=} \partial\partial A$

idea: ∂F is nwd if F closed

$$\partial A = \overline{A} \setminus A^\circ = \overline{A} \cap \left(\overline{A^\circ}\right)^c$$

we say that a map $\overset{cts}{f}$

$f : X \longrightarrow Y$ of topological spaces

is open if it maps open sets to open sets.

a) ex: identity map

b) non-ex

a constant map into a space where singletons are not open

Claim: any map into a space equipped w/ discrete topology ⊃ open

· $f : \mathbb{R} \longrightarrow \mathbb{R} \qquad x \longmapsto x^2$

$$f(\mathbb{R}) = [0, \infty)$$
$$f((-1,1)) = [0, 1)$$

c) Show that if

$f: \mathbb{R} \to \mathbb{R}$ is cts & open

$\Rightarrow f$ is strictly monotone

suppose f not strict mono.

$\Rightarrow \exists x < y$ s.t. $f(x) = f(y)$

go from here . . . . ~ .