REAL ANALYSIS
MATH 205/H140, HW#9

Chapter 8, exercises 67, 68, 70, 75, 78, 81, 89, and the following problems:

Problem 1.

Is l^1 dense in l^∞? If not, what is the closure of l^1 in l^∞?

Problem 2.

Let h be a continuous function on $[0,1]$. Consider $C[0,1]$ - the space of continuous functions on $[0,1]$, equipped with the norm $\|f\| = \max_{x\in[0,1]} |f(x)|$. Let $T : C[0,1] \to \mathbb{R}$ be a linear functional given by

$$T(f) = \int_0^1 h(t)f(t)dt$$

Prove that T is a bounded functional and find $\|T\|$.

Problem 3.

Let $K(s,t)$ be a continuous function on $[0,1] \times [0,1]$. Consider $C[0,1]$ - the space of continuous functions on $[0,1]$, equipped with the norm $\|f\| = \max_{x\in[0,1]} |f(x)|$. Let $T : C[0,1] \to C[0,1]$ be a map given by

$$T(f)(s) = \int_0^1 K(s,t)f(t)dt$$

Prove that T is a bounded linear operator and find $\|T\|$.