REAL ANALYSIS
MATH 205/H140, HW#4

Chapter 3, exercises 5, 8, 11, 19, 23, 34, 46, and the following problems:

Problem 1.

Let d_1, d_2, d_{max} be the metrics on $\mathbb{R}^2$ defined by

$$\begin{aligned} d_1(x,y) &= |x_1 - y_1| + |x_2 - y_2|, \\ d_2(x,y) &= \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}, \\ d_{max} &= \max(|x_1 - y_1|, |x_2 - y_2|), \end{aligned}$$

where $x = (x_1, x_2), y = (y_1, y_2)$. Given a pair of points x and y in $\mathbb{R}^2$, describe the set

$$\{z \in \mathbb{R}^2 \mid \rho(x,y) = \rho(x,z) + \rho(z,y)\}$$

for each choice of the metric $\rho = d_1, d_2, d_{max}$.

Problem 2.

Suppose d', d'' are two metrics on the same set M. Suppose there exits $C > 1$ such that for any $x, y \in M$ we have

$$C^{-1} d'(x,y) \leq d''(x,y) \leq C d'(x,y).$$

Prove that the metrics d' and d'' are equivalent.

Problem 3.

Suppose d' and d'' are two equivalent metrics on M. Is it true that there exits $C > 1$ such that for any $x, y \in M$ one has

$$C^{-1} d'(x,y) \leq d''(x,y) \leq C d'(x,y) \ ?$$

Prove or give a counterexample.