Real Analysis
Math 205/H140, HW#3

Chapter 2, exercises 24, 26, 31, 34, and the following problems:

Problem 1.

Show that $\frac{1}{4}$ belongs to the standard Cantor set $C_{1/3}$.

Problem 2.

Let $f : [0,1] \to [0,1]$ be the Cantor function. Set $g(x) = f(x) + x$, $g : [0,1] \to [0,2]$. What is the total length of the images (under the map g) of all the intervals in the complement to the standard Cantor set?

Problem 3.

Denote by $C_{1/4}$ the set of points in $[0,1]$ that can be represented in base 4 as

$$0.a_1a_2a_3\ldots,\ a_i \in \{0,3\},$$

i.e. without using the digits 1 and 2. Show that $C_{1/4} + C_{1/4}$ does not contain any interval.

Problem 4.

Denote by $C^t_{1/3}$ the shift of the standard Cantor set to the right by $t > 0$, i.e.

$$C^t_{1/3} = \{x + t \mid x \in C_{1/3}\}.$$

Prove that for any $t \in [0,1]$ we have

$$C_{1/3} \cap C^t_{1/3} \neq \emptyset.$$

Problem 5.

Prove that the interval $[0,1]$ has a subset that is non-empty, closed, perfect, and contains only irrational points.