REAL ANALYSIS
MATH 205/H140, HW#1

Chapter 1, exercises 4, 5, 6, 9, 16, 24, and the following problems:

Problem 1.

Suppose $\limsup_{n\to\infty} a_n = a$, and $\limsup_{n\to\infty} b_n = b$. Is it true that

$$\limsup_{n\to\infty}(a_n + b_n) = a + b \quad ?$$

Prove or give a counterexample.

Problem 2.

Suppose $\limsup_{n\to\infty} a_n = a$, and $\limsup_{n\to\infty} b_n = b$. Is it true that

$$\limsup_{n\to\infty} \max(a_n, b_n) = \max(a, b) \quad ?$$

Prove or give a counterexample.

Problem 3.

Suppose $\limsup_{n\to\infty} a_n = a$, and $\limsup_{n\to\infty} b_n = b$. Is it true that

$$\limsup_{n\to\infty} \min(a_n, b_n) = \min(a, b) \quad ?$$

Prove or give a counterexample.

Problem 4.

Suppose $a_n, b_n > 0$ for each $n \in \mathbb{N}$, and $\limsup_{n\to\infty}(a_n)^{\frac{1}{n}} = a$, and $\limsup_{n\to\infty}(b_n)^{\frac{1}{n}} = b$. Find $\limsup_{n\to\infty}(a_n + b_n)^{\frac{1}{n}}$. Explain your answer.