Problem 5.

Let us recall that by $C_{1/3}$ we denoted the standard Cantor set. Let $A \subseteq C(C_{1/3})$ be the set of real valued locally constant functions (a function $f : C_{1/3} \to \mathbb{R}$ is locally constant if for any $x \in C_{1/3}$ there exists an open neighborhood $U \subset C_{1/3}$, $x \in U$, such that $f|_U$ is a constant function). Is A a linear subspace in $C(C_{1/3})$? Is A a subalgebra of $C(C_{1/3})$? Is A dense in $C(C_{1/3})$?

a) subspace? ie, let f, g locally const, is $f+g$ locally const?

- Let $x \in C_{1/3}$ wts $\exists$ nbhd W of x s.t. $f+g$ is const on W
- f L.C. $\Longrightarrow$ $\exists$ nbhd U of x st f const on U

g L.C. $\Longrightarrow$ " V of x s.t. g const on V

$\Longrightarrow$ $f+g$ const on $U \cap V$.

nonempty open since it contains x.

b) algebra? wts $f \cdot g$ L.C.

same proof of (a).

c)

c) dense in $C(C_{1/3})$?

Q: what do L.C. fns look like on $[0,1]$?

A: constant, idea: $[0,1]$ connected.

& can't be L.C. at a jump.

Can you come up w/ a space where the locally const functions are more interesting?

eg. $[0,1] \cup [2,3]$

the LC fns here take some const value on $[0,1]$ & different const on $[2,3]$.

suppose $\overline{LC \text{ functions}} = C(C_{1/3})$

how to prove?

Stone–Weierstrass?

S–W says it's dense if

i) separates pts ($\forall x \neq y, \exists$ LC fn, f st $f(x) \neq f(y)$)

ii) nowhere vanishing

$\hookrightarrow \forall x \in C_{1/3}, \exists$ LC fn that doesn't vanish there.

(ii) is easy: any constant ($\neq 0$) function.

(i) not as easy: $C_{1/3}$ important here

Hint: look at

$C_1 = [0,1]$

$C_2 = [0, \frac{1}{3}] \cup [\frac{2}{3}, 1]$

$\vdots$

other idea: use uniform continuity of $f \in C(C_{1/3})$ to build a locally const function that approximates within ε of f.

Problem 1.

Use the Stone-Weierstrass Theorem to confirm that piecewise-linear continuous functions are dense in $C[0,1]$.

The Stone–Weierstrass Theorem, real scalars 12.9. *Let X be a compact metric space, and let A be a subalgebra of $C(X)$. If A separates points in X and vanishes at no point of X, then A is dense in $C(X)$.*

show PWL fns separate points

& nowhere vanishing.

show PWL are a subalg

(they aren't: eg. $f(x) = x$)

non S-W idea: $f \in C([0,1])$

unifly cts. use this to build

an approxing PWL guy.

Dini's thm: suppose f_n cts

converges to f cts on cpct

space X, where $f_n(x) \leq f_{n+1}(x)$ $\geq$

then $f_n \rightrightarrows f$.