Real Analysis
Math 205B/H140B, HW#8

Chapter 14, exercises 52, 53, 55, 56, and the following problems:

Problem 1.

For the linear functional $L : C[0,1] \to \mathbb{R}$,

$$L(f) = f(0) + f(1/2) + f(1),$$

find a function $\alpha \in BV[0,1]$ such that $L(f) = \int_0^1 f d\alpha$ for all $f \in C[0,1]$.

Problem 2.

For the linear functional $L : C[0,1] \to \mathbb{R}$,

$$L(f) = \int_0^{0.3} f(x)dx - \int_{0.6}^{1} f(x)dx,$$

find a function $\alpha \in BV[0,1]$ such that $L(f) = \int_0^1 f d\alpha$ for all $f \in C[0,1]$.

Problem 3.

For the linear functional $L : C[0,1] \to \mathbb{R}$,

$$L(f) = \sum_{n=1}^{\infty} \int_{\frac{1}{n+1}}^{\frac{1}{n}} \frac{f(x)}{2^n} dx,$$

find a function $\alpha \in BV[0,1]$ such that $L(f) = \int_0^1 f d\alpha$ for all $f \in C[0,1]$.

Problem 4.

For the linear functional $L : C[0,1] \to \mathbb{R}$,

$$L(f) = \sum_{n=1}^{\infty} \int_{\frac{1}{n+1}}^{\frac{1}{n}} (-1)^n \frac{f(x)}{2^n} dx,$$

find a function $\alpha \in BV[0,1]$ such that $L(f) = \int_0^1 f d\alpha$ for all $f \in C[0,1]$.

Problem 5.

For the linear functional $L : C[0,1] \to \mathbb{R}$,

$$L(f) = \int_0^{1/2} \frac{f(x) + 2f(1-x)}{3} dx,$$

find a function $\alpha \in BV[0,1]$ such that $L(f) = \int_0^1 f d\alpha$ for all $f \in C[0,1]$.

Problem 6.

Let f_n, $n = 1, 2, 3, \ldots$, and f be Riemann integrable real-valued functions defined on $[0, 1]$. For each of the following statements, determine whether the statement is true or not:

(a) If $\lim_{n\to\infty} \int_0^1 |f_n(x) - f(x)|\, dx = 0$ then $\lim_{n\to\infty} \int_0^1 |f_n(x) - f(x)|^2\, dx = 0$;

(b) If $\lim_{n\to\infty} \int_0^1 |f_n(x) - f(x)|^2\, dx = 0$ then $\lim_{n\to\infty} \int_0^1 |f_n(x) - f(x)|\, dx = 0$.