

Math 274 - Homework 2

Problem 1. Let $v_1 = (x_1, y_1), \dots, v_n = (x_n, y_n)$ be n vectors in \mathbb{Z}^2 , where each x_i and each y_i is a positive integer that does not exceed $\frac{2^{n/2}}{10\sqrt{n}}$. Show that there exist two disjoint nonempty subsets $I, J \subseteq [n]$ such that $\sum_{i \in I} v_i = \sum_{j \in J} v_j$.

Proof. Consider the random sum $X = \sum_{i=1}^n \epsilon_i v_i$ where each ϵ_i is an iid Bernoulli random variable with success probability $1/2$. We'll show that a sizeable proportion of the possible X_i live in an axis-aligned box centered about the mean of X . If the size of this box is smaller than the number of assignments of the ϵ_i 's that make X land in this box, then there must be two assignments of the ϵ_i 's that give the same realization of X .

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