Math 130B - Integration Review

1. Suppose the joint distribution of the continuous random variables X and Y is given by

$$f(x,y) = \begin{cases} \lambda^2 e^{-\lambda(x+y)}, & \text{if } x, y \geq 0 \\ 0, & \text{otherwise} \end{cases},$$

   for some constant $\lambda > 0$.

   (a) Find the marginal densities f_X and f_Y.

   (b) Find the probability that $X > Y$.

2. Let X and Y be independent continuous random variables with densities f_X and f_Y, respectively. Express the density of XY in terms of the densities of X and Y.

3. Suppose that A, B, C are independent random variables, each being uniformly distributed over $(0, 1)$.

   (a) What is the joint cumulative distribution function of A, B, C?

   (b) What is the probability that the roots of the polynomial $Ax^2 + BX + C$ are all real?

4. If X, Y, Z are independent random variables that are uniformly distributed over $(0, 1)$, compute the probability that the largest of the three is greater than the sum of the other two.

5. Let X and Y be the coordinates of a point uniformly chosen in the circle of radius 1 centered at the origin, i.e., their joint density is

$$f(x,y) = \begin{cases} \frac{1}{\pi}, & \text{if } x^2 + y^2 \leq 1 \\ 0, & \text{otherwise} \end{cases}.$$

   Find the joint density function of the polar coordinates $R = \sqrt{X^2 + Y^2}$ and $\Theta = \tan^{-1}(Y/X)$.