Math 130B - Variance, Covariance

1. If X and Y are independent and identically distributed with mean μ and variance σ^2, find

$$\mathbb{E}[(X-Y)^2].$$

2. Show that $\mathbb{E}[(X-a)^2]$ is minimized when $a = \mathbb{E}[X]$.

3. Let $X_1, \ldots, X_n$ be iid continuous random variables. We say that a record value occurs at time i, $i \leq n$ if $X_i \geq X_k$ for all $k \leq i$. Show that

   (a) $\mathbb{E}[\text{number of record values}] = \sum_{i=1}^{n} 1/i$.

   (b) $\text{Var}[\text{number of record values}] = \sum_{i=1}^{n} (i-1)/i^2$.

4. Suppose that X and Y are identically distributed, but not necessarily independent. Show that $X+Y$ and $X-Y$ are uncorrelated.

5. A multilevel marketing firm operates as follows. Person 1 starts the firm, then recruits person 2. Persons 1 and 2 then compete to recruit person 3 (who is always recruited in the end). Then persons 1, 2 and 3 compete to recruit person 4, and so on. Suppose that when persons 1 through i compete to recruit person $i+1$, they are all equally likely to succeed (but one of them for sure succeeds). This goes on until n people work at the firm.

   (a) Find the expected number of people $1, \ldots, n$ who did not recruit anyone else.

   (b) Come up with an expression for the variance of the number of people who don't recruit anyone.