Math 130B - MGFs and Multivariate Normal RVs

1. The moment generating function of X is given by $M_X(t) = \exp[2e^t - 2]$ and that of Y by $M_Y(t) = (\frac{3}{4}e^t + \frac{1}{4})^{10}$. If X and Y are independent, what are

   (a) $\Pr[X + Y = 2]$?

   (b) $\Pr[XY = 0]$?

   (c) $\mathbb{E}[XY]$.

2. Let X be the value of the first die and Y the sum of the values when two dice are rolled. Compute the joint moment generating function of X and Y.

3. Successive weekly sales, in units of \$1000, have a bivariate normal distribution with common mean 40, common standard deviation 6, and correlation 0.6.

   (a) Find the probability that the total of the next 2 weeks' sales exceeds 90.

   (b) If the correlation was 0.2 rather than 0.6, do you think that this would increase or decrease the answer to (a)?

4. Suppose that Y is a normal random variable with mean μ and variance σ^2, and suppose also that the conditional distribution of X, given that $Y = y$, is normal with mean y and variance 1.

   (a) Argue that the joint distribution of X, Y, is the same as that of $Y + Z, Y$ where Z is a standard normal random variable independent of Y.

   (b) Argue that X, Y has a bivariate normal distribution.

   (c) Find $\mathbb{E}[X]$, $\mathrm{Var}[X]$ and $\mathrm{Cov}(X, Y)$.

   (d) Find $\mathbb{E}[Y \mid X = x]$.

   (e) What is the conditional distribution of Y given that $X = x$.