Quiz 8

Form A | Name ____________________
Math 130B, 5 PM
Please justify all your answers | May 25, 2022
Please also write your full name on the back

1. Suppose X and Y are independent random variables with the following moment generating functions.
$$m_X(t) = e^{2t^2}, \quad m_Y(t) = \frac{3}{3-t}.$$
   (a) What kinds of random variables are X and Y? (There's a table on the back of the quiz).

   (b) Find $\mathbb{E}[(X+Y)^2]$ (there are a couple of ways to do it).

2. Let X and Y be jointly bivariate normal random variables with $\mathrm{Var}(X) = \mathrm{Var}(Y)$. Show that $X+Y$ and $X-Y$ are independent.

Quiz 8

Form B
Math 130B, 6 PM
Please justify all your answers
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Name ______________________

May 25, 2022

1. Suppose X and Y are independent random variables with the following moment generating functions.

$$m_X(t) = \exp[2(e^t - 1)], \quad m_Y(t) = \left(\frac{1}{3}e^t + \frac{2}{3}\right)^5.$$

   (a) What kinds of random variables are X and Y? (There's a table on the back of the quiz).

   (b) Find $\mathbb{E}[(X+Y)^2]$ (there are a couple of ways to do it).

2. Let X and Y be jointly bivariate normal random variables with $\mathrm{Var}(X) = \mathrm{Var}(Y)$. Show that $X+Y$ and $X-Y$ are independent.

Table 7.1 Discrete Probability Distribution.

	Probability mass function, $p(x)$	Moment generating function, $M(t)$	Mean	Variance
Binomial with parameters n, p; $0 \leq p \leq 1$	$\binom{n}{x} p^x (1-p)^{n-x}$ $x = 0, 1, \ldots, n$	$(pe^t + 1 - p)^n$	np	$np(1-p)$
Poisson with parameter $\lambda > 0$	$e^{-\lambda} \frac{\lambda^x}{x!}$ $x = 0, 1, 2, \ldots$	$\exp\{\lambda(e^t - 1)\}$	λ	λ
Geometric with parameter $0 \leq p \leq 1$	$p(1-p)^{x-1}$ $x = 1, 2, \ldots$	$\frac{pe^t}{1-(1-p)e^t}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Negative binomial with parameters r, p; $0 \leq p \leq 1$	$\binom{n-1}{r-1} p^r (1-p)^{n-r}$ $n = r, r+1, \ldots$	$\left[\frac{pe^t}{1-(1-p)e^t}\right]^r$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$

Table 7.2 Continuous Probability Distribution.

	Probability density function, $f(x)$	Moment generating function, $M(t)$	Mean	Variance
Uniform over (a, b)	$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$	$\frac{e^{tb} - e^{ta}}{t(b-a)}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential with parameter $\lambda > 0$	$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$	$\frac{\lambda}{\lambda - t}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Gamma with parameters $(s, \lambda), \lambda > 0$	$f(x) = \begin{cases} \frac{\lambda e^{-\lambda x}(\lambda x)^{s-1}}{\Gamma(s)} & x \geq 0 \\ 0 & x < 0 \end{cases}$	$\left(\frac{\lambda}{\lambda - t}\right)^s$	$\frac{s}{\lambda}$	$\frac{s}{\lambda^2}$
Normal with parameters (μ, σ^2)	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \quad -\infty < x < \infty$	$\exp\left\{\mu t + \frac{\sigma^2 t^2}{2}\right\}$	μ	σ^2