

Math 130B - Midterm Review Problems

1. A model proposed for NBA basketball supposed that when two teams with roughly the same record play each other, the number of points scored in a quarter by the home team minus the number scored by the visiting team is approximately a normal random variable with mean 1.5 and variance 6. Additionally, the model supposes that the point differentials for the four quarters are independent. Suppose we are working within this model.

- (a) What is the probability that the home team wins?
- (b) What is the conditional probability that the home team wins, given that it is behind by 5 points at halftime.
- (c) What is the conditional probability that the home team wins, given that it is ahead by 5 points at the end of the first quarter?

2. Let $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$ be the ordered values of n independent uniform $(0, 1)$ random variables.

- (a) Prove that for $1 \leq k \leq n + 1$,

$$\Pr[X_{(k)} - X_{(k-1)} > t] = (1 - t)^n,$$

where $X_{(0)} \equiv 0$ and $X_{(n+1)} \equiv 1$.

- (b) What is the distribution for the median of the sample? *That is, find the distribution for the median of $X_{(n)} - X_{(1)}$.*

3. Suppose X is a random variable taking only nonnegative integer values. Show that if $\mathbb{E}[X] < 1$, then $\Pr[X = 0] > 0$.

4. Suppose X and Y are jointly continuous random variables with joint pdf given by

$$f(x, y) = ce^{-(x+y)}, \quad 0 \leq y \leq x < \infty$$

- (a) Find the value of the constant c .

- (b) Find the conditional pdf of Y conditioned on $X = x$.

- (c) Are X and Y independent?

- (d) Suppose $U = X + 2Y$ and $V = X - Y$. Find the joint pdf of U and V . Be sure to specify the domain!