Math 130B - Conditional Distributions

1. The joint density function of X and Y is given by

$$f(x,y) = xe^{-x(y+1)}, \quad x > 0,\ y > 0.$$

   (a) Find the conditional density of X, given that $Y = y$, and that of Y, given $X = x$.

   (b) Find the density function of $Z = XY$.

2. Find the distribution of the range of a sample of size 2 from a distribution having density function $f(x) = 2x$, $0 < x < 1$. What about for n samples? *That is, find the distribution of the largest observation minus the smaller.*

3. Let X and Y denote the coordinates of a point uniformly chosen inside the circle of radius 1 centered at the origin. Find the joint density function of the polar coordinates $R = X^2 + Y^2$, $\Theta = \tan^{-1} Y/X$.

4. Suppose that X and Y are independent geometric random variables with the same parameter p.

   (a) Without any computations, what do you think is the value of

$$\Pr[X = i \mid X + Y = n].$$

   (b) Prove your guess.

5. Let $Z_1, Z_2, \ldots, Z_n$ be independent standard normal random variables and let

$$S_n = \sum_{i=1}^{n} Z_i.$$

   (a) What is the conditional distribution of S_n given that $S_k = y$ for $k = 1, \ldots, n$?

   Show that for $1 \leq k \leq n$, the conditional distribution of S_k given that $S_n = x$ is normal with mean xk/n and variance $k(n-k)/n$.