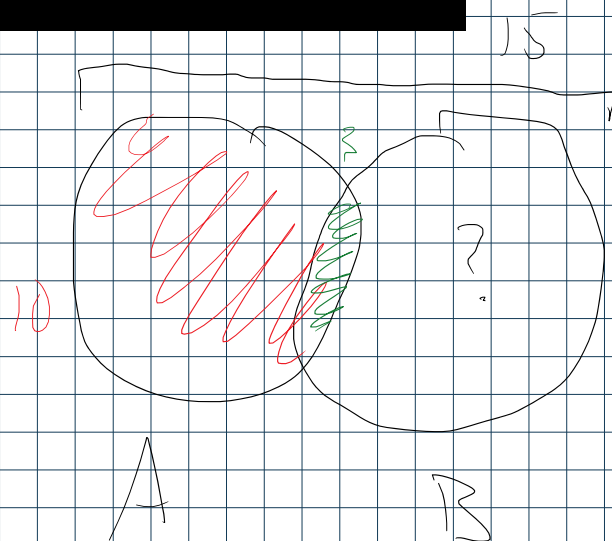


Today: Canvas Worksheet

Files > Discussion Section A > 2-1-

! Don't do all of the multi-part problems !

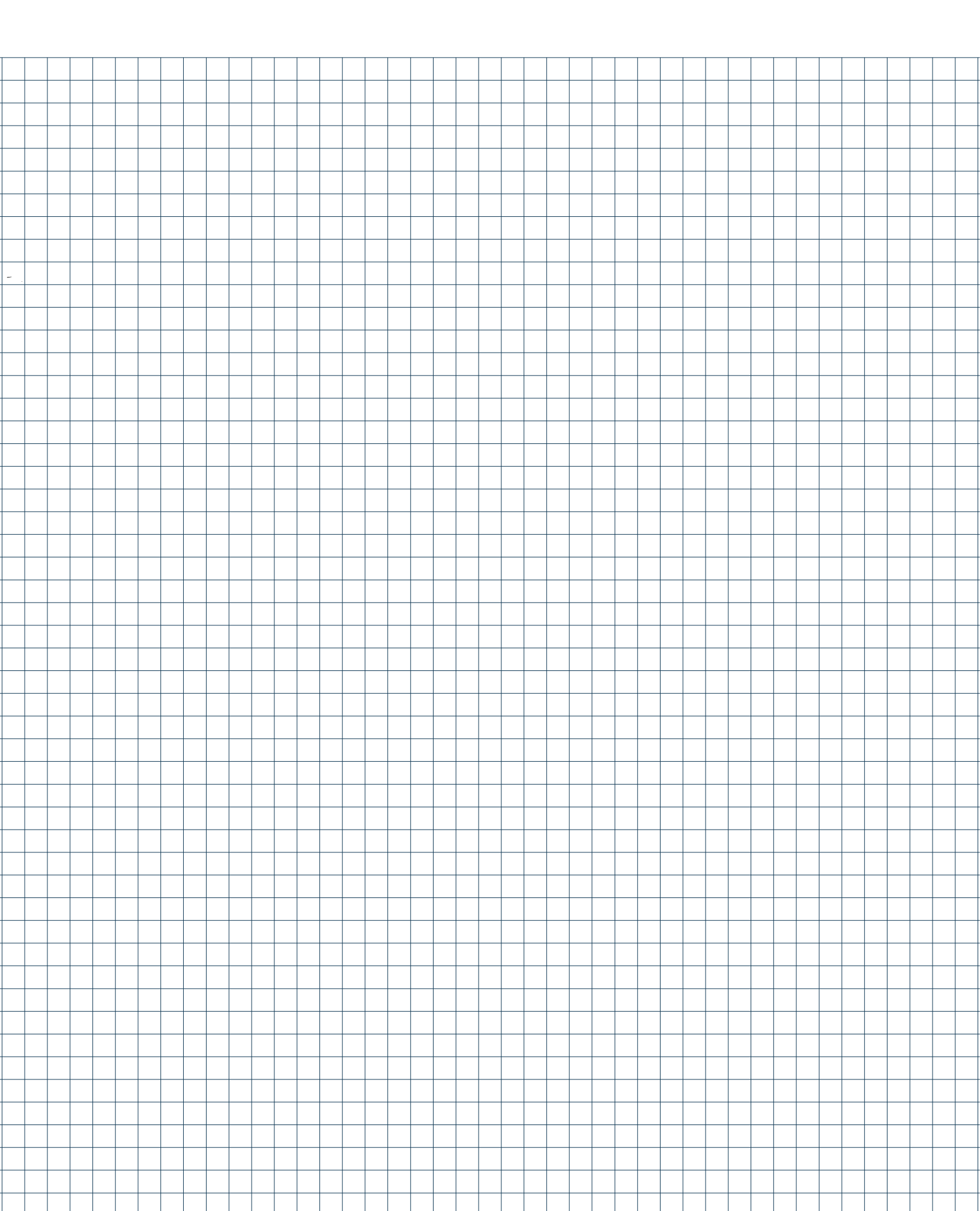
1. Suppose A and B are finite sets. Given that $|A| = 10$, $|A \cup B| = 15$, and $|A \cap B| = 3$, determine $|B|$.



$$|A \setminus B| = 7$$

$$|A \cup B| = |A \setminus B| + |A \cap B| + |B \setminus A|$$
$$15 = 7 + 3 + \downarrow 5$$

$$|B| = |A \cap B| + |B \setminus A|$$
$$= 3 + 5$$



$$\text{---} = 8 \quad \rightarrow \quad \mathbb{Z} = \{\text{integers}\}$$

2. Consider the sets $A = \{a \in \mathbb{Z} : a \text{ is divisible by } 2\}$ and $B = \{b \in \mathbb{Z} : b \text{ is divisible by } 3\}$.
What is the set $A \cap B$?

$$\text{let } C = \left\{ c \in \mathbb{Z} : \begin{array}{l} c \\ \text{is divisible by } 6 \end{array} \right\} = \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$$

$$\text{Claim: } A \cap B = C$$

Proof: (Idea: show $A \cap B \subseteq C$
then show $C \subseteq A \cap B$)

$$\text{Part 1: } A \cap B \subseteq C$$

(we're trying to show that...)

if n is an integer divisible by 2
& by 3, then it is divisible by 6

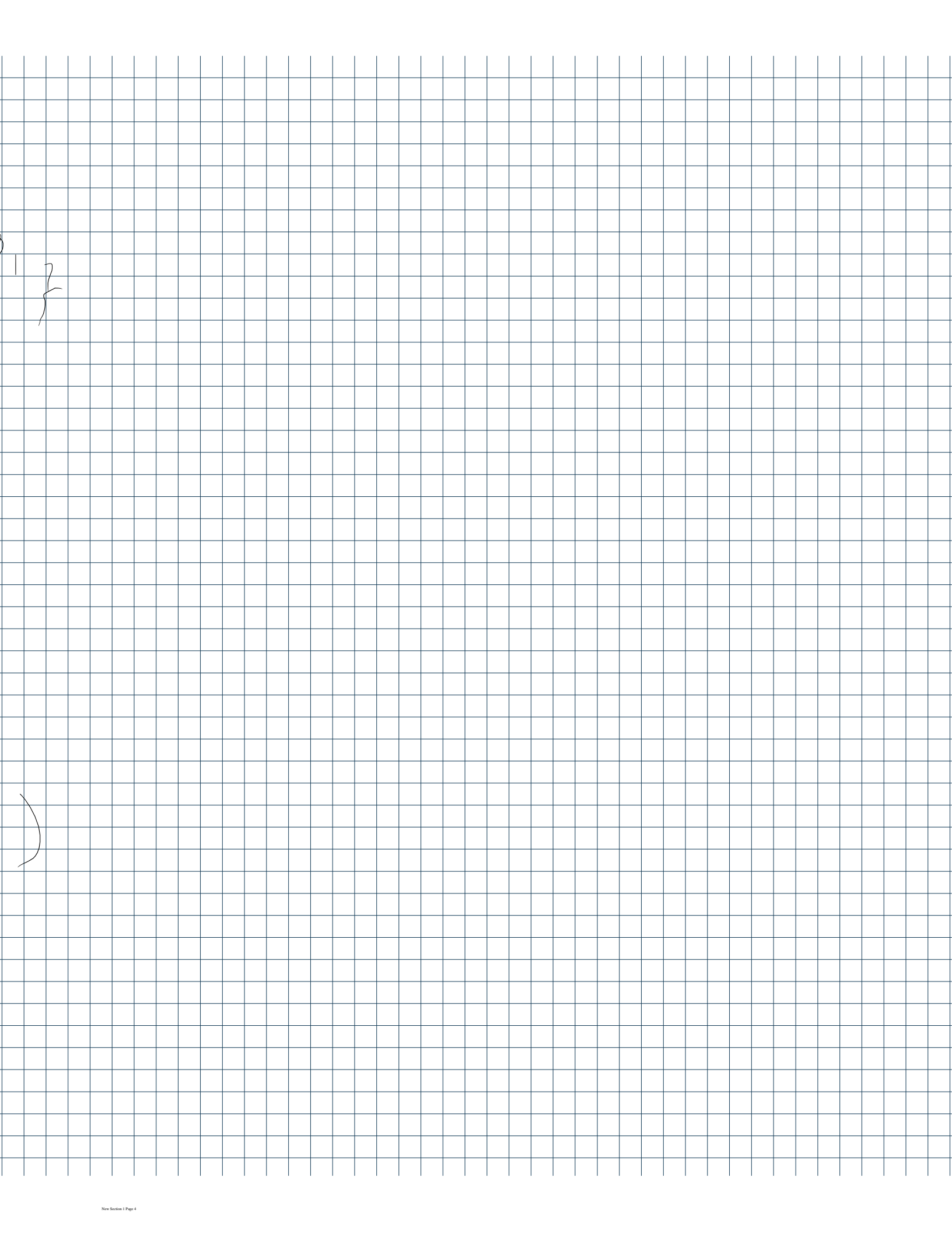
• since n is divisible by 2

$$n = 2 \cdot a$$

• since n is divisible by 3

$$n = 3 \cdot b$$

(if p is prime
... ab)



(if p is prime
 $\wedge p$ divides ab
 $\Rightarrow p$ divides
 a or b)

• since $n = 2a$ is divisible by 3

• because 3 is prime,

3 divides a .

$$\Rightarrow a = 3 \cdot x$$

$$\bullet n = 2 \cdot a = 2(3 \cdot x) = 6x$$

$\Rightarrow n$ is divisible by 6

Part 2 Show $C \subseteq A \cup B$

(ie, show if n is divis. by 6,
 then n is divis by 2 or 3

$$\text{if } n = 6x$$

$$\begin{array}{lll} ab = ba \text{ (commutativity)} & = 2 \cdot (3x) & (n \text{ is divis by } 2) \\ a(bc) = (ab)c \text{ (associativity)} & \xrightarrow{\quad} = 3(2x) & (n \text{ is divis by } 3) \end{array}$$

$$A \cup B = \{ n : n \text{ divis by } 2 \text{ or } n \text{ divis by } 3 \}$$

by 2)

by 3)

4

3 f