Math 13 - Week 7: Number Theory

1. Prove that $\sqrt{3}$ is irrational.

2. Solve the following equations.

   (a) $3x \equiv 4$ in $\mathbb{Z}_{11}$.

   (b) $4x - 8 \equiv 9$ in $\mathbb{Z}_{11}$.

   (c) $3x + 8 \equiv 1$ in $\mathbb{Z}_{10}$.

3. Solve the following equations. How are these problems different from the previous ones. Find *all* solutions.

   (a) $2x \equiv 4$ in $\mathbb{Z}_{10}$.

   (b) $2x \equiv 3$ in $\mathbb{Z}_{10}$.

   (c) $9x \equiv 4$ in $\mathbb{Z}_{12}$.

   (d) $9x \equiv 6$ in $\mathbb{Z}_{12}$.

4. Let x be an integer. Prove that $2 \mid x$ and $3 \mid x$ if and only if $6 \mid x$.

5. Let a and b be positive integers. Prove that a and b are relatively prime if and only if there is no prime p such that $p \mid a$ and $p \mid b$.