Math 13 - Week 3: Functions

1. Let A be the set of even integers and let B be the set of odd integers. Prove that the function $f : A \to B$ defined by $f(x) = x+1$ is a bijection. *Proof writing hint: think to yourself "which things do I need to show in order to prove this?" then break your proof into parts for each.*

2. For each of the following functions, f, find the image of the function, im f.

   (a) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = 2x + 1$.

   (b) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = |x|$.

   (c) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 1/(1 + x^2)$.

3. (a) Let a and b be real numbers and consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = ax + b$. For which values of a and b is f one-to-one? ... onto $\mathbb{R}$?

   (b) Let a, b and c be real numbers and consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = ax^2 + bx + c$. For which values of a and b is f one-to-one? ... onto $\mathbb{R}$?

4. Let A and B be finite sets and let $f : A \to B$.

   (a) If $|A| > |B|$, prove that f is not one-to-one.

   (b) If $|A| < |B|$, prove that f is not onto.

   (c) If f is a bijection, prove that $|A| = |B|$.