Math 130B - Sums of Random Variables

1. Recall that a geometric random variable, X, with success probability p has probability mass function given by

$$\Pr[X = k] = (1-p)^{k-1}p, \quad k \geq 1.$$

(Think of tossing a p-biased coin until you get a head) Let $Z = X_1 + \cdots + X_n$, where the X_i are independent geometric random variables with success probability p. Find the probability mass function for Z.

2. Prove that you can't load two dice in a way such that every sum 2-12 is equally likely to appear in a roll.

3. Suppose X and Y are independent continuous random variables with probability density functions given by

$$f_X(x) = 1 - \frac{1}{2}x, \quad \text{for } 0 \leq x \leq 2$$

$$f_Y(y) = 2 - 2y, \quad \text{for } 0 \leq y \leq 1,$$

respectively. Give the density function for the variable $X + Y$.

4. Suppose n is a positive integer that is not prime. Find discrete random variables X and Y that take values in the nonnegative integers such that $X + Y$ is uniform on $\{0, 1, \ldots, n-1\}$. For a challenge, show that this cannot be done if n is prime.