Math 130B - Joint Random Variables

1. Two fair dice are rolled. Fined the joint probability mass function of X and Y when

   (a) X is the value on the first die and Y is the larger of the two values.

   (b) X is the smallest and Y is the largest value obtained on the dice.

2. The joint density of X, Y is given by

$$f(x,y) = \begin{cases} 3x & \text{if } 0 \leq y \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

   (a) Find the marginal densities f_X and f_Y.

   (b) Compute $Cov(X,Y)$.

3. Let X and Y be independent random variables taking values in the positive integers having the same distribution given by

$$\Pr[X = n] = \Pr[Y = n] = 2^{-n}$$

   for all $n \geq 1$. Find $\Pr[X \text{ divides } Y]$.

4. Let X and Y be independent continuous random variables with densities f_X and f_Y, respectively. Express the density of XY in terms of the densities of X and Y.

5. Suppose that n points are independently chosen at random on the circumference of a circle, and we want the probability that they all lie in a semicircle. That is, we want the probability that there is a line passing through the center of the circle such that all the points are on the same side of that line.

   Let $P_1 \dots, P_n$ be the n points. Let $A^{(n)}$ denote the event that all the points are contained in some semicircle, and let $A_i^{(n)}$ be the event that all the points lie in the semicircle beginning at the point P_i and going clockwise $180°$ for $i = 1, \dots, n$.

   (a) Express $A^{(n)}$ in terms of the $A_i^{(n)}$.

   (b) Find $\Pr[A^{(n)}]$ and describe what happens to this as $n \to \infty$.