Math 13 - Week 2: Sets

1. Suppose A and B are finite sets. Given that $|A| = 10$, $|A \cup B| = 15$, and $|A \cap B| = 3$, determine $|B|$.

2. Consider the sets $A = \{a \in \mathbb{Z} : a \text{ is divisible by } 2\}$ and $B = \{b \in \mathbb{Z} : b \text{ is divisible by } 3\}$. What is the set $A \cap B$?

3. True or false

   (a) If A and B are finite sets, then $A \cap B$ has strictly smaller cardinality than that of A.

   (b) If A is a finite set then A^C is a finite set.

   (c) If A and B are finite sets, then $|A \cup B| \leq \max(|A|, |B|)$.

   (d) $2^{A \cap B} = 2^A \cap 2^B$, where A and B are finite sets

   (e) $2^{A \cup B} = 2^A \cup 2^B$, where A and B are finite sets

   (f) $2^{A \triangle B} = 2^A \triangle 2^B$, where A and B are finite sets

4. Let A be a set. Which of the following are true and which are false?

   (a) $x \in A \iff x \in 2^A$

   (b) $T \subseteq A \iff T \in 2^A$

   (c) $x \in A \iff \{x\} \in 2^A$

   (d) $\{x\} \in A \iff \{\{x\}\} \in 2^A$.

5. For any sets of real numbers A and B, define $AB = \{ab : a \in A \text{ and } b \in B\}$. If $A = \{1, 2\}$ and $B = \{2, 3, 4\}$, what is $|AB|$? What is $|A \times B|$?

6. For any sets of real numbers A and B, define their *sumset* $A+B = \{a+b : a \in A \text{ and } b \in B\}$.

   (a) Suppose $A = \{1, 2, 3, \ldots, 12\}$. What is $|A + A|$?

   (b) Suppose $A = \{2, 4, 6, \ldots, 12\}$. What is $|A + A|$?

   (c) Suppose $A = \{1, 3, 4, 5, 12\}$. What is $|A + A|$?

   (d) Can you come up with a guess for when $|A + A|$ is "big" compared to $|A|$?