Today: Canvas Worksheet

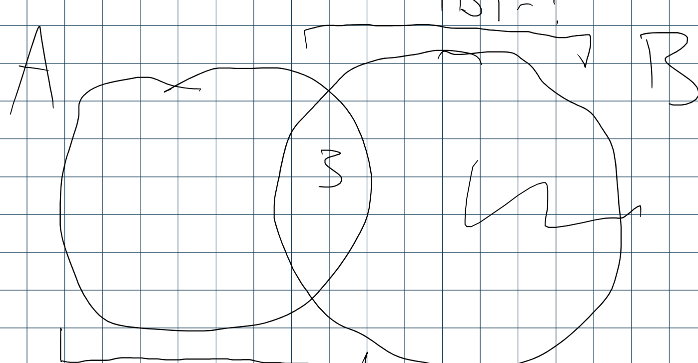
! Don't do every part of the multi-part problems !

Files > Discussion A > 2-1

$$2^A = P(A) = \{\text{subsets of } A\}$$

1. Suppose A and B are finite sets. Given that $|A| = 10$, $|A \cup B| = 15$, and $|A \cap B| = 3$, determine $|B|$.

$|B| = ?$



$|A| = 10$

$|A \cup B| = 15$

no overlap

prove this as an exercise

disjoint uni

$$A \cup B = (A \setminus B) \dot{\cup} (A \cap B) \dot{\cup} (B \setminus A)$$

$$|A \cup B| = |A \setminus B| + |A \cap B| + |B \setminus A|$$

on

$\cap B)$

$\cap B($

$|A \cup B| = 15$

$+ |B \setminus A|$

$A = A \setminus B \,\dot\cup\, A \cap B$

$|A| = |A \setminus B| + |A \cap B| \Rightarrow 15 = 7 + 3 + |B \setminus A|$

$B = \underset{5}{B \setminus A} \,\dot\cup\, \underset{3}{A \cap B}$

$5 = |B \setminus A|$

$\Rightarrow |B| = 8$

2. Consider the sets $A = \{a \in \mathbb{Z} : a \text{ is divisible by } 2\}$ and $B = \{b \in \mathbb{Z} : b \text{ is divisible by } 3\}$. What is the set $A \cap B$?

$\mathbb{Z} \to$ integers $= \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Claim: $A \cap B = \{c \in \mathbb{Z} : c \text{ is divisible by } 6\}$

$=: C$

Idea: show $A \cap B \subseteq C$ & $C \subseteq A \cap B$

Part 1: $A \cap B \subseteq C$

Pf: we want to show that the set of integers divis by 2 & by 3 are also divis by

(if divis by 2 & 3 $\Rightarrow$ divis by 6)

(4 divides $2 \cdot 2 = 4$
b.c. 4 does not)

6)

/6-

divide 2) . If n is divis by 2, then $n = 2a$ for some a

(If p prime divides ab $\Rightarrow$ p divides a or b)

. n is divisible by 3, so 3 divides

. Since 3 is prime & 3 does not divide

3 must divide a.

$\Rightarrow a = 3 \cdot b$ for some b

$\Rightarrow n = 2(3b) = 6b \Rightarrow$ n divis 6

$\square$

Part 2: $C \subseteq A \cap B$

Pf: want to show: if n divis by 6, then n divis by 2 & by 3.

n divis by 6 $\Rightarrow$ $n = 6x$ for some

$= 2 \cdot 3 \cdot x$

$= 2(3x) \Rightarrow$ divis

$= 3(2x) \Rightarrow$ divis

4
||
2a

u 2,

by

x

by 2

by 3

Therefore $A \cap B \subseteq C$ & $C \subseteq A \cap B$

$\Rightarrow A \cap B = C$

3. True or false

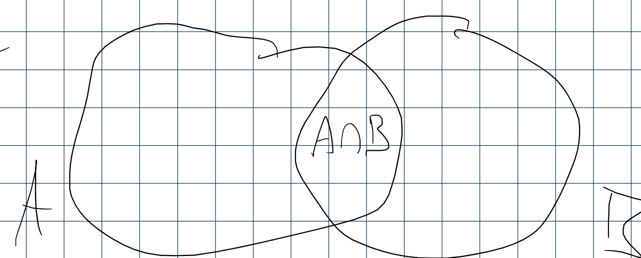
(a) If A and B are finite sets, then $A \cap B$ has strictly smaller cardinality than that of A.
(b) If A is a finite set then A^C is a finite set.
(c) If A and B are finite sets, then $|A \cup B| \leq \max(|A|, |B|)$.
(d) $2^{A \cap B} = 2^A \cap 2^B$, where A and B are finite sets
(e) $2^{A \cup B} = 2^A \cup 2^B$, where A and B are finite sets
(f) $2^{A \triangle B} = 2^A \triangle 2^B$, where A and B are finite sets

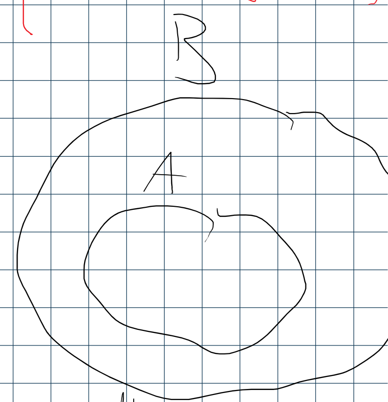
$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 =$

$2^{\{0,1\}} = ???$

if A is a set, $2^A := \mathcal{P}(A) = \{\text{subsets of } A\}$

a) F





$\Downarrow$

$A \cap B = A$

idea behind notation: $|2^A| = 2^{|A|} = |\mathcal{P}(A$

$|AB| \stackrel{?}{=} |A| \cdot |B|$

From class

$$|AB| \stackrel{?}{=} |A|\cdot|B|$$

from class

$$|A \times B| = |A|\cdot|B|$$

$(dog, \text{☺})$

$\{dog, 7\}$ $\{\text{☺}, \text{☹}\}$ different!

$$A \times B = \{(a,b) : a \in A,\ b \in B\}$$

(dog)(

$$AB = \{ab : a \in A, b \in B\}$$

only makes sense if a & b are numbers

example where $|AA| \neq |A|\,|A|$?

(think about $|A+A|$ first)

$$|A+A| \stackrel{?}{=} |A| + |A|$$

$A = \{2,3\}$, $B = \{2,3\}$,

) = ?!

$A = \{2, 3\}$, $B = \{2, 3\}$

$$A + B = \{2+2, 2+3, 3+2, 3+3\}$$
$$= \{4, 5, 6\}$$

$$A = \left\{ \begin{matrix} 2, & 3, & 4, & \ldots & , 13 \\ 3, & 4, & \ldots & & 14 \\ 4, & 5, & \ldots & & , 15 \\ 8, & \ldots & & & , 16 \\ 13, & , & \ldots & & , 24 \end{matrix} \right\} = \{2, \ldots 2$$

23 numbers

$\downarrow$

here.

4)