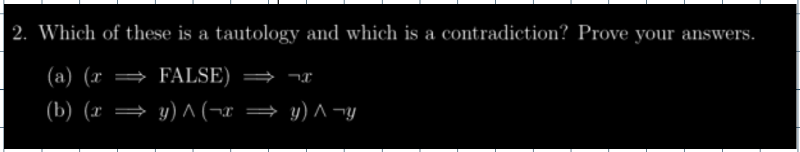
2-2

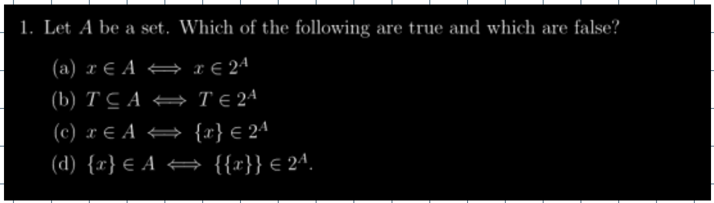
Thursday, January 13, 2022 6:02 PM

Today: Canvas Worksheet.



2. Which of these is a tautology and which is a contradiction? Prove your answers.
   (a) $(x \implies \text{FALSE}) \implies \neg x$
   (b) $(x \implies y) \wedge (\neg x \implies y) \wedge \neg y$

X	Y	$(x \to y) \wedge (\neg x \to y) \wedge \neg y$
T	T	$(T \to T) \wedge (F \to T) \underline{\wedge F}$
T	F	
F	T	
F	F	



1. Let A be a set. Which of the following are true and which are false?
   (a) $x \in A \iff x \in 2^A$
   (b) $T \subseteq A \iff T \in 2^A$
   (c) $x \in A \iff \{x\} \in 2^A$
   (d) $\{x\} \in A \iff \{\{x\}\} \in 2^A$.

a) $2^A = \mathcal{P}(A)$ = the set of all subsets of A.

"if & only if"

$\downarrow$

$x \in A \underline{\Leftrightarrow} x \in 2^A$ "x is in A $\underline{\text{iff}}$ x is

$= F$

2^A "

$\Rightarrow A = 2^A$ never true

since 2^A is alw

bigger than A

d) $\underbrace{\{x\}} \in A \iff \{\{x\}\} \in 2^A$

the set whose one & only element is x

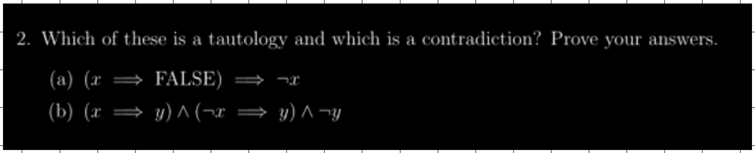
$\Rightarrow$ if $\{x\} \in A$ then $\{\{x\}\} \subseteq A$

$\Rightarrow \{\{x\}\} \in 2^A$

$\Leftarrow \{\{x\}\} \in 2^A$ then $\{\{x\}\} \subseteq A$

$\Rightarrow \{x\} \in A$

True



2. Which of these is a tautology and which is a contradiction? Prove your answers.

(a) $(x \implies \text{FALSE}) \implies \neg x$

(b) $(x \implies y) \wedge (\neg x \implies y) \wedge \neg y$

a1/β

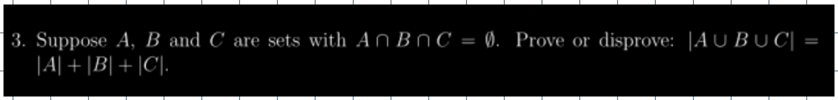
Tautology: sentence that is true for all input

Contradiction: " " false " " "

a)

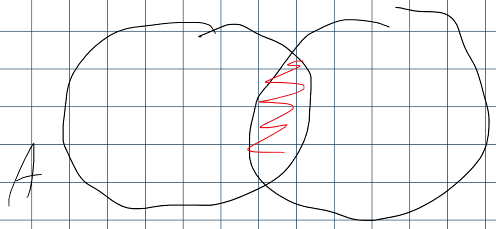
x	$(x \to F) \to \neg x$
T	$(T \to F) \to F = F \to F =$
F	$(F \to F) \to T = T \to T =$

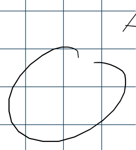
$\Rightarrow$ Tautology.



3. Suppose A, B and C are sets with $A \cap B \cap C = \emptyset$. Prove or disprove: $|A \cup B \cup C| = |A| + |B| + |C|$.

For two sets:





$$|A \cup B| = |A| + |B| - |A \cap B|$$

if $A \cap B = \emptyset$

$$\Rightarrow |A \cup B| = |A| + |B|$$

A

)

= T

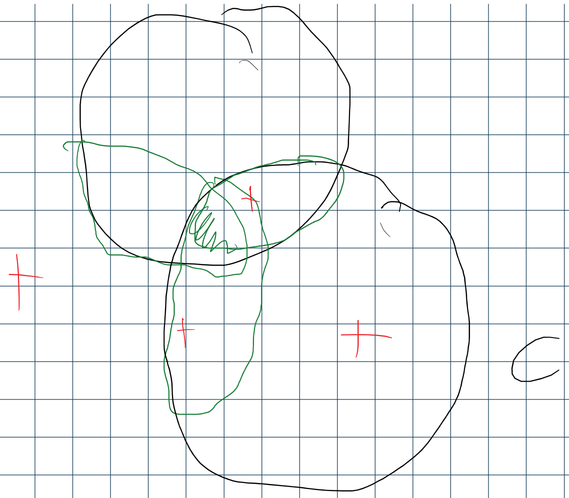
= T

O^{β}

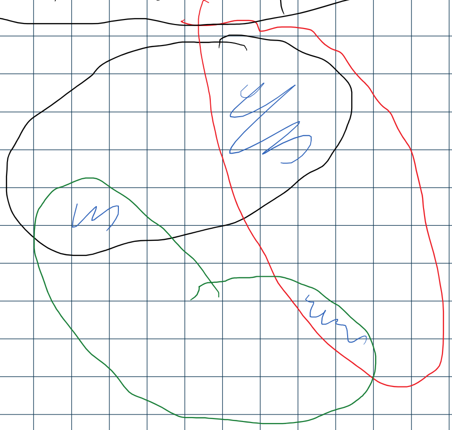
For 3 sets

B

$$\boxed{\begin{aligned}|A\cup B\cup C| = {} & |A|+|B|+|C| \\ & -|A\cap B|-|B\cap C|-|A\cap C| \\ & +|A\cap B\cap C|\end{aligned}}$$



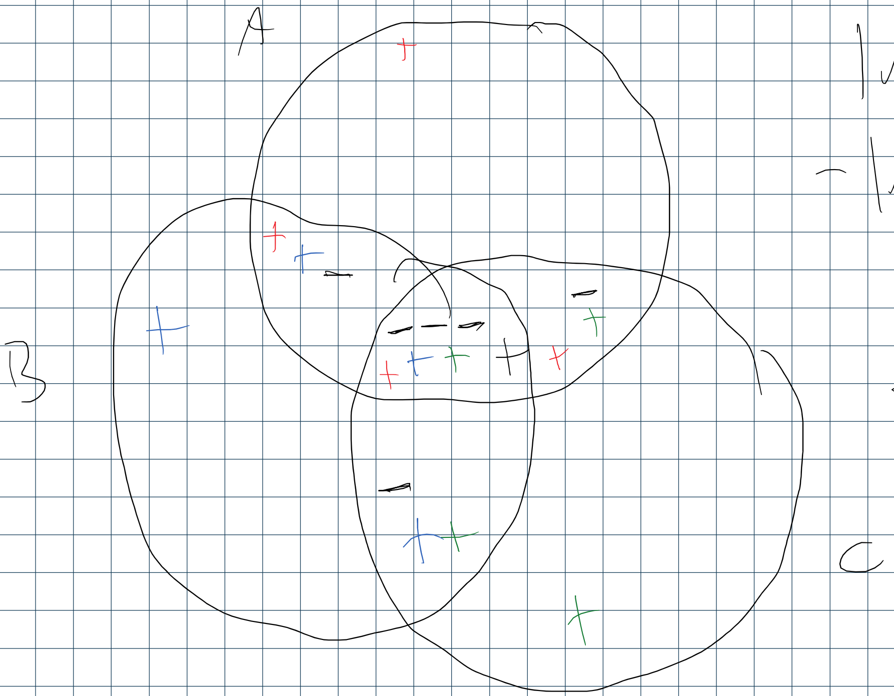
inclusion
- exclusion



$|A\cup B\cup C|$

$= |A|+|B|+|C|$

$-|A\cap B| - |B\cap C| - |A$



$|A|+|B|+|C$

$-|A\cap B| - |A\cap C$

$-|B\cap C|$

$+|A\cap B\cap C$

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