Today: Canvas Worksheet

1. Let A be a set. Which of the following are true and which are false?
   (a) $x \in A \iff x \in 2^A$
   (b) $T \subseteq A \iff T \in 2^A$
   (c) $x \in A \iff \{x\} \in 2^A$
   (d) $\{x\} \in A \iff \{\{x\}\} \in 2^A$.

if A is a set, $2^A = P(A)$
= set of all subsets of A

a) $A = \{1,2,3\}$ $\quad |2^A| = 2^{|A|} = 2^3 = 8$

$2^x > x$

$2^A = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, 1 \neq \{1\}$

if $(x \in A \iff x \in 2^A)$, then $A \subseteq 2^A$ would have exactly the same elements

$\Rightarrow A = 2^A$, a contradiction

so this is false

2. Which of these is a tautology and which is a contradiction? Prove your answers.
   (a) $(x \implies \text{FALSE}) \implies \neg x$
   (b) $(x \implies y) \wedge (\neg x \implies y) \wedge \neg y$

A sentence is a tautology if it is true

for all possible inputs.

- A sentence is a contradiction if it's false for all inputs.

a) $(x \Rightarrow F) \Rightarrow \neg x$

informally "x implies false" implies the opposite of x"

Truth table

x	$(x \to F) \to \neg x$
T	$(T \to F) \to \neg T \equiv F \to F \equiv T$
F	$(F \to F) \to \neg F \equiv T \to T \equiv T$

$\equiv$ tautology

4. Let
$$f = \{(1,2),(2,3),(3,1),(4,7)\} \quad \text{and} \quad g = \{(1,2),(1,3),(4,7)\}.$$
Which of these is a function? For each function, give its domain and range.

"Dirichlet's definition"

A function $f: A \to B$ is a subset of $A \times B$ where each $a \in A$ has a unique

$b \in \mathbb{R}$ s.t. (a, b) is in f.

- f is a function here since each input has exactly one output.
- g is not a function because $(1,2)$ & $(1,3)$ are both in g, i.e. 1 has two distinct outputs

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$\text{Dom } f$ = set of all inputs

$= \{1, 2, 3, 4\}$

$\text{Ran } f = \text{Im } f$ = set of all outputs

(Im f: image)

$= \{2, 3, 1, 7\}$