Math 130B - Conditional Distributions

1. How many simple substitution ciphers (on the English alphabet) are there with the following properties?

   (a) In total.

   (b) That leave no letters fixed.

   (c) That fix at least one letter.

   (d) That fix exactly one letter.

2. (a) Compute gcd(30030, 257).

   (b) Factor 30030 into prime numbers.

   (c) Use the previous two exercises to conclude that 257 is prime.

3. Decrypt this message that was encrypted using a Caesar cipher.

UXENRBWXCUXENFQRLQJUCNABFQNWRCJUCNAJCRXWORWMB

4. Let $a, b, c \in \mathbb{Z}$. Prove the following statements.

   (a) If $a \mid b$ and $b \mid c$ then $a \mid c$.

   (b) If $a \mid b$ and $b \mid a$ then $a = \pm b$.

   (c) If $a \mid b$ and $a \mid c$ then $a \mid (b \pm c)$.

5. Compute $\gcd(291, 252)$ and find integers u and v such that

$$291u + 252v = \gcd(291, 252).$$