Math 13 - Week 2: Sets and Functions

1. Let A be a set. Which of the following are true and which are false?

   (a) $x \in A \iff x \in 2^A$

   (b) $T \subseteq A \iff T \in 2^A$

   (c) $x \in A \iff \{x\} \in 2^A$

   (d) $\{x\} \in A \iff \{\{x\}\} \in 2^A$.

2. Which of these is a tautology and which is a contradiction? Prove your answers.

   (a) $(x \implies \text{FALSE}) \implies \neg x$

   (b) $(x \implies y) \wedge (\neg x \implies y) \wedge \neg y$

3. Suppose A, B and C are sets with $A \cap B \cap C = \emptyset$. Prove or disprove: $|A \cup B \cup C| = |A| + |B| + |C|$.

4. Let
$$f = \{(1,2),(2,3),(3,1),(4,7)\} \quad \text{and} \quad g = \{(1,2),(1,3),(4,7)\}.$$
   Which of these is a function? For each function, give its domain and range.

5. Let $A = \{1,2,3\}$ and $B = \{4,5\}$. Find all functions $f : A \to B$.