Math 130B - Conditional Expectation and Moment Generating Functions

1. A mouse is placed in a maze with two rooms pictured in Figure 1. Starting from room 1, what is the expected number of steps the mouse takes before it reaches the exit?
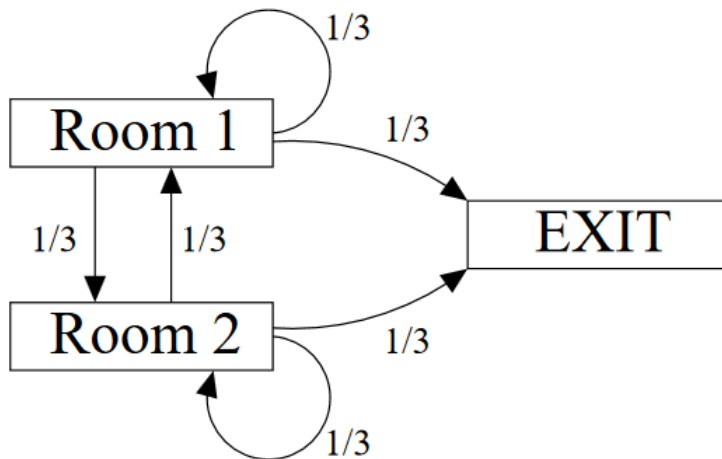

Figure 1: Mouse's actions

2. You run a whale-watching business in San Diego. Every day you are unable to operate your tour due to bad weather with probability p, independently of all other days. You work every day except the bad-weather days.

   Let Y be the number of consecutive days you work between bad-weather days and let X be the total number of customers who attend your tour in this period of Y days. Conditional on Y, suppose the distribution of X is
   $$X \mid Y \sim \text{Pois}(\mu Y).$$

   (a) What kind of random variable is Y. What are its expectation and variance?

   (b) Find the expectation and variance of the number of customers you see between bad weather days.

3. A factory has produced n robots, each of which is faulty with probability ϕ. Each robot is tested to determine whether or not it is faulty. If the robot it faulty, the test detects the fault with probability δ. Let X be the number of faulty robots, and let Y be the number detected as faulty. Under normal assumptions about dependence, show that

$$\mathbb{E}[X \mid Y] = \frac{n\phi(1-\delta) + (1-\phi)Y}{1-\phi\delta}.$$

4. Suppose that a statistician determines that the revenue the biological sciences Starbucks makes in a week is a random variable, X, with moment generating function

$$m_X(t) = \frac{1}{(1-2500t)^4}.$$

Find the standard deviation of the revenue the Starbucks makes in a week.

5. Let X and Y be two independent random variables with respective moment generating functions

$$m_X(t) = \frac{1}{1-5t}, \text{ if } t < 1/5, \quad m_Y(t) = \frac{1}{(1-5t)^2}, \text{ if } t < 1/5.$$

Find $\mathbb{E}[(X+Y)^2]$.

6. True or False? If X and Y are independent exponential random variables with parameters λ_x and λ_y, respectively, then $X + Y$ is an exponential random variable with parameter $\lambda_x + \lambda_y$.