

Math 130B - Homework 2

1. You and three other people are to place bids for an object, with the highest bid winning. If you win, you plan to sell the object immediately for \$10000. How much should you bid to maximize your expected profit if you believe that the bids of the others can be regarded as being independent random variables, taking values between \$7000 and \$10000 uniformly.
2. Suppose a laser pointer sits at a unit distance from the x -axis. We spin the laser about its center and consider the point X at which the beam intersects the x -axis once the beam has stopped spinning. If the beam doesn't point toward the x -axis, just repeat the experiment.
 - (a) Find the probability density function, $f(x)$, of X . The laser setup is illustrated in Figure 1, where θ is the angle the beam makes with the y -axis. Assume that θ is uniformly distributed between $-\pi/2$ and $\pi/2$.

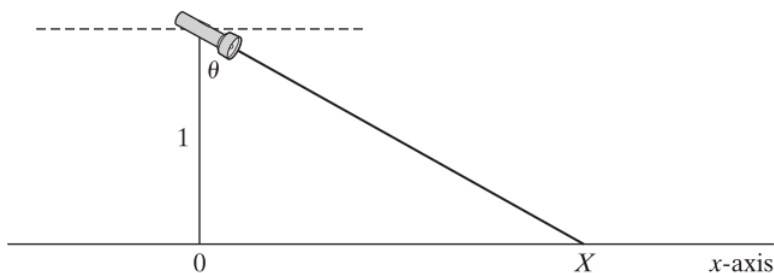


Figure 1: Laser setup

- (b) Find the expectation $E[X]$.
3. The time that it takes to service a car is an exponential random variable with rate 1.
 - (a) If Alice brings her car in at time 0 and Bob brings in his car at time t , what is the probability that Bob's car is ready before Alice's? Assume that service times are independent and begin as soon as the car arrives.
 - (b) If both cars are brought in at time 0, with work starting on Bob's car only when Alice's car has been completely serviced, what is the probability that Bob's car is ready before time 2?
 4. If X_1, X_2, \dots, X_n are independent and identically distributed exponential random variables with parameter λ , compute
 - (a) $\Pr[\min(X_1, \dots, X_n) \leq a]$;
 - (b) $\Pr[\max(X_1, \dots, X_n) \leq a]$;
 5. The joint density function of X and Y is given by

$$f(x, y) = c(x^2 - y^2)e^{-x} \quad \text{if } x \geq 0, |y| \leq x.$$

Find the conditional distribution of Y , given that $X = x$.

6. Let Z_1 and Z_2 be independent standard normal random variables. Show that X, Y has a bivariate normal distribution when $X = Z_1$ and $Y = Z_1 + Z_2$.
7. If X and Y are independent and identically distributed uniform random variables on $(0, 1)$, compute the joint density of
 - (a) $U = X + Y, V = X/Y$;
 - (b) $U = X + Y, V = X/(X + Y)$.
8. Let Z_1, Z_2, \dots, Z_N be independent standard normal random variables and let

$$S_n = \sum_{i=1}^n Z_i.$$

- (a) What is the conditional distribution of S_n given that $S_k = y$, where k is some integer between 1 and n ?
 - (b) Show that for $1 \leq k \leq n$, the conditional distribution of S_k given that $S_n = x$ is normal with mean xk/n and variance $k(n - k)/n$.
9. Suppose T_1, T_2, \dots, T_n are continuous, independent, and identically distributed random variables. For any $k = 1, 2, \dots, n$, find

$$E \left(\frac{T_1 + T_2 + \dots + T_k}{T_1 + T_2 + \dots + T_n} \right).$$

10. Suppose G is a graph with n vertices and e edges. Show that you can partition the vertices of G into two subsets A and B in such a way that at least half of the edges in G cross between A and B .