

## Math 130B - Conditional Distributions

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1. The joint density function of  $X$  and  $Y$  is given by

$$f(x, y) = xe^{-x(y+1)}, \quad x > 0, y > 0.$$

- (a) Find the conditional density of  $X$ , given that  $Y = y$ , and that of  $Y$ , given  $X = x$ .
- (b) Find the density function of  $Z = XY$ .
2. Find the distribution of the range of a sample of size 2 from a distribution having density function  $f(x) = 2x$ ,  $0 < x < 1$ . What about for  $n$  samples? *That is, find the distribution of the largest observation minus the smaller.*
3. Let  $X$  and  $Y$  denote the coordinates of a point uniformly chosen inside the circle of radius 1 centered at the origin. Find the joint density function of the polar coordinates  $R = X^2 + Y^2$ ,  $\Theta = \tan^{-1} Y/X$ .
4. Suppose that  $X$  and  $Y$  are independent geometric random variables with the same parameter  $p$ .
- (a) Without any computations, what do you think is the value of

$$\Pr[X = i \mid X + Y = n].$$

- (b) Prove your guess.
5. Let  $Z_1, Z_2, \dots, Z_n$  be independent standard normal random variables and let

$$S_n = \sum_{i=1}^n Z_i.$$

- (a) What is the conditional distribution of  $S_n$  given that  $S_k = y$  for  $k = 1, \dots, n$ ?

Show that for  $1 \leq k \leq n$ , the conditional distribution of  $S_k$  given that  $S_n = x$  is normal with mean  $xk/n$  and variance  $k(n-k)/n$ .