Math 13 - Week 3: More on Functions

1. Let A and B be finite sets. Prove that the following are equivalent (that is, prove that if any one of these statements is true, then the others are as well).

   (a) If $|A| \leq |B|$.

   (b) There is a one-to-one function $f : A \to B$.

   (c) There is an onto function $g : B \to A$.

2. Use the last exercise to show that if A and B are finite sets, then $|A| = |B|$ if and only if there is a bijection $f : A \to B$.

3. Let $f : A \to B$ and $g : B \to C$ be functions. Prove that:

   (a) If f and g are one-to-one, then so is $g \circ f$.

   (b) If f and g are onto, then so is $g \circ f$.

   (c) If $g \circ f$ is one-to-one, then so is f.

   (d) If $g \circ f$ is onto, then so is g.