Math 13 - Week 6: Modular Arithmetic

1. Which of these assignments are injective, surjective or bijective?

   (a) In 2016, UCI had 33,467 students enrolled. Let f be the function that maps each student to their eight-digit student ID number. (Consider f as a function from the set of students to the set of all eight-digit numbers).

   (b) Let g be the function from the set of ID numbers of current UCI students to the set of current UCI students that maps an ID number to the student it belongs to.

   (c) A padlock company produces 100,000 padlocks in a month. Each padlock is opened with a combination of three numbers, each between 0 and 39 and each lock is made with a random combination. Leg h be the function that maps each padlock to the three-number sequence representing its combination.

2. Find x and y such that $431x + 29y = \gcd(431, 29)$. *I promise it won't take that many steps.*

3. Prove that consecutive integers must be relatively prime (that is, their greatest common divisor is 1).

4. Let a be an integer. Prove that $2a + 1$ and $4a^2 + 1$ are relatively prime.

5. Suppose that a and b are relatively prime integers and that $a \mid c$ and $b \mid c$. Prove that $(ab) \mid c$.