Instructor: Fatih Guvenen

ECO2104H: Problem Set #1

University of Toronto, Department of Economics

Due back: Friday, January 30, by **2:30PM**

Instructions: *You are allowed to work in groups of two students and submit together. You should submit a well-written report in the form of a self-contained executive summary that describes your answer to each question, and this report should include all the output that the questions ask (any table, figure, calculation, etc. as relevant) and you should refer to them as you discuss your answers. You also need to submit all the computer code (Fortran/Julia/Matlab/Python) that generates your results. Please make every effort to produce a complete package, so that I can understand what you did, how you did it, and what you found. Please submit a single zip archive through Quercus that contains all your files and add your last-name(s) to the name of the zip folder.*

Questions indicated by one star "" are required for everybody to answer, including those who audit the class. Questions indicated by two stars "**" are required for Masters students (in addition to 1 star questions). PhD students need to answer all questions.*

PROBLEMS

The first two questions will help you learn some basic but important issues about how computations are carried out on by a computer and how these can affect your results. Before attempting these questions, carefully study Chapter 1.2 of the *Numerical Recipes* book, which covers many other essential topics for computation.

1. ***[15 points] Rounding error:** Using the well-known formula for the roots of a quadratic equation ($x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$) compute the larger root of the following quadratic equation: $ax^2 + bx + c = 0$ for the following values: $a = 1$, $b = 100,000$, and $c = 10^n, n = -1, -2, ..., -8$. Now use the following alternative method. First calculate $q = -\frac{1}{2}\left(b + \text{sign}(b)\sqrt{b^2 - 4ac}\right)$, and then obtain the two roots: $x_1 = q/a$ and $x_2 = c/q$. Notice that the two calculations should give you precisely the same answer if it were not for the rounding error of the computer arithmetic. Also note that in some real life computational problems you will need precision up to the 10th or 12th significant digit (for example, to find the equilibrium price that clears an asset market). How close are the answers for different values of n?

2. **[15 points] Truncation error/Unstable Algorithms:** Consider the "Golden Mean" which is the number given by:
$$\phi = \frac{\sqrt{5} - 1}{2} \approx 0.61803398.$$

(a) First, verify analytically that the powers of this number satisfy the following recursion (Hint: trivial!)

$$\phi^{n+1} = \phi^{n-1} - \phi^n$$

This recursion suggests a seemingly efficient way to compute higher powers of ϕ since it only requires subtraction which is much faster than multiplication (required to exponentiate ϕ directly).

(b) But how well does the clever algorithm described in (a) works in practice? To see this, compute ϕ^n for $n = 2, 3, ..., 20$ using two separate methods. First, write a simple program using the recursion above and the fact that $\phi^0 = 1$ and $\phi^1 = 0.61803398$. Second, simply raise ϕ to the appropriate power. Compare the values obtained using the two methods. How do they compare to each other as n increases? This (somewhat contrived) exercise illustrates the dangers of rounding and truncation errors that combine together to create a dirty mess.

3. ***[70 points] VFI.** Write a computer program to solve the **neoclassical growth model** we saw in class, using value function iteration on a discrete grid, e.g., $\mathcal{K} = \{k_0, k_1, k_2, \ldots, k_N\}$. Let the production function take the form $f(k) = Ak^\alpha + (1-\delta)k$, where $A > 0$, $0 < \alpha < 1$, and $0 \leq \delta \leq 1$. Reasonable values for these parameters would be $\alpha = 0.4$, $\delta = 0.1$, and $A = 1$. Let the utility function be $U(c) = \log(c)$ and assume that the savings choice, k', is restricted to lie on the same discrete capital grid you construct. Start with a small number (say, 11) of equally-spaced grid points, and then increase this number to, 101, and then to 1001, and repeat the problem. Take the zero function as your initial guess for the value function (despite the fact that it is a poor choice from an efficiency stand point, it is an acceptable choice for convergence. We will learn to pick better ones soon). Stopping Rule for VFI: Set the stopping rule for your iteration based on the decision rule convergence — not on value function convergence (We will discuss reasons for this in class). Specifically, stop your VFI iteration when the maximum absolute deviation of the savings rule from the prior iteration is less than 10^{-8}, that is

$$max_{k \in \mathcal{K}} |k'_{n+1}(k) - k'_n(k)| \leq 10^{-8}.$$

(a) [20 points] Using the standard VFI, obtain numerical solution, both the value function and the savings rule.

i. How many iterations it took for VFI to stop?

ii. Plot **both functions** and make sure to choose the scale of your graph and other details so that the figure is informative (not too squeezed, etc.). Comment on the shape of both functions and their properties.

(b) [20 points] Repeat part (a) by changing the stopping tolerance to 10^{-6} and to 10^{-5}. Answer the same questions as in (a).i and (a).ii. [*Hint: When you complete this part, it* *may look like there is something strange with part (b) – that is intentional. Comment on what you find.*]

(c) [20 points] Repeat part (a) for the case of full depreciation ($\delta = 1$). Answer the same questions as in part (a).

   i. Compare the value function and decision rule you obtain numerically here to the analytical (closed-form) solutions we found in class. Plot both of them **and comment.**

   ii. Make a table that shows the iteration numbers for all cases you solved from (a) to (d).