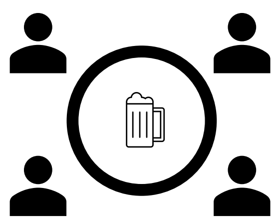
Fingers

Fingers is a beautiful game involving a centrepiece beverage (CB), each player's phalange of choice (POC), and their mind. There can be multiple winners, but only one loser. To begin the game, each player must place their POC on the CB. The object of the game is to guess the correct amount of POC's left on the CB at the end of your turn. The player that poured the CB starts the game by counting down from three. Beginning with said person, after counting down, each player of the game must decide to either leave their POC on the CB or take it off and the player that counted down must say their guess. If the player guesses the correct amount of POC's left on the CB, they win the game and remove themselves from the colosseum. The turns move in a clockwise rotation until there is a loser. The loser must finish the CB and pour another one – the loser of the game, begins the next game in position[1].

Rules

- If you celebrate after you win, you must place your POC back on the CB and return to the game
- Party fouls[2] are called for moments of stupidity such as but not limited to:
    - Guessing an impossible value for the game
        - Ex. calling 0 and leaving your finger on the CB
    - Guessing out of turn
    - Counting down too quickly or too slowly to alter the odds of the game in your favour



The Economics

The game is played with a minimum of 2 players and maximum of n players. For the sake of modelling, a player's decision to leave their POC on the glass will be valued at (1) and taking their POC off will be valued at (0). When a player is the one guessing, they shall be referred to as being in position and when they are not, they shall be refereed to as being a part of the crowd (for games where n>2, the crowd is divided into player A, B…). As each player has a binary choice of either 0 or 1, I am assuming the probability for each player of the game, excluding oneself, to either keep their POC on or remove it from the CB to be 0.50, although in reality people may be more biased towards leaving their finger on the CB or taking it off. The player in position has two decisions to make: 1) whether to leave their POC on the CB or remove it, and 2) guess the value of the game with the best odds to win. To model the second decision, I have derived the following formula:

[1]See economics section for definition of 'in position'

[2] The consequence for a party foul is an automatic loss and public humiliation. If an alleged party foul is contested by the accused, the verdict is put to a vote by all members of the game.

If player in position decides strategy of 1, leaving their POC on the CB:

Expected Value (game) = EV(g) = (n-1)/2 + 1

If player in position decides strategy of 0, removing their POC from the CB:

Expected Value (game) = EV(g) = (n-1)/2

* Note that if (n-1) for the game in question is an odd number, the two closest integers to the calculated value for EV(g) are statistically equally as good.

For example, consider a game with four people. When in position, there are really three people in the game because you are in control of whether you would like to play a 0 or 1. Each of the three other players have a choice of either 0 or 1 and the value of the game, excluding yourself, can be either 0, 1, 2, or 3.

Combinations	Player A	Player B	Player C	Value of the Game
#1	0	0	0	0
#2	1	0	0	1
#3	0	1	0	1
#4	0	0	1	1
#5	1	1	0	2
#6	1	0	1	2
#7	0	1	1	2
#8	1	1	1	3

Statistical Models

[n=2] Here are the results of a statistical model for a two-person game (excluding the person in position):

Game Value	# of Outcomes	% of total
0	1	50%
1	1	50%

[n=3] Here are the results of a statistical model for a three-person game (excluding the person in position):

Game Value	# of Outcomes	% of total
0	1	25%
1	2	50%
2	1	25%

[n=4] Here are the results of a statistical model for a four-person game (excluding the person in position):

Game Value	# of Outcomes	% of total
0	1	13%
1	3	38%
2	3	38%
3	1	13%

[n=5] Here are the results of a statistical model for a five-person game (excluding the person in position):

Game Value	# of Outcomes	% of total
0	1	6%
1	4	25%
2	6	38%
3	4	25%
4	1	6%

[n= 6] Here are the results of a statistical model for a six-person game (excluding the person in position):

Game Value	# of Outcomes	% of total
0	1	3%
1	5	16%
2	10	31%
3	10	31%
4	5	16%
5	1	3%

[n=7] Here are the results of a statistical model for a seven-person game (excluding the person in position):

Game Value	# of Outcomes	% of total
0	1	2%
1	6	9%
2	15	23%
3	20	31%
4	15	23%
5	6	9%
6	1	2%

Pascal's triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
⋰ ⋮ ⋱

Notice that the frequency of each game value and n correspond exactly to the row number n of Pascal's Triangle. For example, row two of Pascal's Triangle corresponds to the set of outcomes when n=2, frequency/number of combinations of each outcome [1, 1] for the expected values of the game [0, 1] and row five of Pascal's Triangle corresponds to the set of outcomes when n=5, frequency/number of combinations of each outcome [1, 4, 6, 4, 1] for the expected values of the game [0, 1, 2, 3, 4].

	n=2	**n=3**	**n=4**	**n=5**	**n=6**	**n=7**
Expected Value of Game	0, 1	0, 1, 2	0, 1, 2, 3	0, 1, 2, 3, 4	0, 1, 2, 3, 4, 5	0, 1, 2, 3, 4, 5, 6
Frequency of Each Outcome	1, 1	1, 2, 1	1, 3, 3, 1	1, 4, 6, 4, 1	1, 5, 10, 10, 5, 1	1, 6, 15, 20, 15, 6, 1
Best Strategy	Either	2	1 or 2	2	2 or 3	3
Probability of Winning	50%	50%	38%	38%	31%	31%

Although the basic formula of halving (n-1) still stands, we stand to gain information on the probability of winning through Pascal. If you are ever in a new environment playing fingers, bring Pascal along with you and some simple math should help you figure out not only the best strategy for each turn but also the probability of you winning.

In short, when you are in position you are not included in the probability calculations because you control your own outcome. Instead, you decide whether you want to be a 1 or a 0 and calculate the odds of what everyone else is going to do.

If you want to be a 1 and leave your finger on: EV(g) = (n-1)/2 + 1

If you want to be a 0 and remove your finger: EV(g) = (n-1)/2

** Note that if (n-1) for the game in question is an odd number, the two closest integers to the calculated value for EV(g) are statistically equally as good*

To calculate your odds of winning with this strategy, count the number of people playing the game (excluding oneself), head to that row in Pascal's Triangle, then find the middle or middle two value(s) in the row. Divide that value by the sum of the whole row and that is your probability of winning.

Liam Hildreth