# Management Center Innsbruck

## Department of Technology & Life Sciences

### Master's program Mechatronics & Smart Technologies

## Report

**composed as part of the course**
**WS 2024 Multibody and Multiphysics Simulation**
**(MECH-M-3-SVM-MKS-ILV)**

**about**

# Task III - Futura Pendulum

**from**


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# Contents

# Chapter 1

# Background

The Furuta pendulum, a variant of the classic pendulum-on-a-cart system, is a well-known benchmark problem in control theory and robotics. In this system, the traditional cart is replaced by a vertical joint around which *Arm 1* rotates. *Arm 1* is connected to *Arm 2* via a rotational joint, forming a pendulum that can swing freely under the influence of gravity. The primary objective of this project is to model and control the Furuta pendulum using Simscape Multibody and Simulink, incorporating a DC motor as a multiphysics component to actuate the system.

The project is divided into several key tasks. First, a Simscape Multibody model of the Furuta pendulum is constructed, with the DC motor modeled as a multiphysics component. The dynamic behavior of the system is then analyzed under the influence of gravity, with an initial angular displacement applied to *Arm 2* to move it away from its stable equilibrium position. This allows for the observation of the system's natural response without external control.

Next, a control system is introduced to stabilize the pendulum in its upright position. A full-state regulator or Linear Quadratic Regulator (LQR) controller is employed to generate the control signal (voltage) for the DC motor, which in turn applies the necessary torque to stabilize the system. The robustness of the controller is tested by introducing a disturbance force at the tip of *Arm 2*, simulating real-world perturbations.

The system parameters are defined based on a global coordinate system, with gravity acting in the negative $z$-direction. The long, slender arms of the Furuta pendulum allow for simplifications in the inertia tensors, as the principal axes of each arm align with the global coordinate system. Geometric representations of the components are chosen as needed, ensuring the model accurately reflects the physical system.

This project not only provides a practical application of multibody dynamics and control theory but also serves as a foundation for further exploration into advanced control strategies and system robustness.

# Chapter 2

# Methods

## Methods

The physical system of the Furuta pendulum was modeled as a series of solid bodies connected via revolute joints. Rigid frame transformations were employed to translate the system into the appropriate coordinate system. A base frame was established using a mechanism configuration block, a world frame block, and a solver configuration block, ensuring the system was properly grounded and simulated. The DC motor, a critical component of the system, was modeled as a multiphysics component to accurately capture its electromechanical behavior.

An initialization script was written to define the system parameters, including the geometric and inertial properties of the arms, joint configurations, and motor characteristics. To validate the model, the pendulum was offset by an initial angle of 5 degrees, and the uncontrolled motion of the system was observed. This step ensured that the system followed expected physical behavior under the influence of gravity, confirming the correctness of the model.

The Simulink model of the system, shown in Figure 2.1, was constructed to simulate the dynamics of the Furuta pendulum. The model includes the multibody components, the DC motor, and the control system. Additionally, the Mechanism Explorer view, depicted in Figure 2.2, provides a visual representation of the physical system, including the orientation of the arms and joints.

Once the model was validated, the system was linearized around the upright position (joint position of 180 degrees) using the Simulink Linearization Manager. The system matrices $A$, $B$, $C$, and $D$ were extracted from the linearized model. These matrices describe the state-space representation of the system and were used for controller design. A Linear Quadratic Regulator (LQR) controller was designed to stabilize the pendulum in its upright position. The Matlab LQR Command was used to calculate the K matrix from Q and R.

Here, $Q$ is a diagonal matrix weighting the state variables, and $R$ is a scalar weighting the control effort. The LQR gain matrix $K$ was computed using the `lqr` function in

MATLAB. The system states were then multiplied by the calculated gain $K$, and the resulting control signal was fed as a voltage input to the DC motor. This closed-loop control system was designed to stabilize the pendulum in its upright position.

To test the robustness of the controller, an external torque step function was applied to Joint 2, simulating a disturbance force at the tip of *Arm 2*. This step ensured that the controller could effectively reject disturbances and maintain stability under perturbed conditions. The results of these tests are discussed in the following sections.

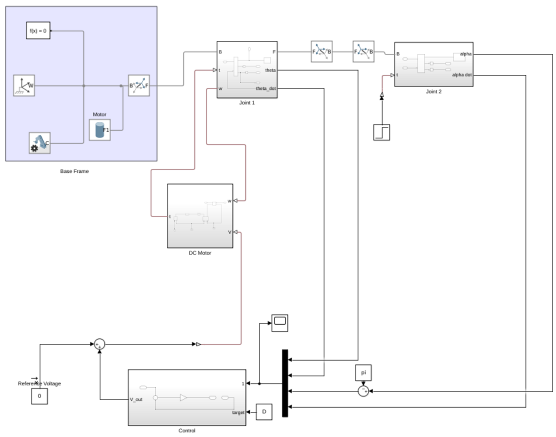

**Figure 2.1.** Simulink model of the Furuta pendulum system.

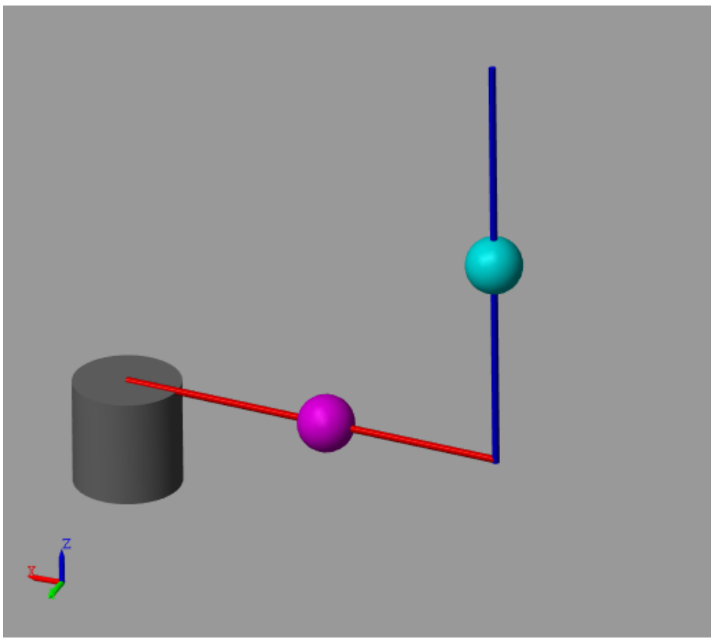

**Figure 2.2.** Mechanism Explorer view of the Furuta pendulum system.

# Chapter 3

# Results and Conclusion

## Results

The performance of the Furuta pendulum system under both initial angular displacement and external torque disturbance was analyzed using simulation results. The response of the system in these scenarios is illustrated in Figures 3.1 and 3.2.

Figure 3.1 depicts the system's behavior when Joint 2 is initially offset by 10 degrees. The results indicate that the system successfully returns to its stable upright position, demonstrating the effectiveness of the LQR controller in stabilizing the pendulum. Joint 2 quickly responds to the control input, bringing the pendulum back to equilibrium with minimal oscillations.

Figure 3.2 presents the system's response to an external torque disturbance applied at Joint 2. The results show that despite the perturbation, the system manages to return to stability, highlighting the robustness of the implemented controller. However, a notable observation is that while Joint 2 exhibits a rapid response to the control action, Joint 1 experiences a slow drift and does not respond as effectively to the target state. This suggests that the current controller may not be optimally tuned for Joint 1, leading to sluggish corrective actions in that axis.

## Conclusion

The implemented LQR controller successfully stabilizes the Furuta pendulum in both an initial angular displacement scenario and in response to an external torque disturbance. The system remains stable and demonstrates good robustness to external perturbations. However, an area for improvement lies in the response of Joint 1, which exhibits slow drift and lacks responsiveness compared to Joint 2.

This behavior could be attributed to several factors:

- The weighting matrices and in the LQR design may not be optimally tuned to balance control efforts between the two joints.

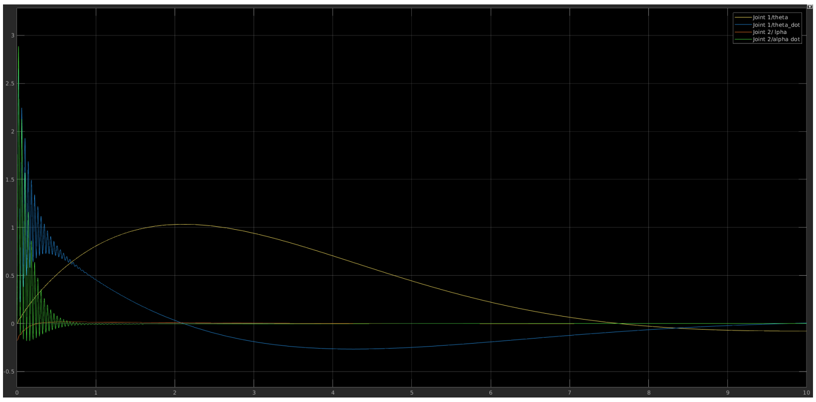


**Figure 3.1.** System response when Joint 2 is initially offset by 10 degrees.

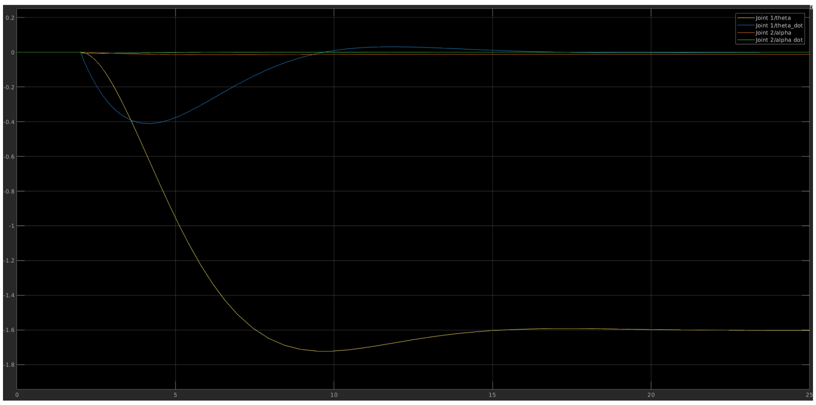


**Figure 3.2.** System response when an external torque disturbance is applied.

- The system dynamics may introduce coupling effects where the control input has a stronger influence on Joint 2 than on Joint 1.
- The actuator dynamics of the DC motor may contribute to slower response times in Joint 1.

To enhance system performance, the following improvements could be considered:

- Fine-tuning the and matrices to ensure better responsiveness in Joint 1 without overloading the actuator.
- Implementing a state observer or augmented state-feedback control to compensate for slow drifts.
- Exploring alternative control strategies such as optimal gain-scheduling or nonlinear control approaches to improve stability and responsiveness.

Overall, the project demonstrates the successful application of multibody modeling and control design in stabilizing the Furuta pendulum. Further refinements in controller design and system modeling could yield improved performance, particularly in addressing the slow drift observed in Joint 1.

# List of Figures