Management Center Innsbruck

Department of Technology & Life Sciences

Master's program Mechatronics & Smart Technologies

Report

composed as part of the course
WS 2024 Multibody and Multiphysics Simulation
(MECH-M-3-SVM-MKS-ILV)

about

Final Project

from

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Chapter 1

Background

1.1 Task Overview

The primary objective of this project is to design and analyze a vibration absorber system for a beam structure subjected to harmonic loading. The beam, which represents a simplified machine structure, experiences vibrations due to an external harmonic force acting on a point mass attached to the beam. The goal is to mitigate these vibrations using both passive and active vibration absorbers.

The project involves several key steps:

- Modeling the beam structure using Simscape Multibody, considering its flexibility and boundary conditions.
- Deriving an equivalent Single Degree of Freedom (SDOF) model to simplify the analysis of the beam's vibrational behavior.
- Designing a passive vibration absorber based on the Den Hartog method, which is a well-established approach for tuning absorbers to specific frequencies.
- Developing an active vibration absorber using either an electromechanical or hydraulic actuator, which allows for real-time tuning of the absorber's parameters to adapt to varying excitation frequencies.
- Comparing the performance of the system without any absorber, with a passive absorber, and with an active absorber across the first three natural modes of the beam.

1.2 Significance of Vibration Control

Vibration control is a critical aspect of mechanical engineering, particularly in the design of structures and machinery. Excessive vibrations can lead to several issues, including:

- **Structural Damage**: Prolonged exposure to vibrations can cause fatigue failure, leading to cracks and eventual breakdown of the structure.

- **Operational Inefficiency**: Vibrations can reduce the efficiency of machinery by causing misalignment, increased wear and tear, and energy losses.

By implementing a vibration absorber, the amplitude of these vibrations can be significantly reduced, thereby enhancing the longevity, efficiency, and safety of the structure. Passive absorbers are simple and cost-effective but are limited to specific frequencies. Active absorbers, on the other hand, offer greater flexibility by adapting to changing conditions, making them suitable for more dynamic environments.

Chapter 2

Methods

2.1 Beam Modeling in Simulink and Simscape Multibody

The beam was modeled as a fixed-free (cantilever) undamped system in Simulink and Simscape Multibody. The parameters of the beam, including geometric and material properties, were defined in a MATLAB initialization script. The flexible I-beam element from Simscape Multibody was used to represent the beam, and a frame was placed at the midpoint of the beam to apply the harmonic force.

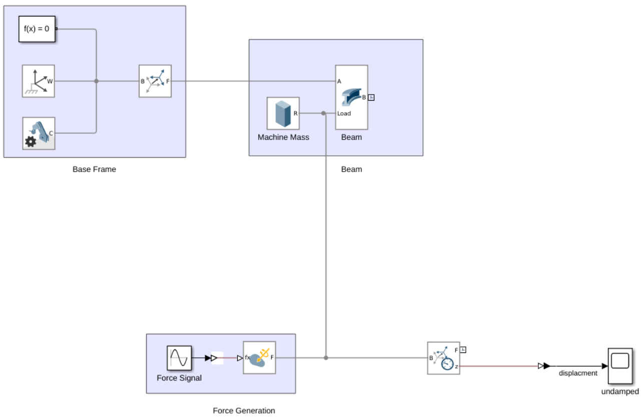

Figure 2.1. Simulink model of the undampened system.

The beam's natural frequencies were calculated using the following formula for a fixed-free beam:

$$\omega_n = \sqrt{\frac{EI_y}{\rho A L^4}} \cdot (\beta L)^2$$

where:

- E is the Young's modulus,

- I_y is the moment of inertia of the beam,
- ρ is the material density,
- A is the cross-sectional area,
- L is the length of the beam,
- βL is the dimensionless frequency parameter for the fixed-free beam, with the first few values being $\beta L = [1.875, 4.694, 7.855, 10.996]$.

The first three natural frequencies were calculated and used to assess the system's response. A sinusoidal force was applied at the midpoint of the beam with a frequency equal to the first natural frequency of the system.

2.2 Equivalent Single Degree of Freedom (SDOF) System

To simplify the analysis, the beam system was reduced to an equivalent SDOF system. The equivalent mass m_{eq} and stiffness k_{eq} were calculated using the mode shape of the beam. The mode shape for a fixed-free beam is given by:

$$w(x) = C_n \left(\cosh\left(\frac{\beta x}{L}\right) - \cos\left(\frac{\beta x}{L}\right) - \alpha_n \left(\sinh\left(\frac{\beta x}{L}\right) - \sin\left(\frac{\beta x}{L}\right) \right) \right)$$

where:

- C_n is a normalization constant,
- $\alpha_n = \frac{\cosh(\beta L)+\cos(\beta L)}{\sinh(\beta L)+\sin(\beta L)}$,
- β is the frequency parameter for the first mode.

The equivalent mass m_{eq} was calculated by integrating the square of the mode shape over the length of the beam:

$$m_{eq} = m_m + \rho A \int_0^L w(x)^2 \, dx$$

where m_m is the machine mass attached to the beam. The equivalent stiffness k_{eq} was calculated using the second derivative of the mode shape:

$$k_{eq} = EI_y \int_0^L \left(\frac{d^2 w(x)}{dx^2} \right)^2 \, dx$$

The natural frequency of the equivalent SDOF system was then calculated as:

$$\omega_{eq} = \sqrt{\frac{k_{eq}}{m_{eq}}}$$

2.3 Passive Absorber Design

A passive vibration absorber was designed using Den Hartog's method, which provides optimal parameters for the absorber mass, stiffness, and damping. The absorber mass m_t was chosen as a fraction of the equivalent mass m_{eq}, typically $m_t = 0.05m_{eq}$. The optimal tuning ratio r_{opt} and damping ratio ζ_{opt} were calculated as:

$$r_{opt} = \frac{1}{\sqrt{2}}, \quad \zeta_{opt} = \frac{1}{2}\frac{m_t/m_{eq}}{1-r_{opt}^2}$$

The optimal damping coefficient c_{opt} and spring stiffness k_t were then determined:

$$c_{opt} = 2\zeta_{opt}\sqrt{m_t k_{eq}}, \quad k_t = m_t\omega_t^2$$

where $\omega_t = r_{opt}\omega_{eq}$ is the tuned frequency of the absorber. The passive absorber was implemented in Simscape Multibody as a prismatic joint with internal damping and spring stiffness, and a mass attached to it.

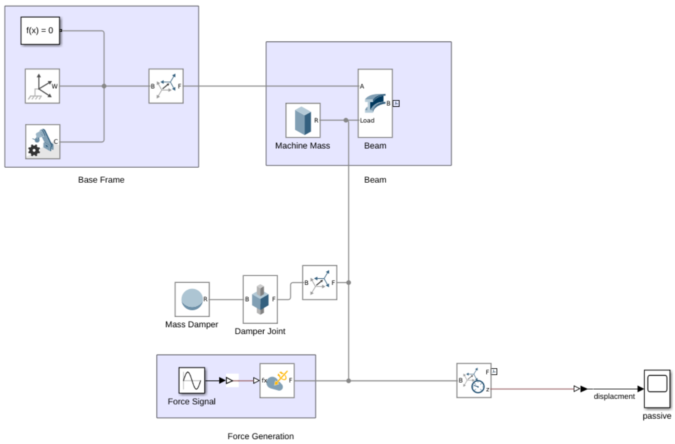

Figure 2.2. Simulink model of the passive dampened system.

2.4 Active Absorber Design

The active absorber was modeled using a Voice Coil Actuator (VCA) as a multiphysics component in Simulink. A Proportional-Derivative (PD) controller was designed to control the actuator force $u(t)$. The controller gains K_P and K_D were calculated using the following equations:

$$K_P = \Re\left(\frac{-(Ls+R)(m_t s^2 + r_t s + k_t) - M^2 s}{M}\right)$$

$$K_D = \frac{1}{\Omega}\Im\left(\frac{-(Ls+R)(m_t s^2 + r_t s + k_t) - M^2 s}{M}\right)$$

where:

- L and R are the inductance and resistance of the actuator,
- m_t, r_t, and k_t are the mass, damping, and stiffness of the absorber,
- M is the actuator constant,
- $s = i\Omega$ is the complex frequency, with Ω being the excitation frequency.

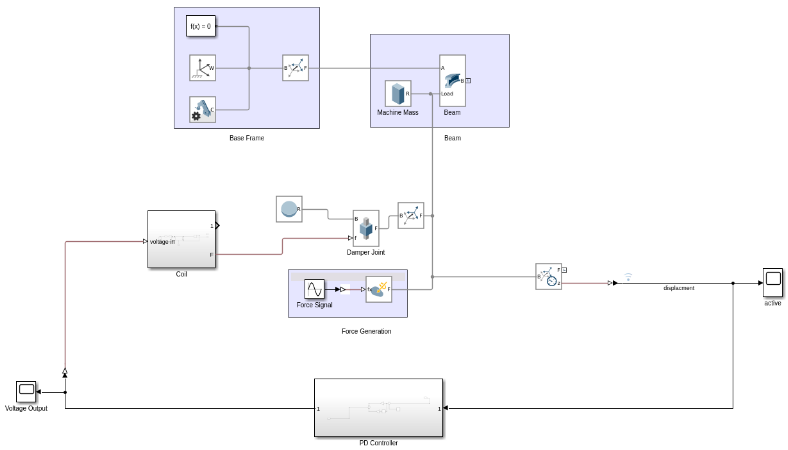

Figure 2.3. Simulink model of the active dampened system.

The system performance was assessed for the undamped beam, the beam with a passive absorber, and the beam with an active absorber. The results were compared to determine the effectiveness of each vibration control strategy.

Chapter 3

Results and Conclusion

3.1 Results

The system responses for the undampened, passive dampened, and active dampened systems were analyzed and compared. The results are presented below, along with the corresponding figures.

1. **Undampened System Response**: The undampened system freely oscillated with a maximum magnitude of -2.5×10^{-3} m and settled into a steady-state oscillation of -2×10^{-3} m. The response is shown in Figure 3.1.

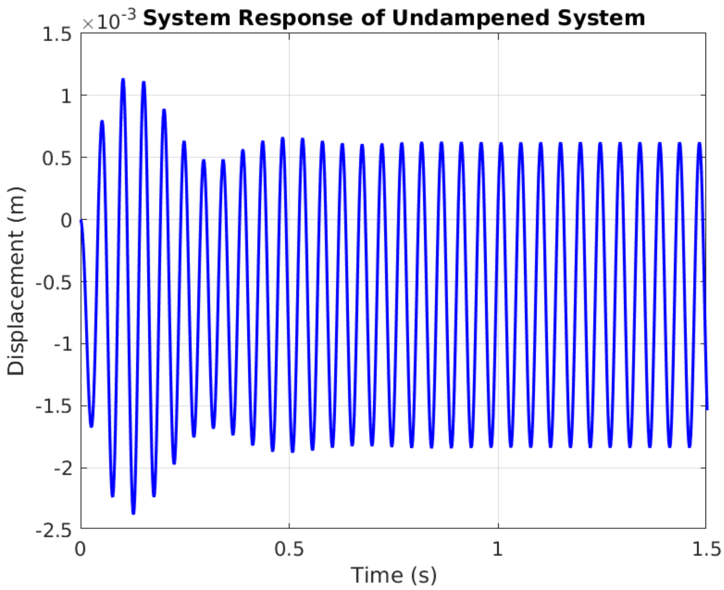

Figure 3.1. System response of the undampened system.

2. **Passive Dampened System Response**: The passive dampened system oscillated with a maximum magnitude of -2×10^{-3} m and settled into a steady-state oscillation of -1.4×10^{-4} m. The response is shown in Figure 3.2.

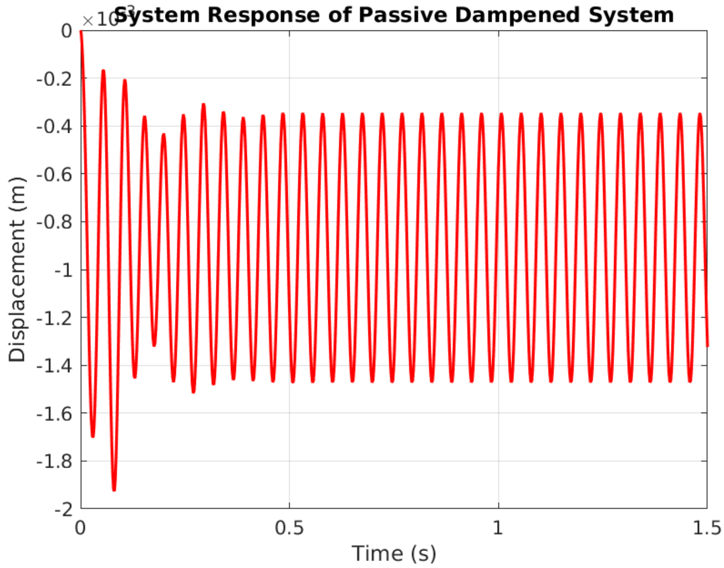

Figure 3.2. System response of the passive dampened system.

3. **Active Dampened System Response**: The active dampened system oscillated with a maximum magnitude of -1.75×10^{-3} m and settled into a steady-state oscillation of -0.75×10^{-3} m. The response is shown in Figure 3.3.

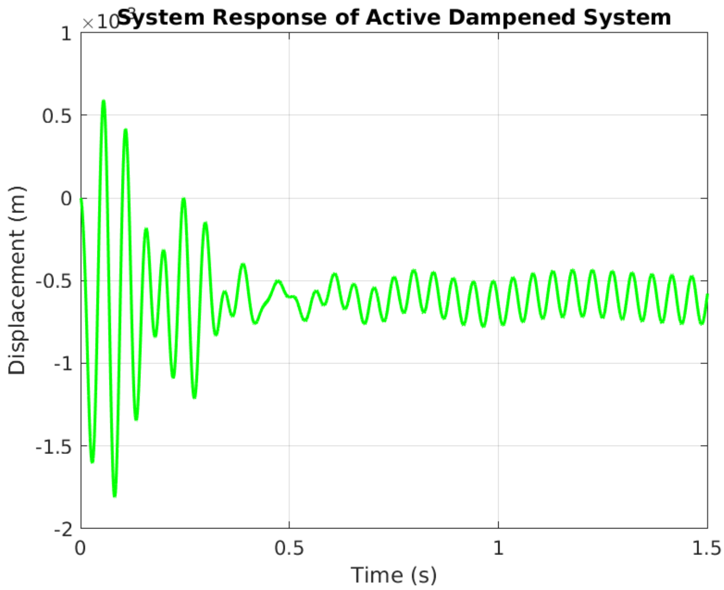

Figure 3.3. System response of the active dampened system.

4. **Comparison of System Responses**: A comparison of the undampened, passive dampened, and active dampened systems is shown in Figure 3.4. The active dampened system demonstrates superior performance in reducing oscillations, while the passive dampened system provides a simpler alternative with significant vibration reduction.

The undampened system, although simple, exhibits dangerous oscillations that could lead to structural failure.

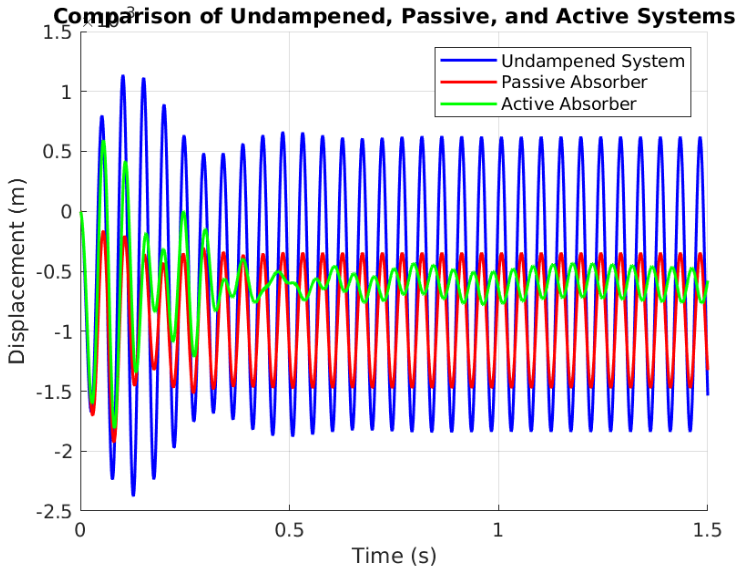

Figure 3.4. Comparison of the undampened, passive dampened, and active dampened system responses.

3.2 Conclusion

The results demonstrate the effectiveness of vibration control strategies in reducing oscillations in a beam structure subjected to harmonic loading. The key findings are summarized as follows:

1. **Undampened System**: - The undampened system exhibited large oscillations with a maximum magnitude of -2.5×10^{-3} m and settled into a steady-state oscillation of -2×10^{-3} m. - While this system is simple, the large oscillations could lead to structural damage or failure, making it unsuitable for practical applications.

2. **Passive Dampened System**: - The passive dampened system reduced the maximum oscillation magnitude to -2×10^{-3} m and settled into a steady-state oscillation of -1.4×10^{-4} m. - This system provides a significant improvement over the undampened system with relatively low complexity, making it a cost-effective solution for vibration control.

3. **Active Dampened System**: - The active dampened system further reduced the maximum oscillation magnitude to -1.75×10^{-3} m and settled into a steady-state oscillation of -0.75×10^{-3} m. - Although more complex, the active system offers superior performance and adaptability, making it ideal for applications requiring precise vibration control.

4. **Comparison**: - The active dampened system is clearly superior in reducing oscillations, but its complexity and cost may not be justified for all applications. -

The passive dampened system strikes a balance between performance and complexity, making it a practical choice for many scenarios. - The undampened system, while simple, is not suitable for applications where vibrations must be controlled.

In conclusion, the choice of vibration control strategy depends on the specific requirements of the application. For systems where simplicity and cost are critical, the passive dampened system is recommended. For applications requiring high performance and adaptability, the active dampened system is the best choice. The undampened system should be avoided in scenarios where vibrations could lead to structural damage or operational issues.

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