Final Project Description

Multibody & Multiphysics Simulation

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Chapter 1

Introduction

As has been detailed in the lecture slides the final grade for the lecture Multibody & Multiphysics Simulation is comprised of a theoretical part and a practice part, that are both equally weighted. The practice sessions are assessed based on two projects, Task III contributing 20 %, and the final project that is described in this document, contributing 80 %.

The project's ultimate aim is the modelling of a vibration absorber system for a simple machine on a beam structure using Simscape. One such example is shown in Fig. 1.1 and has been discussed in detail in the course Machine Dynamics [1]. Please refer back to these course documents for a refresher on the topic if required.

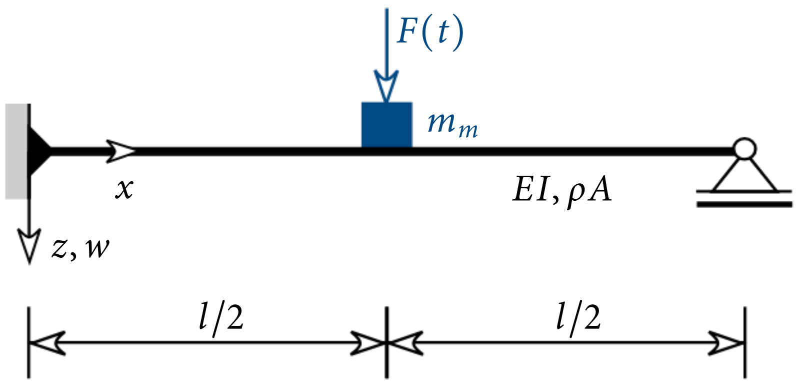


Fig. 1.1: Simple model of a machine on a beam structure [1].

Each group will get their own beam structure with a different beam cross section or boundary conditions. Therefore, a vibration analysis has to be carried out for the beam type and load case, based on which a vibration absorber system can be designed. The flexible beam structure shall be modelled using Simscape Multibody, whether as lumped-parameter model or reduced order model is up to the reader. Each group is comprised of up to 3 students, which can work together on one simulation.

Chapter 2

Vibration Absorber

2.1 Problem Outline

The aim of the presented final project is to design a vibration absorber system, both in a passive as well as an active manner, for a beam under the impact of a harmonic load. This load $F(t)$ will act on a „machine" that is represented by a point mass of mass m_m, as seen in Fig. 1.1. The machine excitation occurs at the natural frequency of the system with a given amplitude.

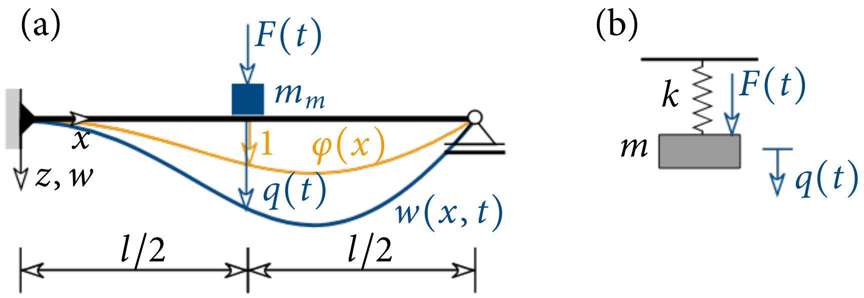


Fig. 2.1: (a) First eigenmode and vertical vibration of the beam, (b) equivalent SDOF model. [1]

The system modelled as such shall be evaluated and compared for its response in the first 3 mode for the following different cases:

- without absorber
- with passive absorber
- with active absorber

2.2 Simscape multibody model

The first part of the task will be setting up a system in Simscape, representing the beam as a deformable body. This can be done by making use of the Flexible Beam components or the Reduced Order Model (ROM) methods, as detailed in the slides.

Depending on the support configuration selected for the task, different joints shall be utilised in order to appropriately define the system's boundary conditions. The placement of the support or joints shall be realised with extreme care, as the base and follower frame of them **must** be aligned at the start of the simulation.

2.3 Equivalent SDOF model

After a flexible beam model has been established an initial passive absorber can be designed to dampen undesired oscillations due to the harmonic loading. This can be done by converting the system into its equivalent SDOF system, as shown in Fig. 2.1. This gives an equivalent stiffness and mass, based on which the vibration absorber can be developed.

This can be easily done by employing the Ritz approach for approximating the vertical vibration of the beam by its eigenmode of interest, as $w(x,t) \approx q(t)\varphi(x)$. The full procedure is detailed in [1], [2] or a number of other text books.

An example of the calculation procedure for a pinned-free beam is given in the following MATLAB Listing 2.1. Of particular importance are the solutions of the frequency equation βl (`beta_L`), which are also available in tables in many text books such as [2].

List. 2.1: Determine equivalent SDOF from beam

```
%% Parameters
%A rectangular beam is considered in this example
E = 210 * 1e9; %Pa
b = 50e-3; %in m
h = 5e-3; %in m

Iy = b*h^3 / 12; %in m^4
A = b*h; %in m^2

rho = 7850; %in kg/m^3
l = 1; %in m

%Machine parameters
m_m = 1; %in kg

%% Natural frequencies
beta_L = [3.9266, 7.0685, 10.2101, 13.3518]';
fn = sqrt((E*Iy)/(rho*A*l^4)) .* beta_L.^2 ./ (2*pi)

%% Mode shapes
%Ritz approach:
%w(x) = A sin(beta x) + B cos(beta x) + C sinh(beta x) + D cosh(beta x)
%Apply the coundary conditions (pinned-free) & simplify the expression to solve
%w(x) = C_n * (sin(beta x) + alpha_n * sinh(beta x))

% Calculation of Constant C_n within the mode shape equation depending on the
% beam support case scaling the mode wm(x) to 1 at the center of the beam,
bl = beta_L(1);
alpha_n = sin(bl)/sinh(bl);
```

```
x_cen = l/2;
wn_cen = 1 * (sin(bl/l*x_cen) + alpha_n * sinh(bl/l*x_cen));
C_n = 1 / (wn_cen);

x = linspace(0, l, 200);
w_n = C_n * (sin(bl/l*x) + alpha_n * sinh(bl/l*x));
plot(x, w_n);

%% Equivalent SDOF
syms x_sym

%Mode shape as symbolic function
w_x_sym = C_n * (sin(bl/l*x_sym) + alpha_n * sinh(bl/l*x_sym));

%Contribution to the kinetic energy
T_sym_beam = int(w_x_sym^2, x_sym, [0, 1]);
T_beam = double(T_sym_beam);

m_beam = T_beam*rho*A*l;
m_beam_system = m_m + m_beam;

%Contribution to the potential energy
w_xx = diff(diff(w_x_sym));
V_sym_beam = int(w_xx^2, x_sym, [0, 1]);
V_beam = double(V_sym_beam);
k_beam = V_beam * E*Iy/(l^3);

%Differ because of m_m
omega_beam = sqrt(k_beam/m_beam)
omega_beam_system = sqrt(k_beam/m_beam_system)
```

2.4 Passive Absorber

Once the equivalent SDOF model is found a passive vibration absorber can be designed as shown in Fig. 2.2. In this case the passive absorber in combination with the beam system acts as a 2-DOF model.

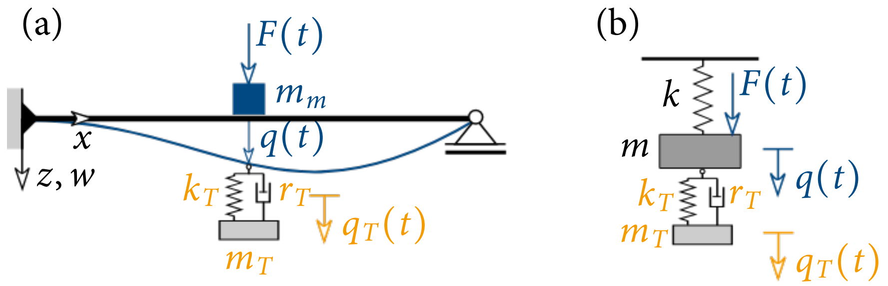


Fig. 2.2: (a) Mounted absorber to system, (b) equivalent two-degrees-of-freedom model [1]

A solution for an optimal design of the absorber mass, stiffness and damping coefficients has been proposed nearly a century ago by Den Hartog and has also been discussed at length in the

Machine Dynamics lecture [1].

2.5 Active Absorber

Passive vibration absorbers are very simple to design and build, but have the fundamental disadvantages that they are only designed for excitations of one frequency Ω. If the absorber could change its parameters m_T, k_T and r_T good performance could be achieved for many more cases. This is achieved by introducing an actuator that can tune the absorber behaviour in real time.

This actuator conceptually exerts a force $u(t)$ between the two masses of Fig. 2.2, therefore acting in parallel to k_T and r_T. As $u(t)$ can change with arbitrary dynamics it can act like any combination of spring and damper. Depending on the working principle of this actuator a Simscape multiphysics system can be created that models this force $u(t)$.

2.5.1 Electromechanical Actuator

As Task II has already expected you to create am electromechanical voice coil actuator a similar approach could be used for the active absorber. Literature (e.g. [3]) already proposes many control schemes for such actuators in the use case of active vibration control.

The main idea of many of these systems is to tune the controller gains depending on the system dynamics of the actuator (parameterized by R, L and M) and the damper system (parameterized by $m_{T,a}$, $r_{T,a}$ and $k_{T,a}$) to have a natural frequency that corresponds to the excitation frequency Ω.

For the approach discussed in [3] of using a PD-controller that acts on the absorber displacement $q_{T,a}(t)$ as an error, and outputs the actuator voltage the gains could be derived as follows:

$$
\begin{aligned}
K_P &= \mathfrak{R}\left(\frac{-(Ls+R)(m_{T,a}s^2 + r_{T,a}s + k_{T,a}) - M^2 s}{M}\right)\\
K_D &= \frac{1}{\Omega}\mathfrak{I}\left(\frac{-(Ls+R)(m_{T,a}s^2 + r_{T,a}s + k_{T,a}) - M^2 s}{M}\right)
\end{aligned}
$$

Herein R and L are the actuator resistance and inductance respectively, M is the actuator constant of proportionality and the condition $s = i\Omega$ must hold.

2.5.2 Hydraulic Actuator

An alternative approach could be using a hydraulic actuator instead of an electromechanical one. Also this case the design of the control system is in principle the same as for the electromechanical actuator. One just has to consider different actuator dynamics that describe how the controller output (e.g. a valve opening in %) cause an actuator force $u(t)$.

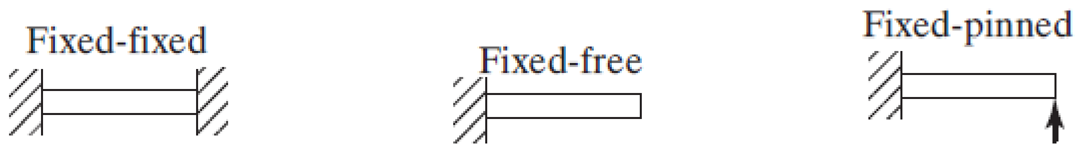
2.6 Summary of the objectives

1. Understand the system behaviour and properly assess the components which will be needed to construct the system.
2. Gather the starting parameters of your system from the "selected" beam configuration.
3. Build the Simscape-Multibody system **without** any absorber.
4. Derive the parameters of the equivalent SDOF system.
5. Design a passive absorber according to Den Hartog.
6. Select one actuator type and design an active absorber. This consists of a multiphysics component **and** a controller.
7. Test all systems with an harmonic force with a frequency equal to the first 3 natural modes of the structure.
8. Compare the results of the 3 load cases and the 3 systems. It is a good idea to normalize results and graphs to be independent of the input force magnitude.
9. Find out the best vibration absorber solution and discuss the results.

Chapter 3

Set-up

As there are 3 groups in the course each group will receive a different set of boundary conditions for their beam, as shown in Fig. 3.1.



(a) Boundary conditions gr. 1. (b) Boundary conditions gr. 2. (c) Boundary conditions gr. 3.

Fig. 3.1: Boundary conditions for each group.

The geometry of the beam will be identical for all groups and will be a classical, simplified I-beam with the following parameters:

Table 3.1: Parameters of the beam.

Parameter	Symbol	Value	Unit
Height (parallel to z-axis)	h	55	mm
Width (parallel to y-axis)	w	65	mm
Thickness	t	5	mm
Beam length	L	1.5	m
Load and mass location	$a = L/2$	0.75	m
Young Modulus	E	210	GPa
Poisson ratio	ν	0.33	-
Material density	ρ	7850	$\mathrm{kg\,m^{-3}}$
Machine mass	m_m	10	kg

Chapter 4

Handing in the project

The objective of this project is creating a **functioning** Simscape simulation. This is most often only possible if the simulation model is prepared well and structured in a clean fashion. An example of what **not** to create is shown in Fig. 4.1.

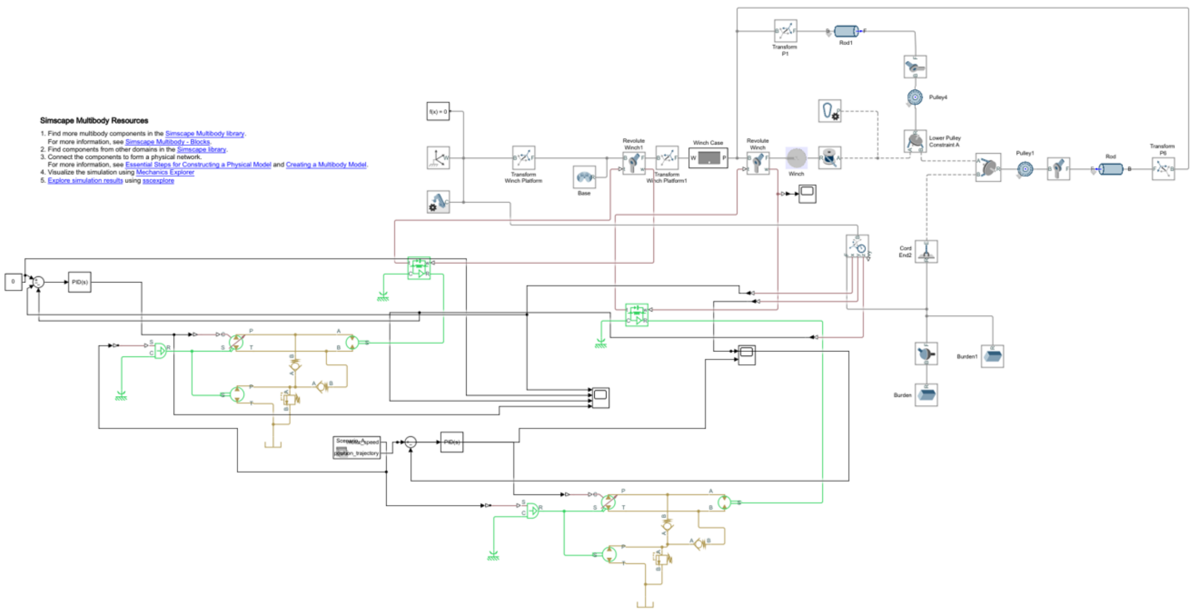


Fig. 4.1: Unclean model. How **NOT** to hand it in.

To some extent a clean model also acts as its own kind of documentation, as it allows a user to follow along with what is simulated.

Nevertheless, a report is still the best tool to show and compare the simulation results for all developed vibration absorbers at all frequencies of interest. It further is used to document implementation details such as the developed control strategy for the active vibration absorber. Additionally, the results must also be discussed and contextualized to the problem.

All of these aspects do not require an extensive report, so make sure to keep it short and concise.

Bibliography

[1] F.-J. Falkner, *Machine Dynamics Lecture Notes*, 2023.

[2] S. S. Rao, *Mechanical Vibrations*, 2011.

[3] C. Rincon, J. Alencastre, and R. Rivera, *Analytical modelling of an active vibration absorber for a beam*, 2023.