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CSCI 3656

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Homework 5

1.

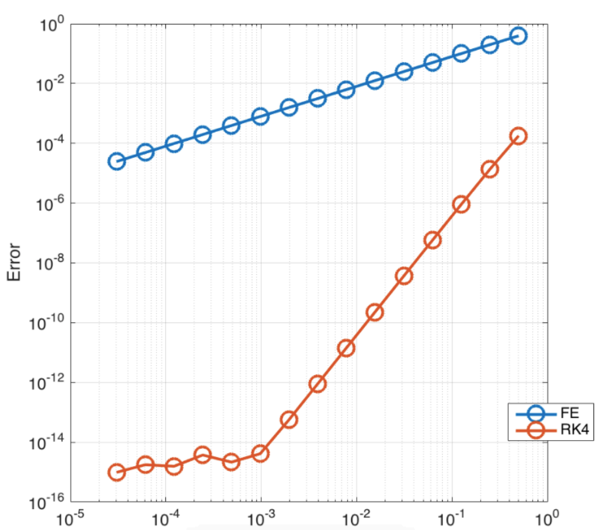
A.

$y'(t) + y(t) = \sin(t) \qquad y(0) = 0$

$y(t) = \frac{1}{2}(e^{-t} + \sin(t) - \cos(t)) \qquad y'(t) = -\frac{1}{2}e^{-t} + \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t)$

$y'(t) + y(t) = \left(\cancel{\tfrac{1}{2}e^{-t}} + \tfrac{1}{2}\sin(t) - \cancel{\tfrac{1}{2}\cos(t)}\right) + \left(\cancel{-\tfrac{1}{2}e^{-t}} + \cancel{\tfrac{1}{2}\cos(t)} + \tfrac{1}{2}\sin(t)\right)$

$y'(t) + y(t) = \sin(t)$



B. The forward Euler method produced greater error than the Runge-Kutta method as seen in the ideal plot above. The asymptotic regime for forward Euler ranges from $10^{-4.8}$ to

$10^{-0.5}$. The convergence rate of this method is the slope of the line within the asymptotic regime. For forward Euler, the rate of convergence is about 1.

C. The asymptotic regime for Runge-Kutta is the range of values in which the slope of the line is constant, and in this example, the range is from 10^{-3} to $10^{-0.5}$. As stated before, the rate of convergence is the slope of the line within the asymptotic regime. For this method, the rate of convergence is approximately 4.

2.

A.

$$y'''(t) + y''(t) + 4y'(t) + 4y(t) = 4t^2 + 8t - 10 \qquad t[0,5]$$

$$y(0) = -3 \qquad y'(0) = -2 \qquad y''(0) = 2$$

$$y(t) = -\sin(2t) + t^2 - 3$$

$$y'(t) = -2\cos(2t) + 2t$$

$$y''(t) = 4\sin(2t) + 2$$

$$y'''(t) = 8\cos(2t)$$

$$\rightarrow 8\cos(2t) + (4\sin(2t) + 2) + 4(-2\cos(2t) + 2t) + 4(-\sin(2t) + t^2 - 3) = 4t^2 + 8t - 10$$

$$\rightarrow \cancel{8\cos(2t)} + \cancel{4\sin(2t)} + \cancel{2} + \cancel{-8\cos(2t)} + \cancel{8t} + \cancel{-4\sin(2t)} + \cancel{4t^2} \cancel{-12} = \cancel{4t^2} \cancel{+8t} \cancel{-10}$$

$$0 = 0 \quad \therefore \text{equivalent}$$

$$\therefore y(t) = -\sin(2t) + t^2 - 3$$

B.

$$U_1(t) = y(t)$$

$$U_1'(t) = y'(t) = U_2(t)$$

$$U_2'(t) = y''(t) = U_3(t)$$

$$U_3'(t) = y'''(t)$$

$$= 4t^2 + 8t - 10 - 4y(t) - 4y'(t) - y''(t)$$

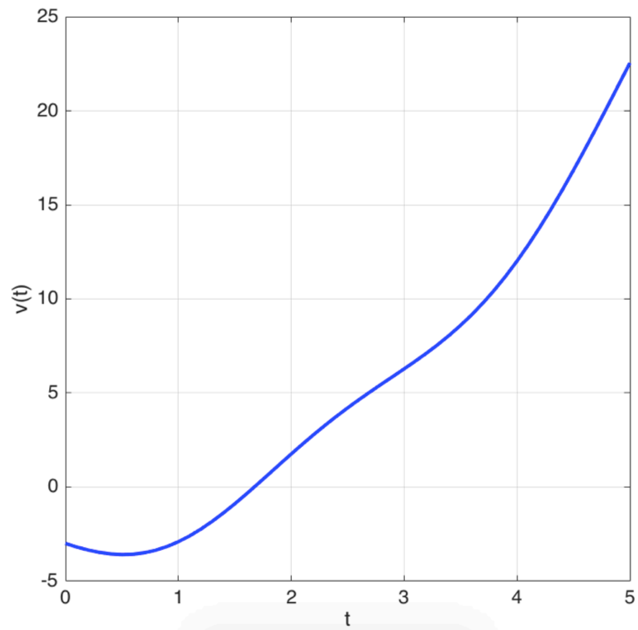
$$= 4t^2 + 8t - 10 - 4U_1(t) - 4U_2(t) - U_3(t)$$

System

$$U = \begin{bmatrix} U_1'(t) \\ U_2'(t) \\ U_3'(t) \end{bmatrix} = \begin{bmatrix} U_2(t) \\ U_3(t) \\ 4t^2 + 8t - 10 - 4y(t) - 4U_2(t) - U_3(t) \end{bmatrix} \Rightarrow$$

Initial Conditions

$$\begin{bmatrix} U_1(0) \\ U_2(0) \\ U_3(0) \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \\ 2 \end{bmatrix}$$



C. The plot above shows the ideal representation of ode45's approximations as a function of time. ODE45 returns an approximation of the solution differential equation. This approximation is similar to the real solution on the interval from 0 to 5.

Plots are borrowed