CSCI3656: NUMERICAL COMPUTATION
Test 1, Part 1: Due Wednesday, Oct. 11, 3:00pm

You may collaborate in groups of **at most** three, but you must turn in your own writeup. Please list the names of the people you collaborated with and any external resources you used. Please submit an electronic version via Moodle that includes code and plots. **Submissions will shut off at 3:00pm on Wednesday, October 11, and no late submissions will be accepted.**

1. Write a function called `vandermonde` that takes two arguments, n and d, where n is the number of points and d is degree of polynomial, and returns the Vandermonde matrix of size $n \times (d+1)$. The values $x_1, \dots, x_n$ that you use should be evenly spaced in the interval $[-1, 1]$, with $x_1 = -1$ and $x_n = 1$. As a check,

$$\texttt{vandermonde(3,2)} \text{ returns } \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad \texttt{vandermonde(4,1)} \text{ returns } \begin{bmatrix} 1 & -1 \\ 1 & -1/3 \\ 1 & 1/3 \\ 1 & 1 \end{bmatrix}.$$

2. Let A_n be the $n \times n$ Vandermonde matrix computed with your function `vandermonde(n,n-1)`. For $n = 2^k + 1$ with $k = 1, \dots, 8$, plot the condition number of A_n versus n. Note that the Matlab function `cond` computes the condition number of a matrix. Use a logarithmic scale on the vertical axis of your plot. (Hint: look up the function `semilogy` for the plot.) *What's going on?!* What does this tell you about interpolation on evenly spaced points?

3. Consider the function

$$f(x) \;=\; 2 + x + x\sin(2\pi x), \quad x \in [-1, 1]. \tag{1}$$

Using your `vandermonde` function, compute the coefficients of a least-squares-fit degree-3 polynomial from $n = 33$ evenly spaced samples of the function. In other words, your data is

$$x_i = -1 + 2\,\frac{i-1}{n-1}, \qquad y_i = f(x_i), \quad i = 1, \dots, n, \tag{2}$$

where $n = 33$. Make a plot of both $f(x)$ and the cubic polynomial approximation on 100 evenly spaced points.

4. For d from 1 to 31, compute the least-squares coefficients of a polynomial of degree d using the function $f(x)$ from Equation (1) sampled at 33 evenly spaced points in $[-1, 1]$. In other words, your data is the same as in Problem 3. For each d compute the largest error in the polynomial approximation over 100 evenly spaced points. In other words, if $p_d(x)$ is your polynomial of degree d, the error e_d is

$$e_d \;=\; \max_{1 \le j \le 100} \frac{|f(x_j) - p_d(x_j)|}{|f(x_j)|}, \tag{3}$$

where

$$x_j \;=\; -1 + 2\,\frac{j-1}{99}, \quad j = 1, \dots, 100.$$

Plot the error e_d versus d on a log scale (that is, use `semilogy`). Interpret the error behavior as a function of the polynomial degree.

5. For d from 1 to 31, compute the condition number of the matrix you used to compute the least-squares polynomial coefficients in the previous part. Plot the condition number versus d on a log scale. How does this compare to the plot you made in the Problem 2?