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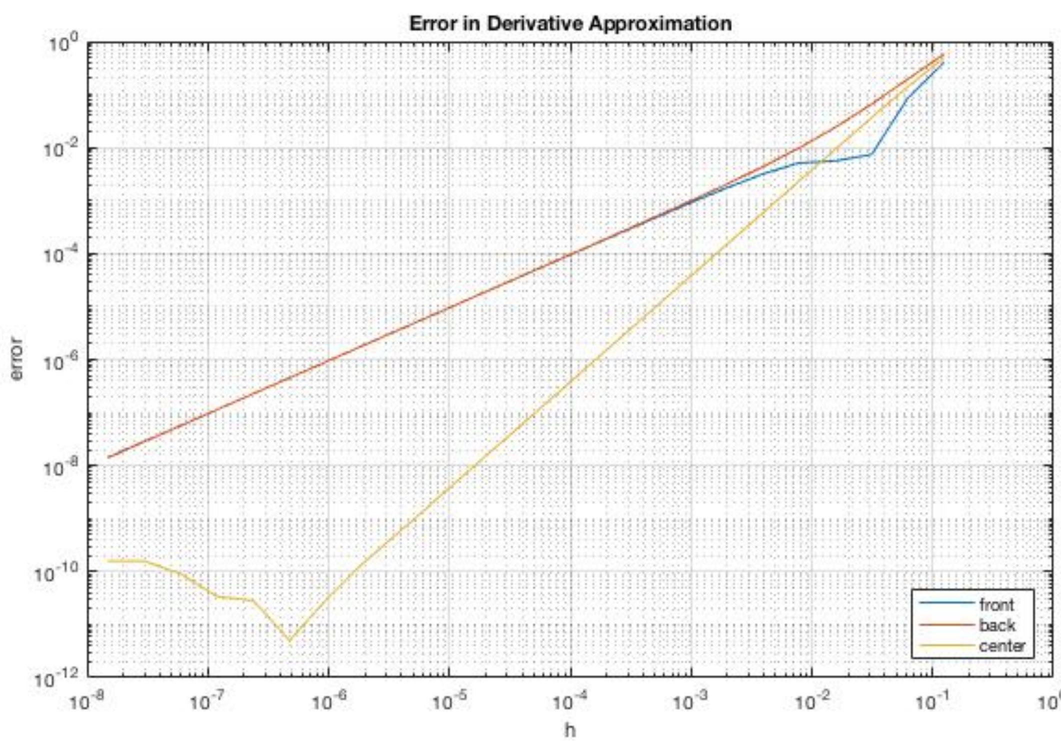
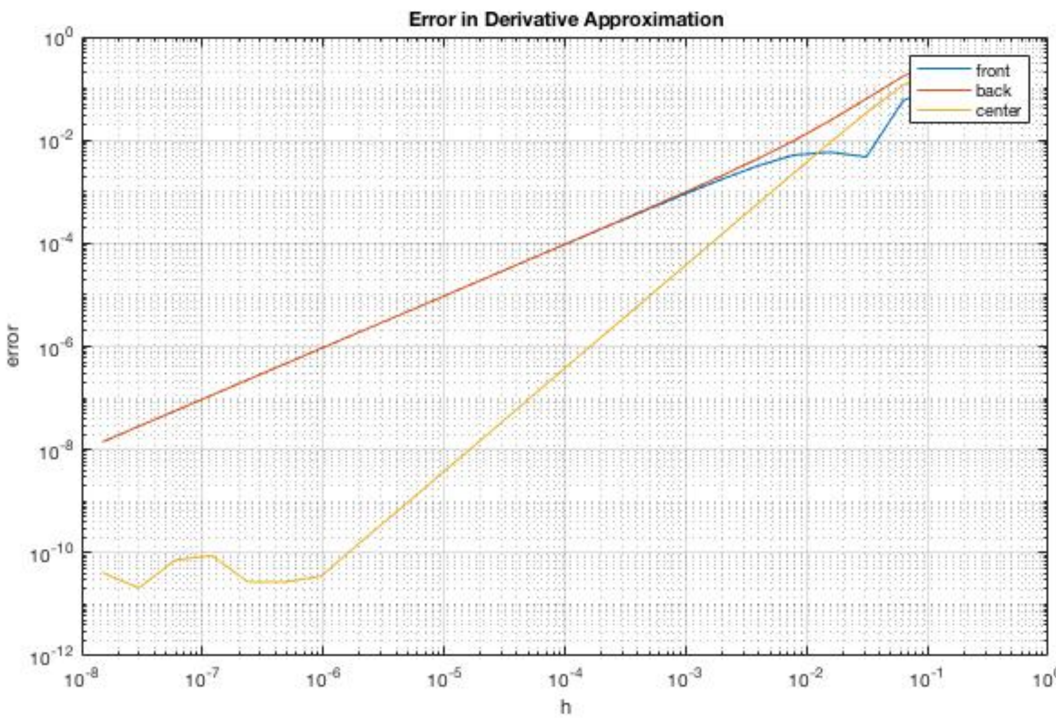
Homework 4

1. The asymptotic regime is the region of values of *h* in which the slope of the function is constant. The values of *h* that respond to the asymptotic regime in the one-sided forward difference are the values ranging from 3 to 22. The values of *h* that respond to the asymptotic regime in the one-sided backward difference are the values ranging from 3 to 19. The values of *h* that respond to the asymptotic regime in the central difference are the values ranging from 8 to 26. The rate of convergence is simply the slope of the lines within the asymptotic regime. Analyzing the apparent values of the error vs. the values of h, the rate of convergence of the forward difference is 1, the backward difference is 1, and the center difference is 2.



2. The process of analyzing the behavior of the approximations, the majority of the process remained the same as in problem 1. The only difference was that instead of manually calculating the derivative of the function, the given approximation was used instead. This resulted in an almost identical graph with very slight differences. The forward and backward

differences produces nearly identical lines; however, the center difference shows a fluctuation for smaller *h* rather than a large dip like in the first problem. The asymptotic regime as well as the rate of convergence for the forward and backward approximations remains the same. The asymptotic regime for the center difference changes to 9 to 25, but the rate of convergence remains the same.



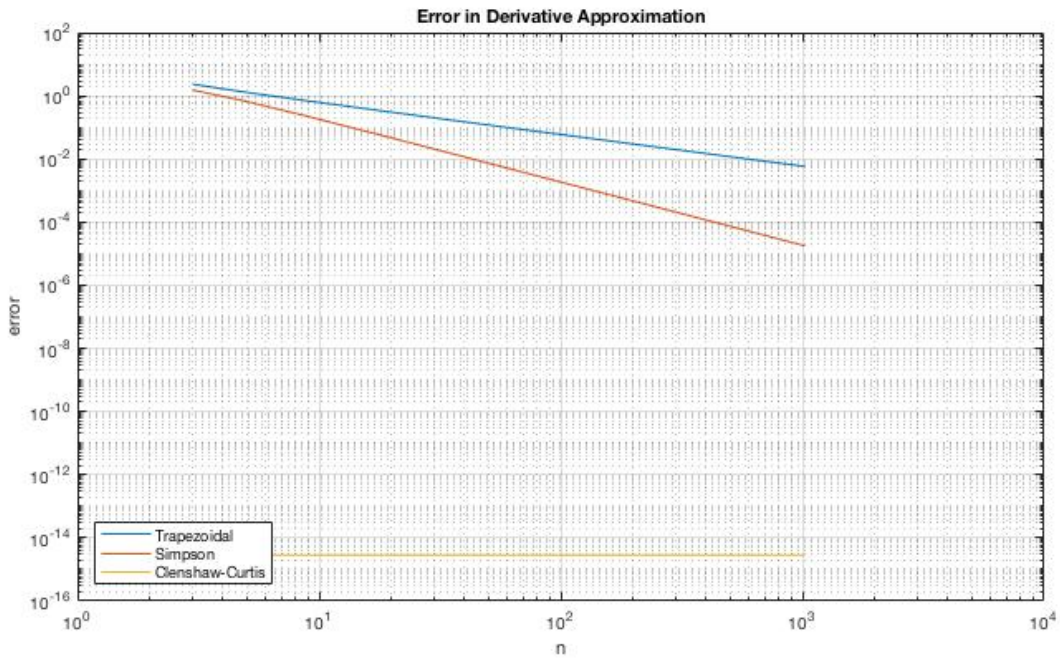
3. Implementing the various rules

   A. The work to calculate the definite integral:

$f(x) = \cos(3\pi x)$

$$\int_{1}^{1.5} \cos(3\pi x)\,dx = \frac{\sin(3\pi x)}{3\pi}\Bigg]_{1}^{1.5} = \frac{\sin(3\pi(1.5))}{3\pi} - \frac{\sin(3\pi(1))}{3\pi} = 0.1061\ldots$$

B. After many attempts to properly shift and scale [((b-a)/2)xj + (b-a)/2] for the Clenshaw-Curtis approximation, we were unable to get a working piece of code to produce the correct line. We did manage to get Simpson's and the trapezoidal rule to plot properly.



C. The asymptotic regime for both the Trapezoidal and Simpson's seems to be the whole range of *n*'s used ranging from 3 to 2^{10}. The rates of convergence for Trapezoidal rules is approximately -2, and the rate of convergence for the Simpson's rule is approximately -4. Based on the plot provided by the professor, the asymptotic regime for Clenshaw-Curtis exists within the range of 2^3 to 2^4. The rate of convergence is heavily negative at a value of approximately -36 by eying the values.

4. The rate of converge will differ from the first function because the function only exists at three values of *f(x)* and is not continuous (only returns values of 0, 1, or 2) as opposed to the continuous function used in three that exists everywhere.