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**Electrical Engineering and Computer Science**  
**EECS 358 - INTRODUCTION TO PARALLEL COMPUTING**

**Lecture 9**  
**Dist. Memory Message Communication**

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# Outline

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- One-to-all Broadcast
- All-to-all Broadcast
- One-to-all Scatter
- All-to-all Scatter

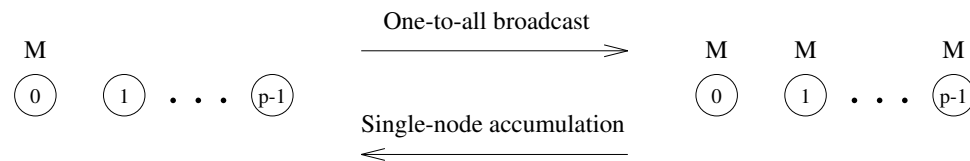
# One-to-all Broadcast

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- Algorithms often require a processor to send identical data to all other processors or a subset of processors. This operation is called a one-to-all broadcast or singlenode broadcast
- At the start of a singlenode broadcast, a processor has  $m$  words of data that needs to be sent, at the end there are  $p$  copies of this data, one on each processor
- The dual of a broadcast operation is a all-to-one reduction or singlenode reduction
- At the start of a singlenode reduction each processor has  $m$  words of data, the reduction combines all the data from processors using an associative operator to produce  $m$  words at the receiver
- Naive singlenode broadcast or reduction using  $p - 1$  steps
- Scatter is similar to broadcast, but each node gets a different data element

# One-to-all Broadcast

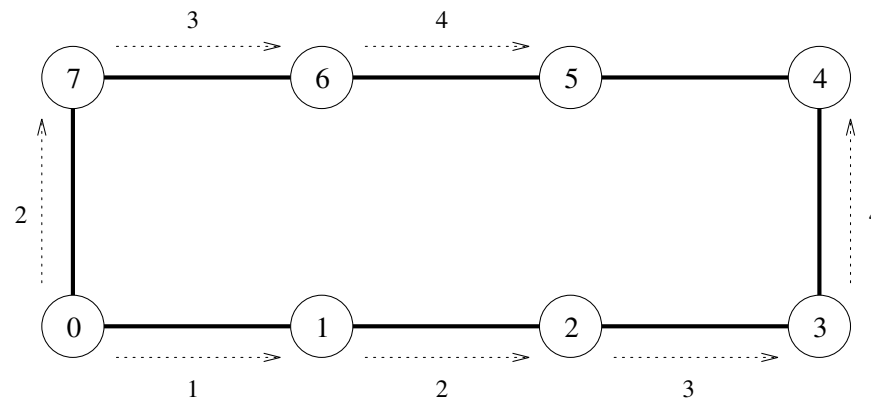
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# Store-and-forward Routing on Ring

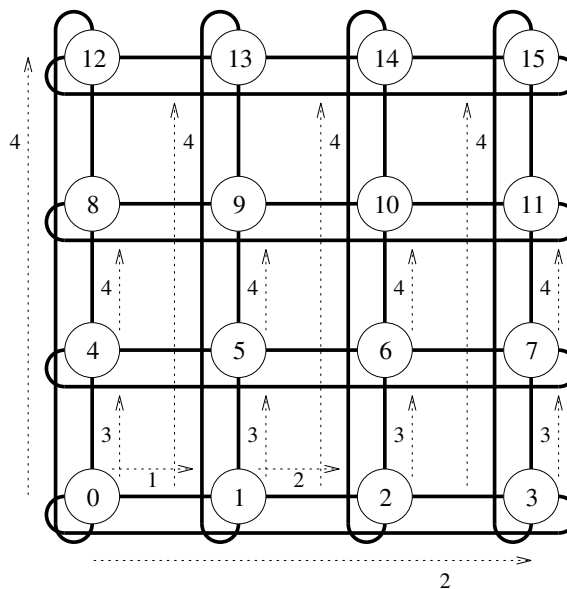
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- Source sends message on both outgoing links in first two steps
- All other processors receive on a link and transmit on other link
- $\lceil \frac{p}{2} \rceil$  steps and  $(t_s + t_w m) \lceil \frac{p}{2} \rceil$  cost



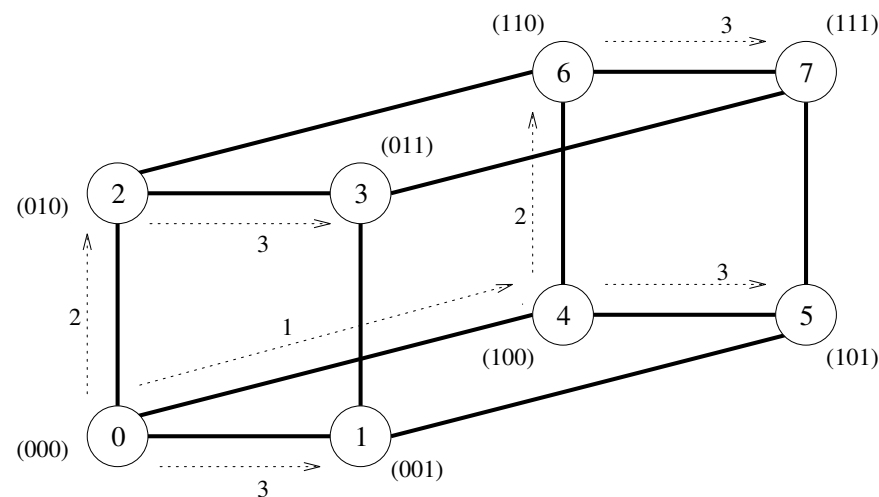
# Store-and-forward Routing on 2d Torus

- Each row or column of the torus can be regarded as a ring: Use ring method for the row to which the sending processor belongs then use ring method for every column
- $2\lceil\frac{\sqrt{p}}{2}\rceil$  steps and  $2(t_s + t_w m)\lceil\frac{\sqrt{p}}{2}\rceil$  cost



# Store-and-forward Routing on Hypercube

- Takes  $\log(p)$  steps for a  $p$  processor hypercube
- In the  $i$ th step, all processors that have the message transmit it to the neighbouring processor that differs in the  $i$ th most significant bit
- $(t_s + t_w m) \log(p)$  cost



# Cut-through Routing on Ring

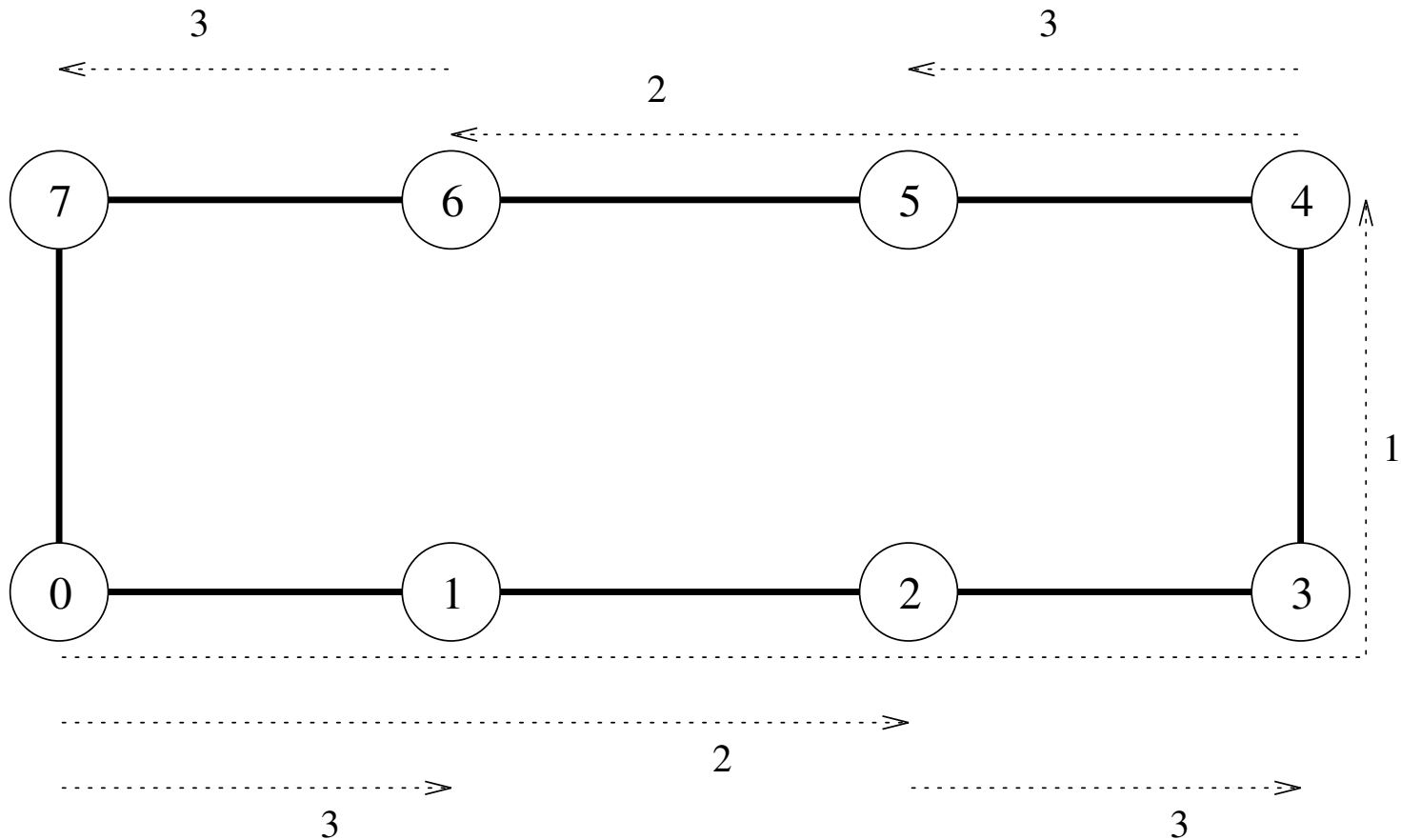
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- Algorithm similar to one used for hypercube; takes  $\log(p)$  steps
- In step  $i$ , message is sent to processor at a distance  $\frac{p}{2^i}$
- All messages flow in the same direction
- $t_s \log(p) + t_w m \log(p) + t_h(p - 1)$  cost



# Cut-through Routing on Ring

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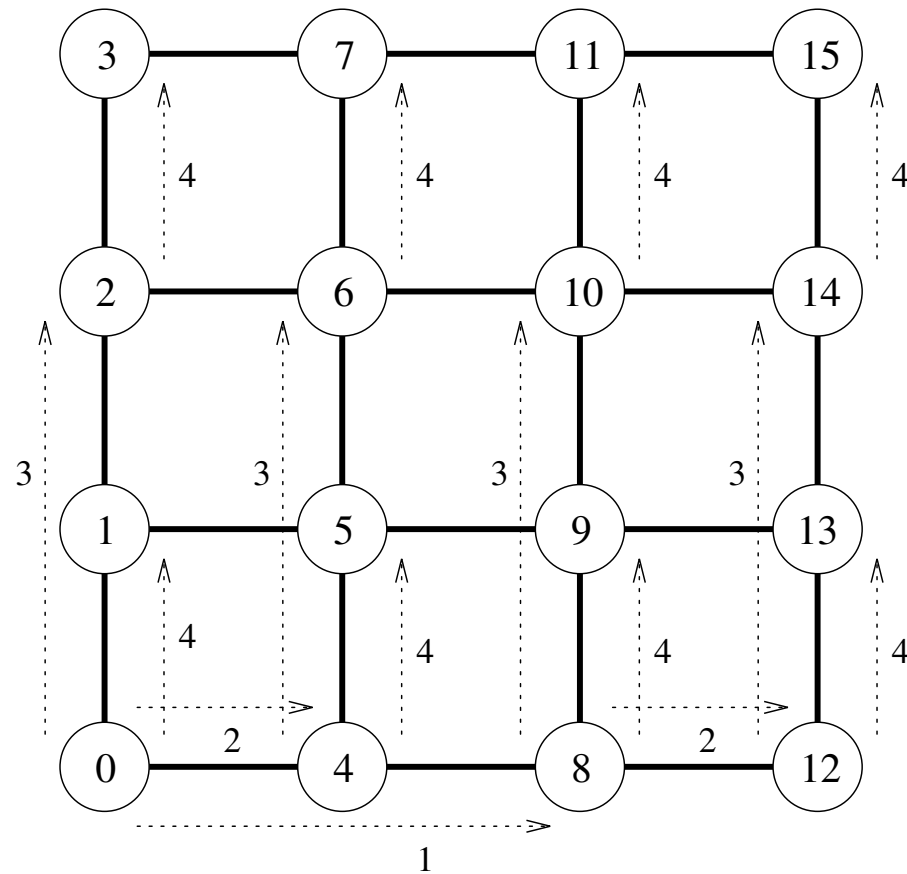
# Cut-through Routing on 2d Torus

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- Apply ring algorithm for the processor row of sender
- Now use ring algorithm for all processor columns
- $2 \log(\sqrt{p})$  steps
- $2(t_s + t_w m) \log(\sqrt{p}) + 2t_h(\sqrt{p} - 1)$  cost

# Cut-through Routing on 2d Torus

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# Cut-through Routing on Hypercube

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- For hypercube, cut-through does not provide benefits because of the use of only single link communications

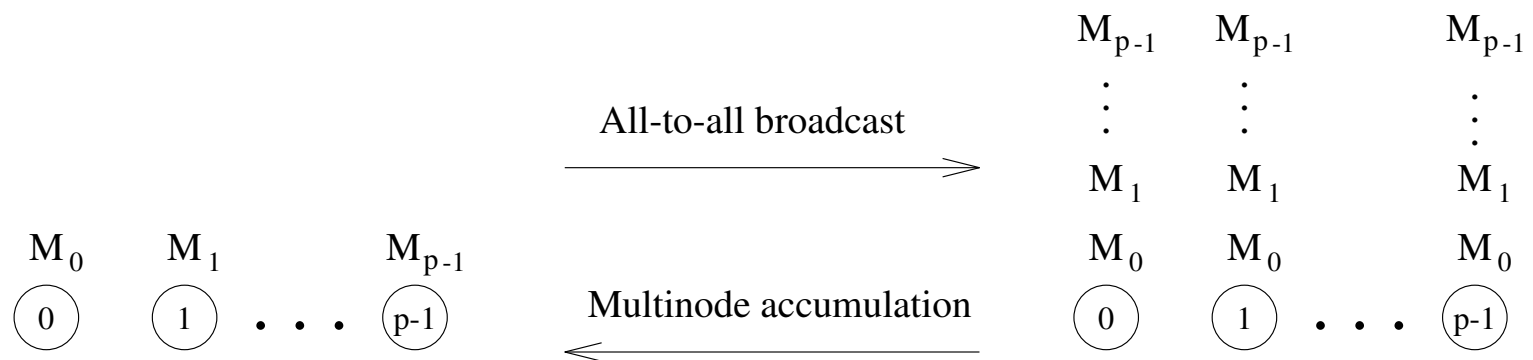
# All-to-all Broadcast

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- Algorithms often require each processor to send identical data to all other processors or a subset of processors. This operation is called a all-to-all broadcast or a multinode broadcast
- At the start of a multinode broadcast each processor has  $m$  words of data; at the end each processor has a copy of the  $m$  words that originated at each of the other processors
- The dual of this operation is a all-to-all reduction or a multinode reduction
- At the start of a multinode reduction each processor has  $m$  words of data, the reduction combines all the data from processors using an associative operator to produce  $m$  words that are available at all the processors
- Naive multinode broadcast or reduction using  $p$  singlenode broadcasts

# All-to-all Broadcast

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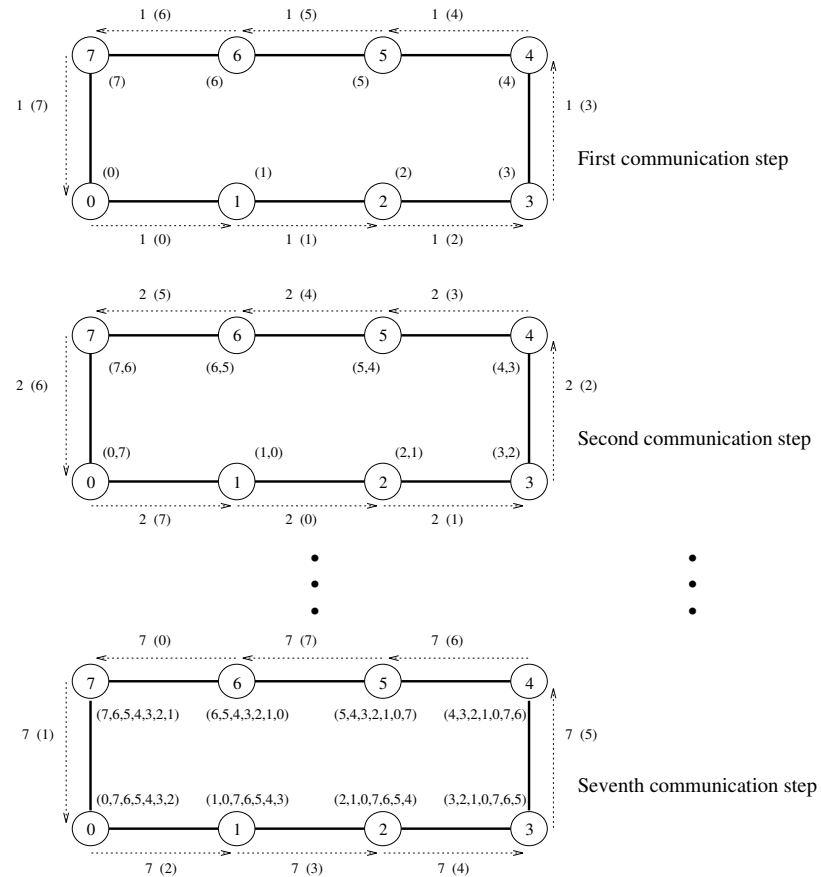


# Store-and-forward Routing on Ring

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- Every processor sends its message to the next processor on the ring in the first step
- In every subsequent step, all processors receive a message from the previous processor and send it to the next processor after retaining a copy for themselves
- $p - 1$  steps
- $(t_s + t_w m)(p - 1)$  cost

# Store-and-forward Routing on Ring



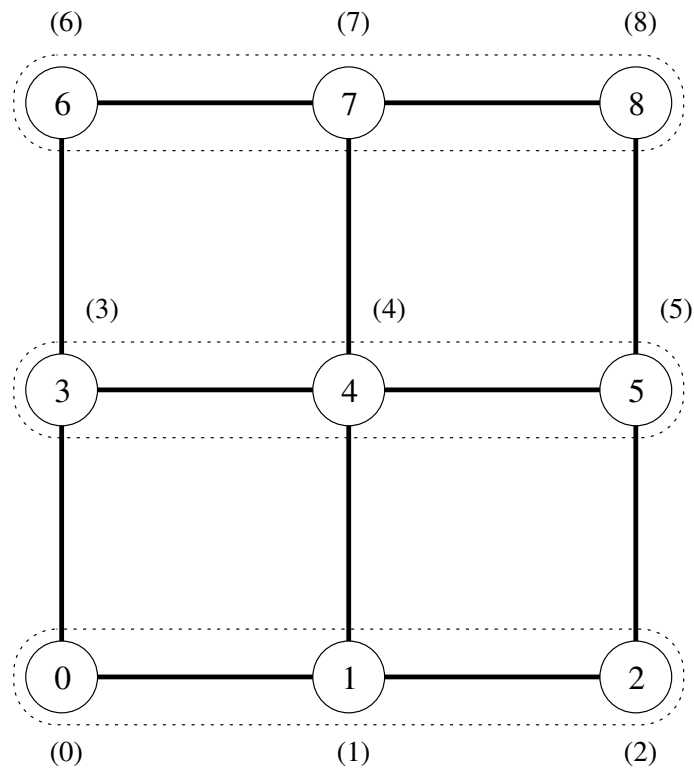


# Store-and-forward Routing on 2d Torus

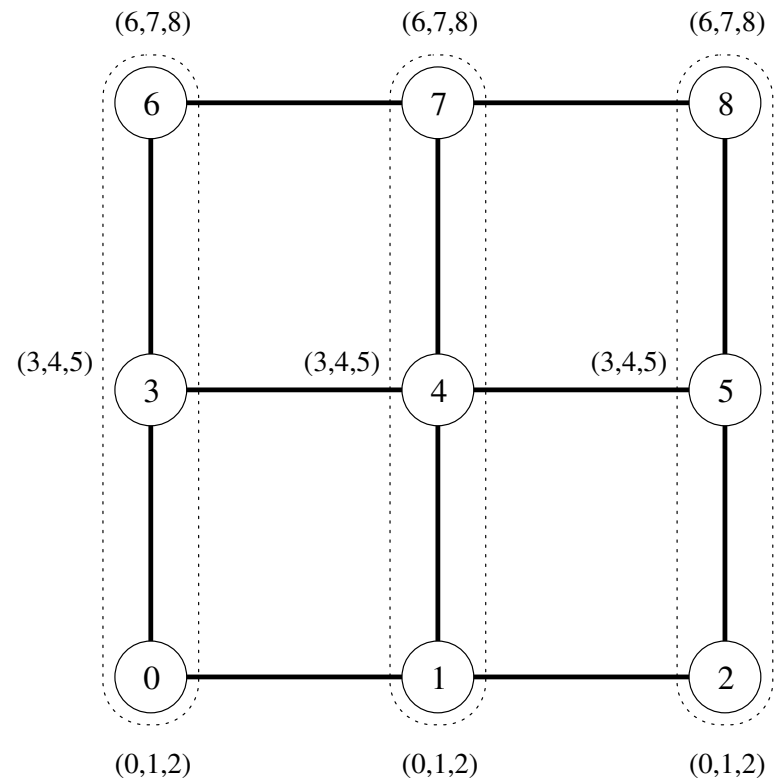
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- Use ring method for each row of the torus; compose all  $\sqrt{p}$  messages received into a single message and use the ring method for every column
- $2(\sqrt{p} - 1)$  steps
- $2t_s(\sqrt{p} - 1) + t_w m(p - 1)$  cost

# Store-and-forward Routing on 2d Torus



(a) Initial data distribution



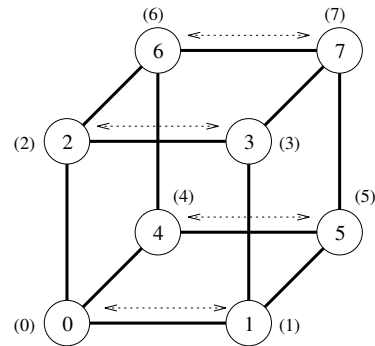
(b) Data distribution after rowwise broadcast

# Store-and-forward Routing on Hypercube

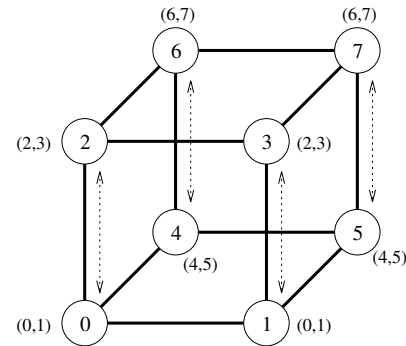
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- Takes  $\log(p)$  steps for a  $p$  processor hypercube
- In the  $i$ th step, every processor exchanges messages with the neighbouring processor that differs in the  $i$ th most significant bit. At each step larger messages are built out of smaller messages for subsequent steps
- $t_s \log(p) + t_w m(p - 1)$  cost

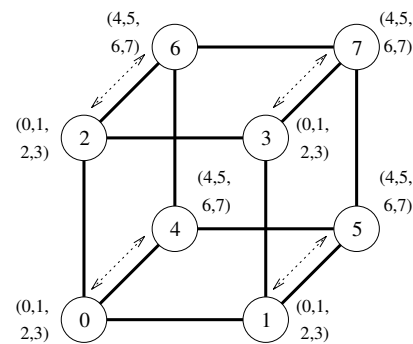
# Store-and-forward Routing on Hypercube



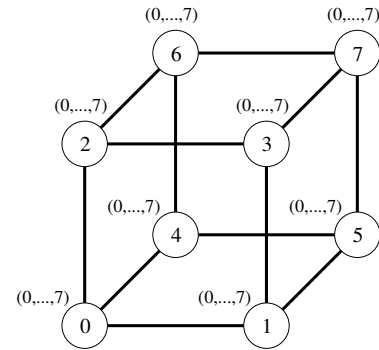
(a) Initial distribution of messages



(b) Distribution before the second step



(c) Distribution before the third step



(d) Final distribution of messages

# Cut-through Routing

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- Cut-through routing does not provide any benefits over store-and-forward for all-to-all broadcasts
- The one-to-all algorithm used for the ring and torus creates contention in the all-to-all case which renders it useless

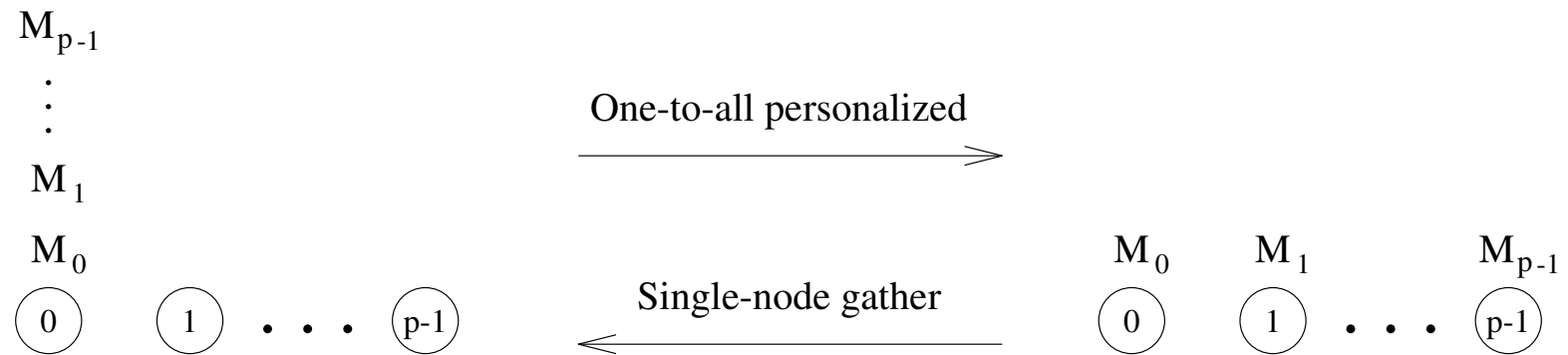
# One-to-all Scatter

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- Algorithms often require a processor to send different data to each of the other processors or each of a subset of processors
- This operation is called a one-to-all personalized communication or singlenode scatter
- At the start of a singlenode scatter, the source processor has  $p - 1$  messages of  $m$  words each that need to be sent to each of the other processors; at the end the other processors each have  $m$  words
- The dual of this operation is a all-to-one personalized communication or singlenode gather
- At the start of a singlenode gather each processor has  $m$  bytes of data, the gather combines all the data from processors to produce  $m(p - 1)$  words at the receiver
- Naive singlenode scatter or gather using  $p - 1$  steps

# One-to-all Scatter

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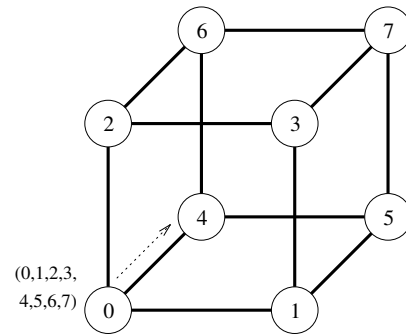
# Store-and-forward Routing on Hypercube

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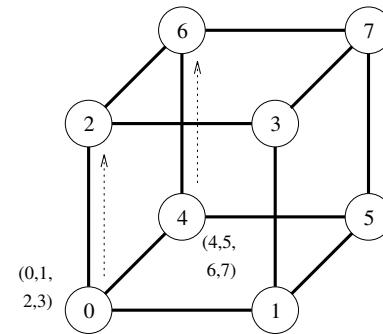
- Takes  $\log(p)$  steps for a  $p$  processor hypercube
- In the  $i$ th step, all processors that have messages transmit half of them to the neighbouring processor that differs in the  $i$ th most significant bit
- $t_s \log(p) + t_w m(p - 1)$  cost



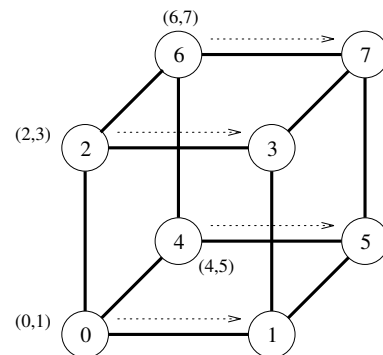
# Store-and-forward Routing on Hypercube



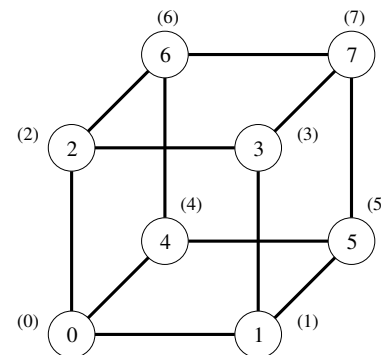
(a) Initial distribution of messages



(b) Distribution before the second step



(c) Distribution before the third step



(d) Final distribution of messages

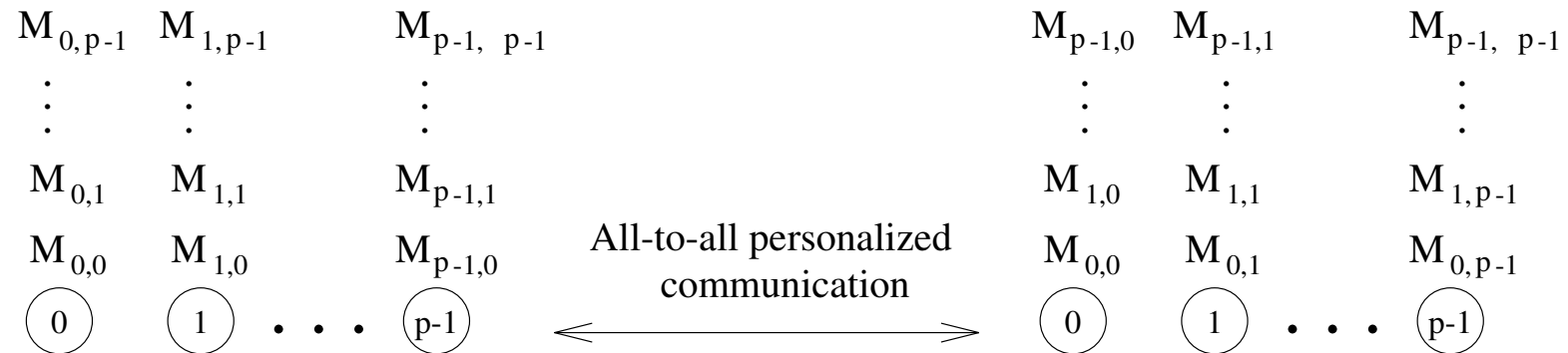
# All-to-all Scatter

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- Algorithms often require each processor to send different data to each of the other processors or each of a subset of processors
- This operation is called a all-to-all personalized communication or a multinode scatter
- At the start of a multinode scatter each processor has  $(p-1)m$  words of data; at the end each processor has a copy of the  $m$  words that originated at each of the other processors,  $(p-1)m$  words in all
- Naive multinode scatter using  $p$  singlenode scatters

# All-to-all Scatter

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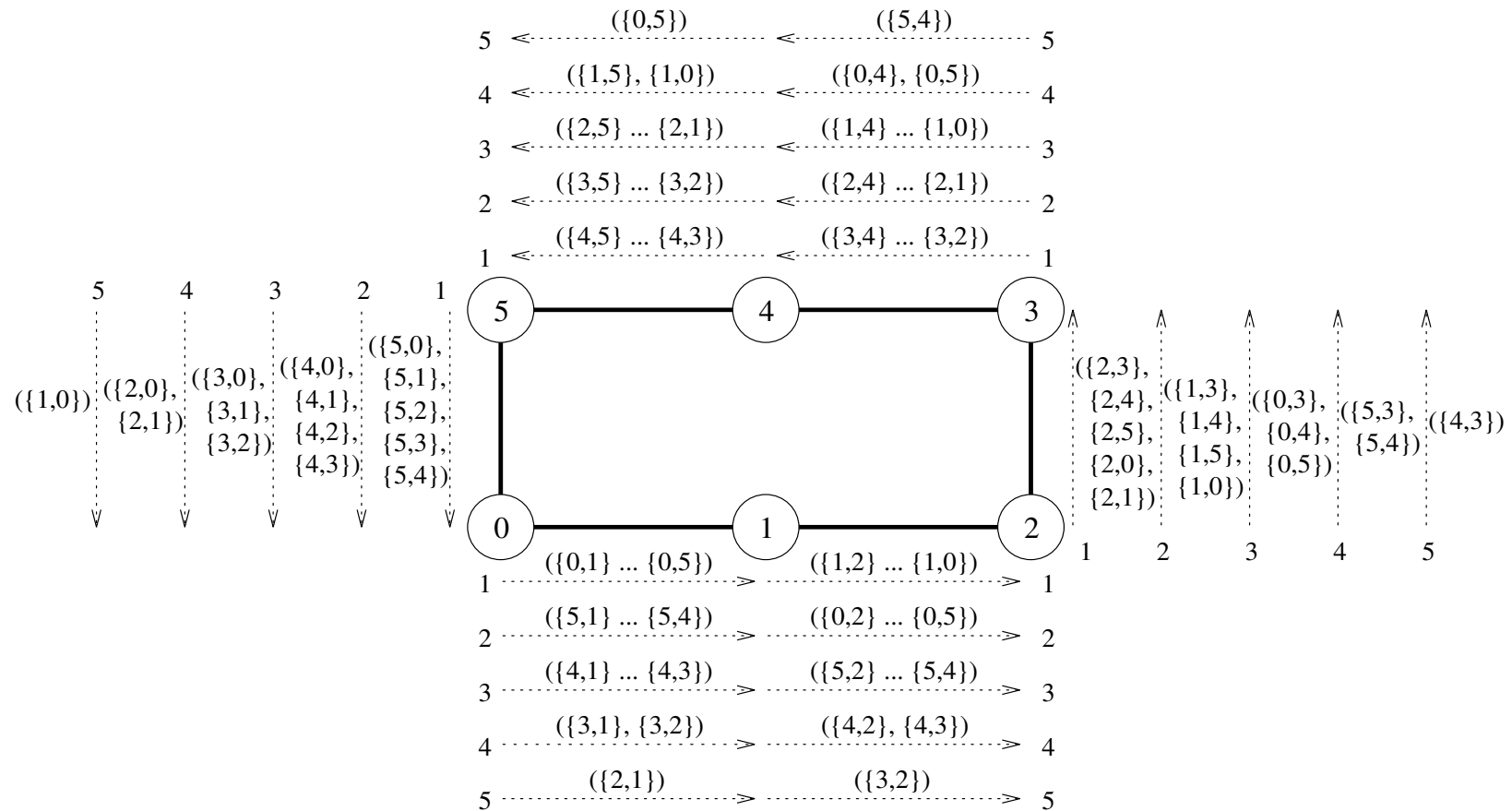


# Store-and-forward Routing on Ring

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- Every processor consolidates all the data to be sent and sends a single message to the next processor on the ring in the first step
- In every subsequent step, processors retain part of the message received for themselves and send the rest to the next processor on the ring
- $p - 1$  steps
- $(t_s + \frac{1}{2}t_w mp)(p - 1)$  cost

# Store-and-forward Routing on Ring

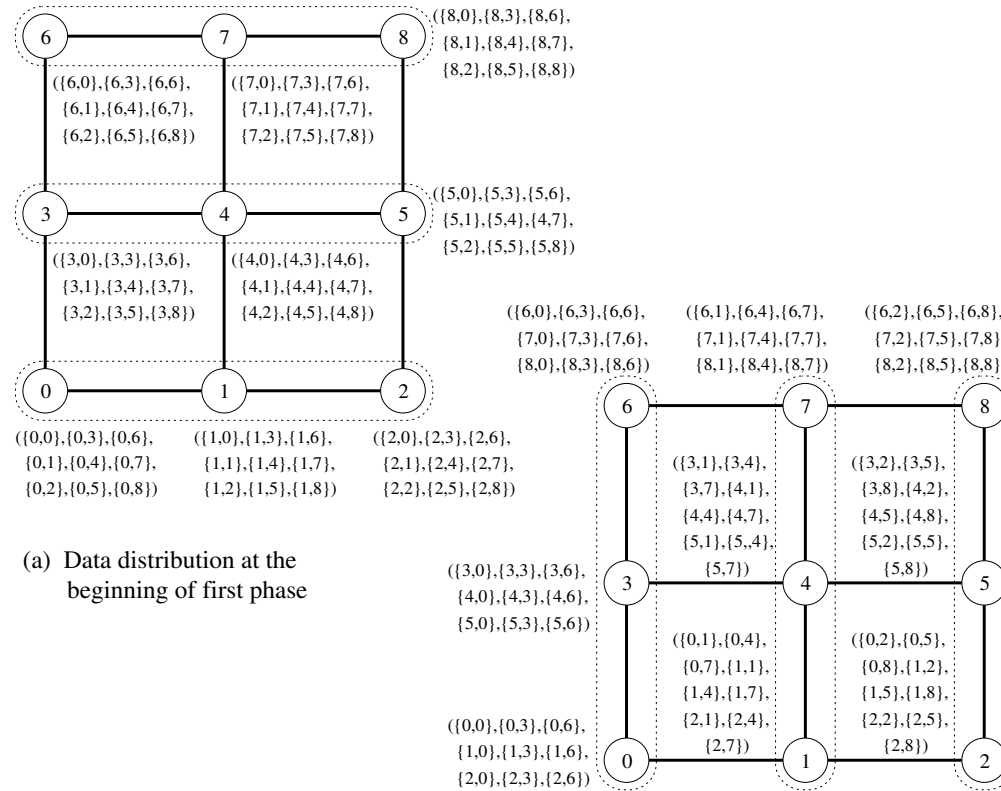


# Store-and-forward Routing on 2d Torus

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- Processors consolidate messages into groups meant for each processor column
- The ring method is applied to each processor row independently
- Now the messages in each processor are sorted into groups meant for each processor row
- The ring method is applied to each processor column independently
- $2(\sqrt{p} - 1)$  steps
- $(2t_s + t_w mp)(\sqrt{p} - 1)$  cost

# Store-and-forward Routing on 2d Torus



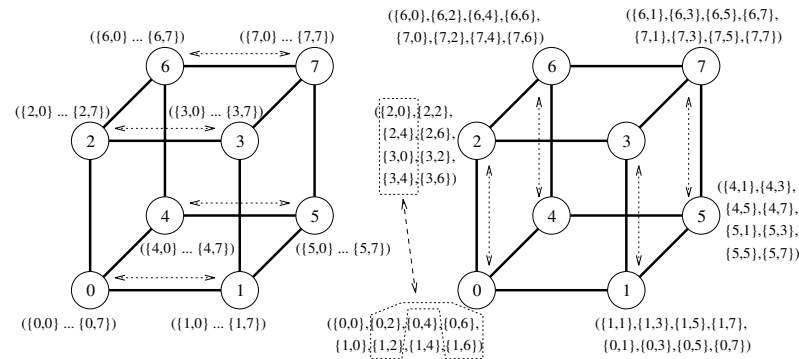
# Store-and-forward Routing on Hypercube

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- Takes  $\log(p)$  steps for a  $p$  processor hypercube
- In the  $i$ th step, every processor exchanges messages with the neighbouring processor that differs in the  $i$ th least significant bit
- At each stage, every processor holds  $p$  messages,  $\frac{p}{2}$  of these are consolidated into a single message for exchange with a neighbouring processor
- $(t_s + \frac{1}{2}t_w mp) \log(p)$  cost

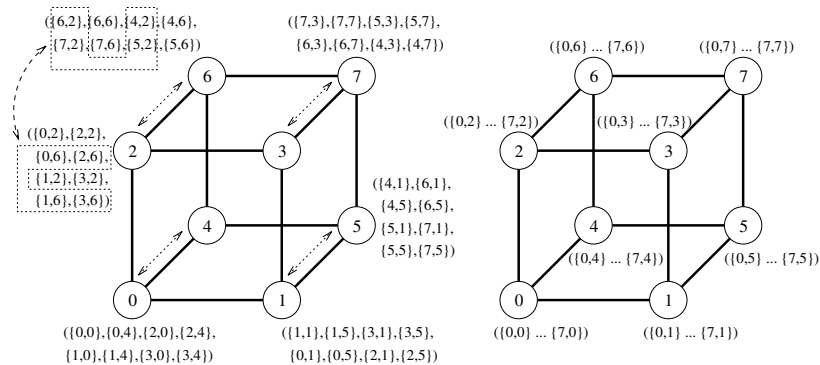


# Store-and-forward Routing on Hypercube



(a) Initial distribution of messages

(b) Distribution before the second step



(c) Distribution before the third step

(d) Final distribution of messages

# Summary

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- One-to-all Broadcast
- All-to-all Broadcast
- One-to-all Scatter
- All-to-all Scatter