n_queens_LOWSLEY

November 3, 2020

1 Solving the n-Queens Problem using Local Search

Points: 10

1.1 The n-Queens Problem

Goal: Find an arrangement of n queens on a $n \times n$ chess board so that no queen is on the same row, column or diagonal as any other queen.

State space: An arrangement of the queens on the board. We restrict the state space to arrangements where there is only a single queen per column. We represent a state as an integer vector of length n, each number representing for one column (from left to write) the row the queen is located in. We will call a state a "board."

Objective function: The number of pairwise conflicts (i.e., two queens in the same row/column/diagonal).

The optimization problem is to find the optimal arrangement a^* of n queens on the board:

$$a^* = \text{argmin}_a[\text{conflicts}(a)]$$

$$\text{s.t. } a \text{ contains only one queen per column}$$

Note that for this problem there is always an arrangement a^* with $\text{conflicts}(a) = 0$.

Local move: Move one queen to a different position in its column.

1.2 Helper functions

[228]:

```
import numpy as np
%matplotlib inline
import matplotlib.pyplot as plt
from matplotlib import colors
import math
from operator import itemgetter
import random
import pandas as pd
import time
```

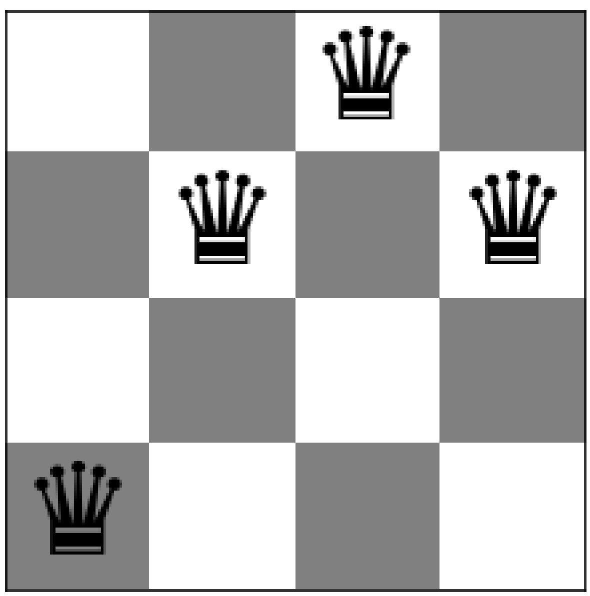
```python
def random_board(n):
    """Creates a random board of size n x n. Note that only a single queen is␣
↪placed in each column!"""

    return(np.random.randint(0,n,size = n))


def conflicts(board):
    """Caclulate the number of conflicts, i.e., the objective function."""

    board = np.array(board)

    n = len(board)
    conflicts = 0

    # check horizontal (we do not check vertical since the state space is␣
↪restricted to one queen per col)
    for i in range(n): conflicts += math.comb(np.sum(board == i), 2)
    #print(f"Horizontal conflicts: {conflicts}")

    # check for each queen diagonally up and down (only to the right side of␣
↪the queen)
    for j in range(n):
        q_up = board[j]
        q_down = board[j]

        for jj in range(j+1, n):
            q_up -= 1
            q_down += 1
            if board[jj] == q_up: conflicts += 1
            if board[jj] == q_down: conflicts += 1
        #print(f"Conflicts after queen {j}: {conflicts}")

    return(conflicts)


def show_board(board, cols = ['white', 'gray']):
    """display the board"""

    n = len(board)

    # create chess board display
    display = np.zeros([n,n])
    for i in range(n):
        for j in range(n):
            if (((i+j) % 2) != 0):
```

```
                display[i,j] = 1

    cmap = colors.ListedColormap(cols)
    fig, ax = plt.subplots()
    ax.imshow(display, cmap = cmap,
              norm = colors.BoundaryNorm(range(len(cols)+1), cmap.N))
    ax.set_xticks([])
    ax.set_yticks([])

    # place queens. Note: i and j are switched. Unicode u265B is a black queen
    for j in range(n):
        plt.text(j, board[j], u"\u265B", fontsize = 48,
                 horizontalalignment = 'center',
                 verticalalignment = 'center')

    print(f"Board with {conflicts(board)} conflicts.")
    plt.show()
```

1.3 Create a board

[101]:
```
board = random_board(4)

show_board(board)
print(f"Queens (left to right) are at rows: {board}")
print(f"Number of conflicts: {conflicts(board)}")
```

```
Board with 3 conflicts.
```

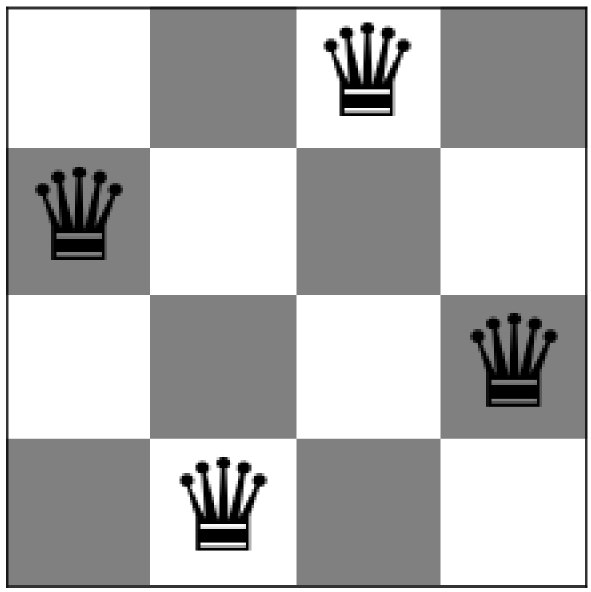


```
Queens (left to right) are at rows: [3 1 0 1]
Number of conflicts: 3
```

A board 4×4 with no conflicts:

[102]:
```
board = [1,3,0,2]
show_board(board)
```

```
Board with 0 conflicts.
```



1.4 Steepest-ascend Hill Climbing Search [3 Points]

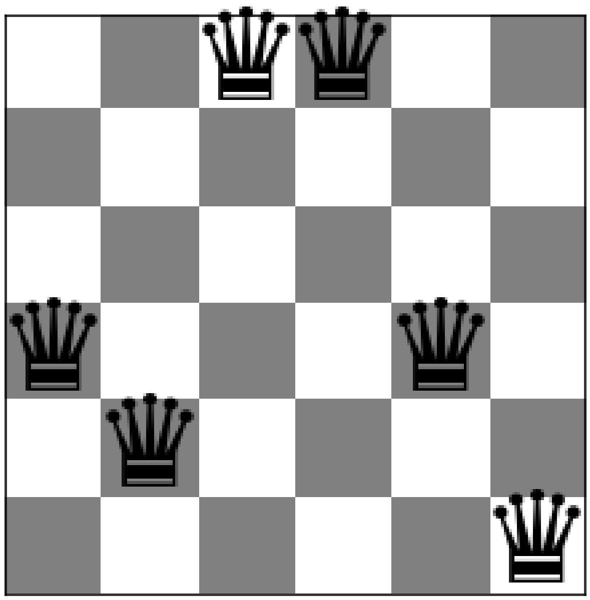
Calculate the objective function for all local moves (move each queen within its column) and always choose the best among all local moves.

[103]:
```
def steepest_ascent_get_neighbors(board):
    # define list of neighbors to populate
    neighbors = []
    # loop through each column
    for col in range(len(board)):
        # for each column, store a list of possible alternative moves
        states = []
        # loop through each row of a column
        for pos in range(len(board)):
            # if pos is current queen position then skip since we are already␣
 ↪there
```

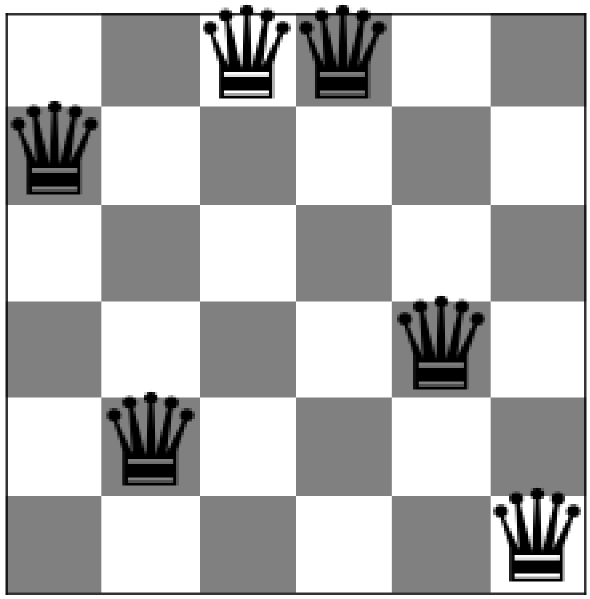
```
            if(board[col] == pos): continue
            # if not, copy over board values and move queen to new state
            new_state = board.copy()
            new_state[col] = pos
            # calculate the conflict score of this state
            new_state_score = conflicts(new_state)
            # add it to our list of states
            states.append((new_state_score,new_state.copy()))
        # taking the best state for a given column and save it off
        min_state_score = min(states, key = itemgetter(0))
        # add it to our list of neighbors
        neighbors.append((min_state_score[0],min_state_score[1].copy()))
    return(neighbors)

# Code goes here
def steepest_ascent_hill(board = None, size = 4, debug = False,␣
 ↪max_steps=10000):
    # if board not specified, generate one
    if(board == None):
        current = random_board(size)
    else:
        current = board
    # obtain score of current board
    current_score = conflicts(current)
    # debug
    if(debug):
        print("Starting board...")
        show_board(current)
    # run algo until solution is found or max_steps/stop_after is reached
    for step in range(max_steps):
        # obtain neighbors
        neighbors = steepest_ascent_get_neighbors(current)
        # obtain best neighbor
        min_neighbor_score = min(neighbors, key = itemgetter(0))
        # if best neighbor is better than current score, update it
        if(min_neighbor_score[0] < current_score):
            # debug
            if(debug):
                print("# STEEPEST ASCENT - NEW HIGH SCORE - ")
                print("NEW -> ",min_neighbor_score[0], " for ",␣
 ↪min_neighbor_score[1])
                print("OLD -> ",current_score, " for ", current)
                show_board(min_neighbor_score[1])
            # assign new values
            current_score = min_neighbor_score[0]
            current = min_neighbor_score[1]
        # if next best found neighbor is not better
```

```
        else:
            # stop and return current
            if(debug): print("Ending board...")
            return((current_score,current))

result = steepest_ascent_hill(size = 6, debug = True)
show_board(result[1])
```

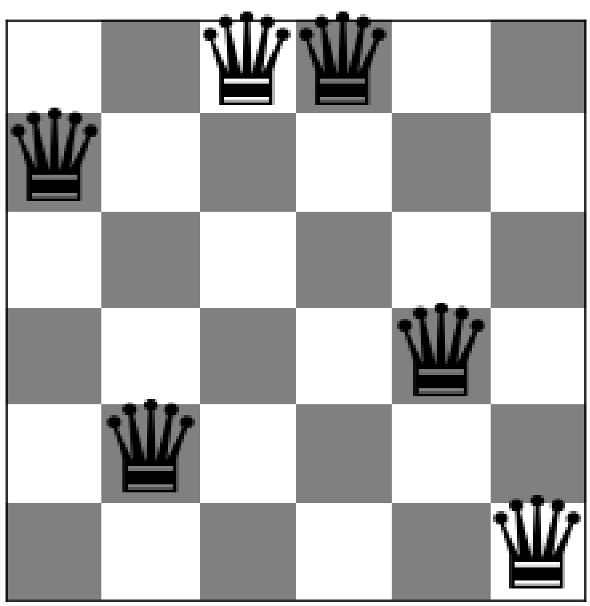
```
Starting board…
Board with 4 conflicts.
```



```
# STEEPEST ASCENT - NEW HIGH SCORE -
NEW ->  1  for  [1 4 0 0 3 5]
OLD ->  4  for  [3 4 0 0 3 5]
Board with 1 conflicts.
```



Ending board…
Board with 1 conflicts.



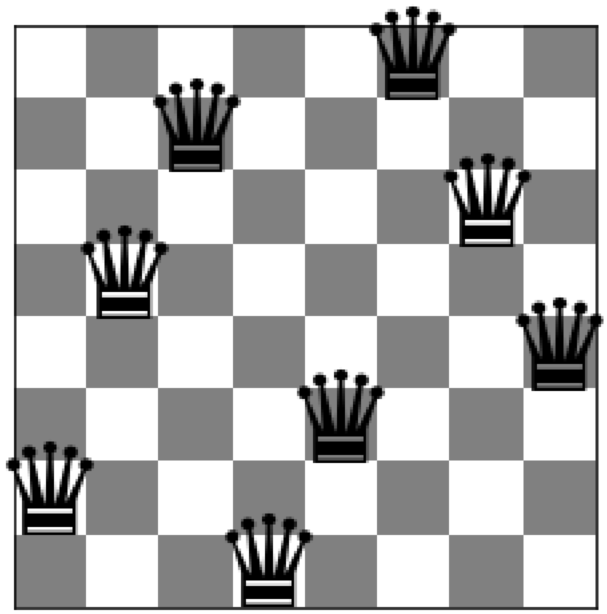
1.5 Steepest-ascend Hill Climbing Search with Random Restarts [1 Point]

Steepest-ascend hill climbing will often end up in local optima. Restart the algorithm up to 100 times with a random board to find a better (hopefully optimal) solution.

```
[104]: def steepest_ascent_hill_random(board = None, size = 4, debug = False,␣
    ↪max_steps=10000, restarts=100):
        # create board if not supplied
        if(board == None):
            board = random_board(size)
        # set up initial best values
        best_score = conflicts(board)
        best_solution = board
        # run through number of random restarts
        for i in range(restarts):
            if(debug): print("STARTING RESTART ", i)
            # run steepest ascent with random board each time
            result = steepest_ascent_hill(size=size, debug = False,␣
    ↪max_steps=max_steps)
            # if the score of the result is better, replace it
            if(result[0] < best_score):
                # debug
                if(debug):
                    print("# RANDOM RESTART - NEW HIGH SCORE - ")
                    print("NEW -> ",result[0], " for ", result[1])
                    print("OLD -> ",best_score, " for ", best_solution)
                # assign new values
                best_score = result[0]
                best_solution = result[1]
            # if solution has zero conflicts, return
            if(best_score == 0):
                return((best_score,best_solution))
        # if no optimal solution found, return best found
        return((best_score,best_solution))

    result = steepest_ascent_hill_random(size = 8, debug = True, restarts=50)
    show_board(result[1])
```

```
STARTING RESTART  0
# RANDOM RESTART - NEW HIGH SCORE -
NEW ->  2  for  [3 6 7 2 0 5 4 1]
OLD ->  8  for  [1 6 5 1 3 4 6 1]
STARTING RESTART  1
# RANDOM RESTART - NEW HIGH SCORE -
NEW ->  1  for  [4 0 5 7 1 3 6 2]
OLD ->  2  for  [3 6 7 2 0 5 4 1]
STARTING RESTART  2
STARTING RESTART  3
```

```
STARTING RESTART  4
STARTING RESTART  5
STARTING RESTART  6
# RANDOM RESTART - NEW HIGH SCORE -
NEW ->  0  for  [6 3 1 7 5 0 2 4]
OLD ->  1  for  [4 0 5 7 1 3 6 2]
Board with 0 conflicts.
```



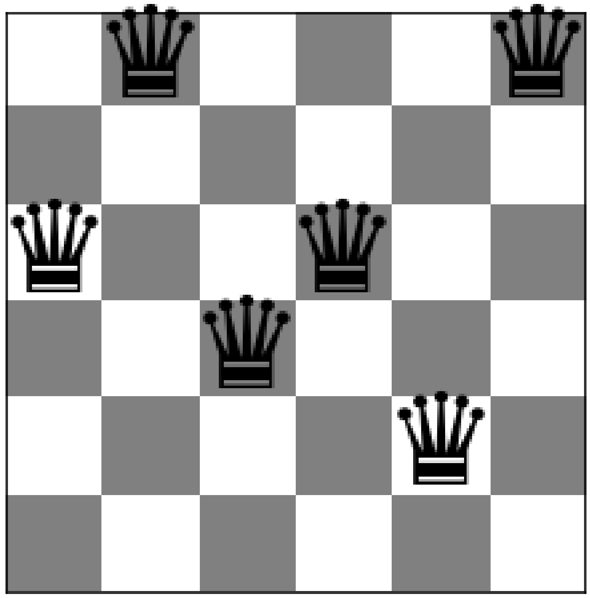
1.6 Stochastic Hill Climbing [1 Point]

Chooses randomly from among all uphill moves.

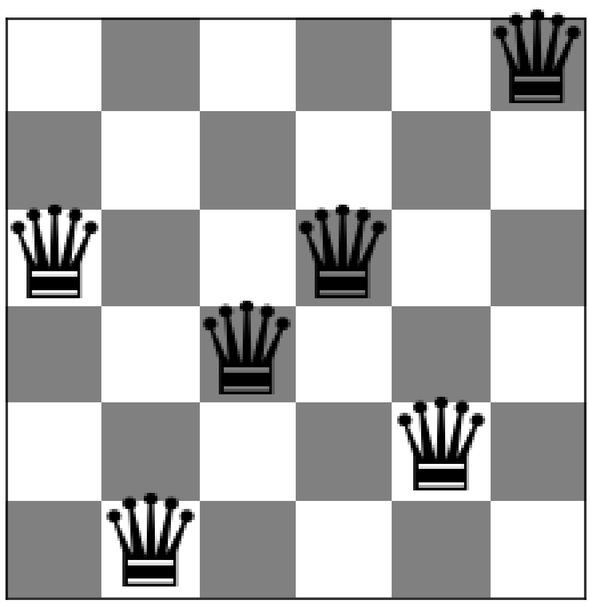
[105]:
```python
def stochastic_get_neighbors(board, score):
    # define list of neighbors to populate
    neighbors = []
    # loop through each column
    for col in range(len(board)):
        # loop through each row of a column
        for pos in range(len(board)):
            # if pos is current queen position then skip since we are already␣
↪there
            if(board[col] == pos): continue
            # if not, copy over board values and move queen to new state
            new_state = board.copy()
            new_state[col] = pos
            # calculate the conflict score of this state
```

```
            new_state_score = conflicts(new_state)
            # if we have a state with better score, add to neighbors list
            if(new_state_score < score):
                neighbors.append((new_state_score,new_state.copy()))
    return(neighbors)


def stochastic_hill(board = None, size = 4, debug = False, max_steps=10000):
    # if board not specified, generate one
    if(board == None):
        current = random_board(size)
    else:
        current = board
    # obtain score of current board
    current_score = conflicts(current)
    # debug
    if(debug):
        print("Starting board...")
        show_board(current)
    # run algo until solution is found or max_steps/stop_after is reached
    for step in range(max_steps):
        # obtain neighbors with better score than current
        neighbors = stochastic_get_neighbors(current, current_score)
        # if at least one better neighbor found
        if(len(neighbors) != 0):
            # obtain random neighbor from list and assign to current
            new_neighbor = random.choice(neighbors)
            # debug
            if(debug):
                print("# STEEPEST ASCENT - NEW HIGH SCORE - ")
                print("NEW -> ",new_neighbor[0], " for ", new_neighbor[1])
                print("OLD -> ",current_score, " for ", current)
                show_board(new_neighbor[1])
            # assign new values
            current_score = new_neighbor[0]
            current = new_neighbor[1]
        # if no better neighbor found at all
        else:
            # stop and return current
            if(debug): print("Ending board...")
            return((current_score,current))

result = stochastic_hill(size = 6, debug = True)
show_board(result[1])
```

```
Starting board…
Board with 6 conflicts.
```

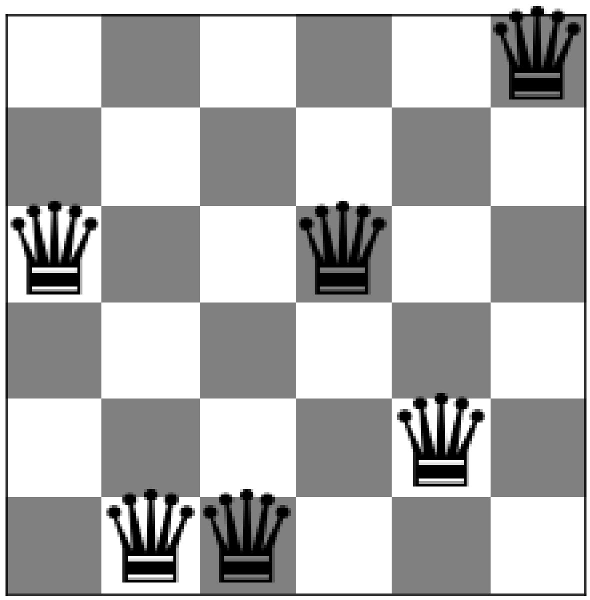


```
# STEEPEST ASCENT - NEW HIGH SCORE -
NEW ->  4  for  [2 5 3 2 4 0]
OLD ->  6  for  [2 0 3 2 4 0]
Board with 4 conflicts.
```

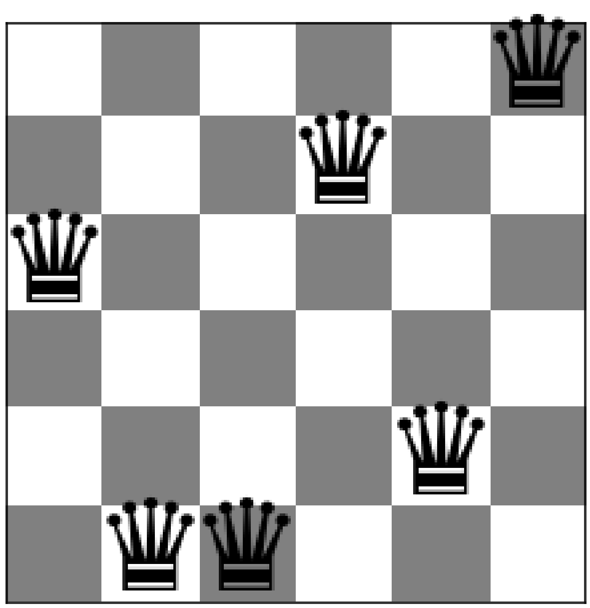


```
# STEEPEST ASCENT - NEW HIGH SCORE -
```

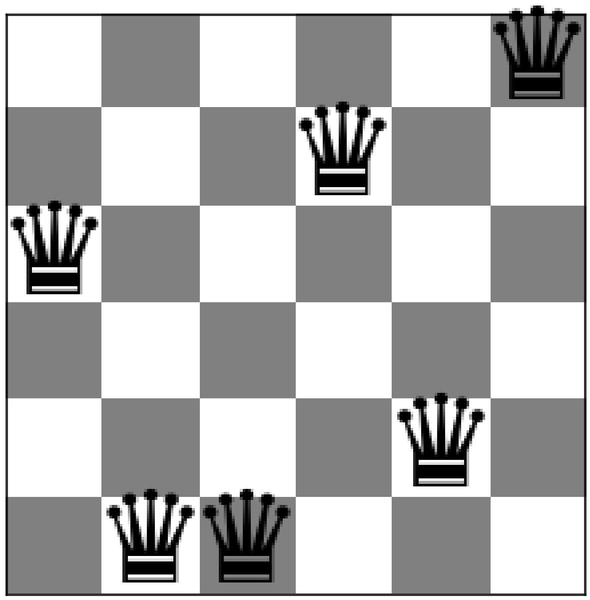
```
NEW ->  3  for  [2 5 5 2 4 0]
OLD ->  4  for  [2 5 3 2 4 0]
Board with 3 conflicts.
```



```
# STEEPEST ASCENT - NEW HIGH SCORE -
NEW ->  1  for  [2 5 5 1 4 0]
OLD ->  3  for  [2 5 5 2 4 0]
Board with 1 conflicts.
```



Ending board…
Board with 1 conflicts.



1.7 First-choice Hill Climbing [1 Point]

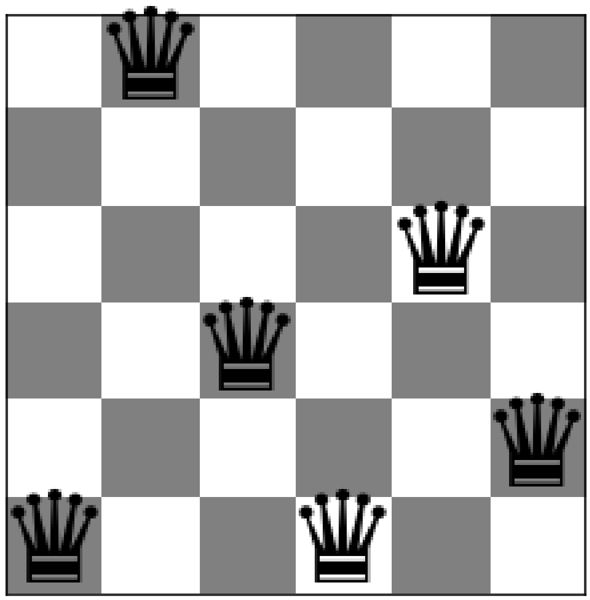
First-choice hill climbing is a type of stochastic hill climbing that generates one random local neighbor at a time and accept it if it has a better objective function value than the current state.

[106]:
```
def first_choice_get_neighbor(board,size):
    # copy board for altering
    new_neighbor = board.copy()
    # obtain random queen to move
    col = random.randint(0, size-1)
    # obtain random position to move to
    pos = random.randint(0, size-1)
    # ensure it is not same location, if it is keep generating new locations
    while(new_neighbor[col] == pos):
        pos = random.randint(0, size-1)
    # assign queen to new location
    new_neighbor[col] = pos
    # obtain score
    new_score = conflicts(new_neighbor)
    # return new neighbor
    return((new_score,new_neighbor))
```

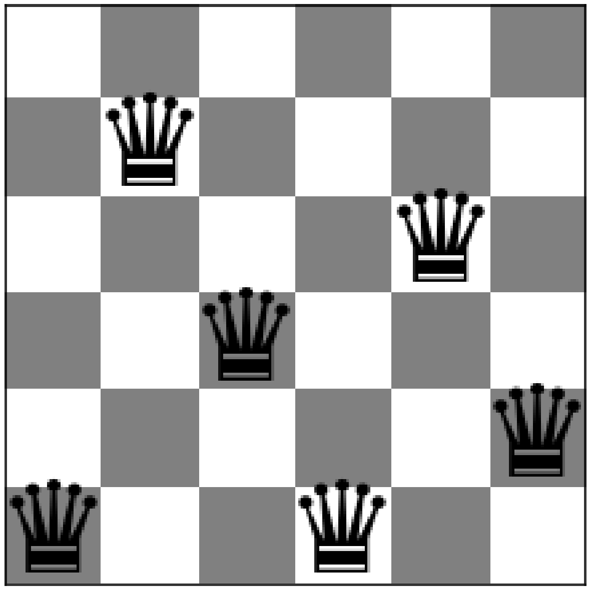
```python
def first_choice_hill(board = None, size = 4, debug = False, max_steps=10000,␣
↪stop_after=1000):
    # if board not specified, generate one
    if(board == None):
        current = random_board(size)
    else:
        current = board
        size = len(current)
    # obtain score of current board
    current_score = conflicts(current)
    # debug
    if(debug):
        print("Starting board...")
        show_board(current)
    # value to keep track of number of steps that have occured since the last␣
↪best found solution
    steps_since_best = 0
    # run algo until solution is found or max_steps/stop_after is reached
    for step in range(max_steps):
        # return current if no better solution is found after stop_after steps
        if(steps_since_best >= stop_after):
            if(debug): print("No better solution found after {} steps.".
↪format(stop_after))
            return((current_score,current))
        if(current_score == 0):
            return((current_score,current))
        # obtain neighbors with better score than current
        neighbor = first_choice_get_neighbor(current, size)
        # if at least one better neighbor found
        if(neighbor[0] < current_score):
            # debug
            if(debug):
                print("# STEEPEST ASCENT - NEW HIGH SCORE - ")
                print("NEW -> ",neighbor[0], " for ", neighbor[1])
                print("OLD -> ",current_score, " for ", current)
                show_board(neighbor[1])
            # assign new values
            current_score = neighbor[0]
            current = neighbor[1]
            # reset our steps since best found solution
            steps_since_best = 0
        # if no better neighbor found, increment our steps since best solution␣
↪found
        steps_since_best = steps_since_best + 1
    if(debug): print("Ending board...")
    return((current_score,current))
```

```
result = first_choice_hill(size = 6, debug = True)
show_board(result[1])
```

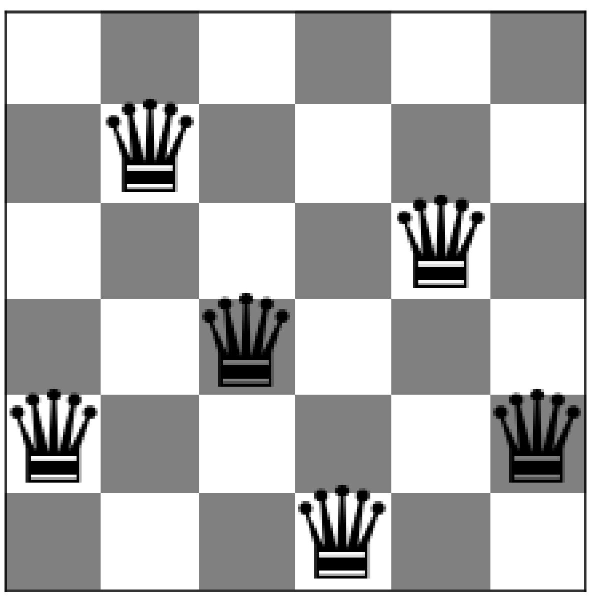
```
Starting board…
Board with 3 conflicts.
```



```
# STEEPEST ASCENT - NEW HIGH SCORE -
NEW ->  2  for  [5 1 3 5 2 4]
OLD ->  3  for  [5 0 3 5 2 4]
Board with 2 conflicts.
```

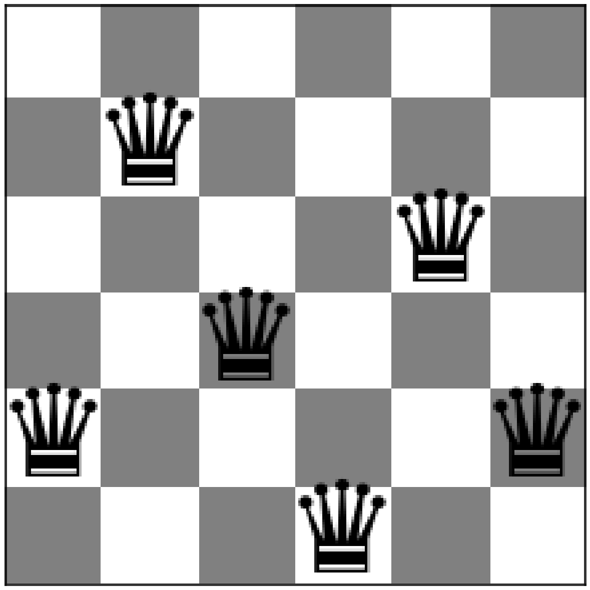


```
# STEEPEST ASCENT - NEW HIGH SCORE -
NEW ->  1  for  [4 1 3 5 2 4]
OLD ->  2  for  [5 1 3 5 2 4]
Board with 1 conflicts.
```



```
No better solution found after 1000 steps.
```

```
Board with 1 conflicts.
```



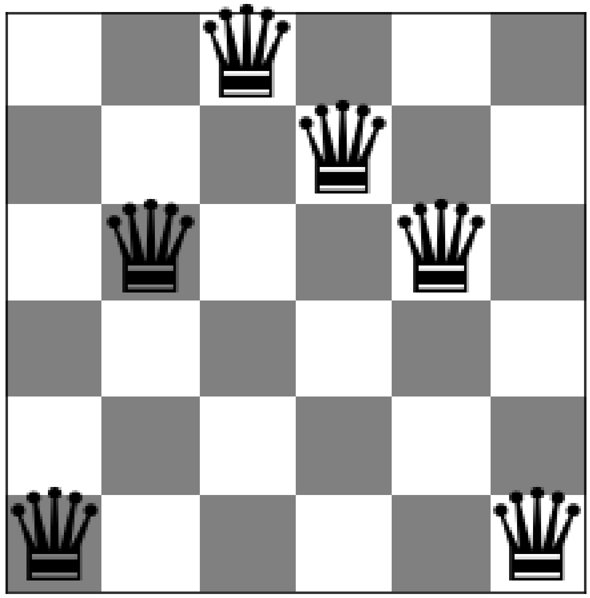
1.8 Simulated Annealing [2 Points]

You have to experiment with the annealing schedule.

[234]:
```python
def simulated_annealing_get_neighbor(board,size):
    # copy board for altering
    new_neighbor = board.copy()
    # obtain random queen to move
    col = random.randint(0, size-1)
    # obtain random position to move to
    pos = random.randint(0, size-1)
    # ensure it is not same location, if it is keep generating new locations
    while(new_neighbor[col] == pos):
        pos = random.randint(0, size-1)
    # assign queen to new location
    new_neighbor[col] = pos
    # obtain score
    new_score = conflicts(new_neighbor)
    # return new neighbor
    return((new_score,new_neighbor))

def simulated_annealing(board = None, size = 4, T0 = None, alpha = 0.999,␣
 ↪epsilon = 1e-3, debug = False):
# if board not specified, generate one
    if(board == None):
```

```
        current = random_board(size)
    else:
        current = board
        size = len(current)
    # obtain score of current board
    current_score = conflicts(current)
    # debug
    if(debug):
        print("Starting board...")
        show_board(current)
    # use the current board conflict score to set T0
    if T0 is None:
        T0 = (size^2) / 2
    # set T and t
    T = T0
    t = 0
    while T > epsilon:
        # calculate temperature from schedule
        T = T0 * alpha ** t
        # obtain neighbor
        neighbor = simulated_annealing_get_neighbor(current, size)
        # calculated delta E
        deltaE = neighbor[0] - current_score
        # check if the new solution is better
        if deltaE < 0 or np.random.rand() < math.exp(-deltaE/T):
            current = neighbor[1]
            current_score = neighbor[0]
            if debug:
                print(f"step: {t} \t temp: {T:5.3f} \t deltaE: {deltaE:+3.3f}␣
↪\t new conflict score: {current_score:3.3f}")
            if(current_score == 0):
                if(debug): print("Ending board...")
                return((current_score, current))
        t += 1
    if(debug): print("Ending board...")
    return((current_score, current))

result = simulated_annealing(size=6, debug=True)
show_board(result[1])
```

```
Starting board…
Board with 5 conflicts.
```



```
step: 0          temp: 2.000     deltaE: +1.000          new conflict score:
6.000
step: 1          temp: 1.998     deltaE: -2.000          new conflict score:
4.000
step: 3          temp: 1.994     deltaE: -1.000          new conflict score:
3.000
step: 6          temp: 1.988     deltaE: +0.000          new conflict score:
3.000
step: 10         temp: 1.980     deltaE: +0.000          new conflict score:
3.000
step: 15         temp: 1.970     deltaE: +2.000          new conflict score:
5.000
step: 16         temp: 1.968     deltaE: -1.000          new conflict score:
4.000
step: 18         temp: 1.964     deltaE: +0.000          new conflict score:
4.000
step: 21         temp: 1.958     deltaE: +2.000          new conflict score:
6.000
step: 22         temp: 1.956     deltaE: +0.000          new conflict score:
6.000
step: 23         temp: 1.955     deltaE: -1.000          new conflict score:
5.000
step: 24         temp: 1.953     deltaE: +1.000          new conflict score:
6.000
step: 25         temp: 1.951     deltaE: -1.000          new conflict score:
5.000
```

```
step: 26         temp: 1.949     deltaE: +0.000          new conflict score:
5.000
step: 27         temp: 1.947     deltaE: +0.000          new conflict score:
5.000
step: 28         temp: 1.945     deltaE: -1.000          new conflict score:
4.000
step: 29         temp: 1.943     deltaE: -1.000          new conflict score:
3.000
step: 30         temp: 1.941     deltaE: +0.000          new conflict score:
3.000
step: 31         temp: 1.939     deltaE: +0.000          new conflict score:
3.000
step: 33         temp: 1.935     deltaE: -1.000          new conflict score:
2.000
step: 36         temp: 1.929     deltaE: +0.000          new conflict score:
2.000
step: 38         temp: 1.925     deltaE: +1.000          new conflict score:
3.000
step: 44         temp: 1.914     deltaE: -1.000          new conflict score:
2.000
step: 47         temp: 1.908     deltaE: -1.000          new conflict score:
1.000
step: 48         temp: 1.906     deltaE: +1.000          new conflict score:
2.000
step: 53         temp: 1.897     deltaE: +1.000          new conflict score:
3.000
step: 54         temp: 1.895     deltaE: -2.000          new conflict score:
1.000
step: 55         temp: 1.893     deltaE: +2.000          new conflict score:
3.000
step: 58         temp: 1.887     deltaE: +1.000          new conflict score:
4.000
step: 59         temp: 1.885     deltaE: -1.000          new conflict score:
3.000
step: 62         temp: 1.880     deltaE: +0.000          new conflict score:
3.000
step: 63         temp: 1.878     deltaE: +1.000          new conflict score:
4.000
step: 66         temp: 1.872     deltaE: +1.000          new conflict score:
5.000
step: 68         temp: 1.868     deltaE: -2.000          new conflict score:
3.000
step: 69         temp: 1.867     deltaE: -1.000          new conflict score:
2.000
step: 76         temp: 1.854     deltaE: +0.000          new conflict score:
2.000
step: 80         temp: 1.846     deltaE: +0.000          new conflict score:
2.000
```

```
step: 84        temp: 1.839     deltaE: +0.000          new conflict score:
2.000
step: 85        temp: 1.837     deltaE: +0.000          new conflict score:
2.000
step: 87        temp: 1.833     deltaE: +2.000          new conflict score:
4.000
step: 89        temp: 1.830     deltaE: +0.000          new conflict score:
4.000
step: 90        temp: 1.828     deltaE: +3.000          new conflict score:
7.000
step: 91        temp: 1.826     deltaE: -2.000          new conflict score:
5.000
step: 92        temp: 1.824     deltaE: +0.000          new conflict score:
5.000
step: 93        temp: 1.822     deltaE: -2.000          new conflict score:
3.000
step: 95        temp: 1.819     deltaE: +1.000          new conflict score:
4.000
step: 96        temp: 1.817     deltaE: +1.000          new conflict score:
5.000
step: 97        temp: 1.815     deltaE: -1.000          new conflict score:
4.000
step: 100       temp: 1.810     deltaE: -1.000          new conflict score:
3.000
step: 101       temp: 1.808     deltaE: +0.000          new conflict score:
3.000
step: 106       temp: 1.799     deltaE: -1.000          new conflict score:
2.000
step: 111       temp: 1.790     deltaE: +1.000          new conflict score:
3.000
step: 112       temp: 1.788     deltaE: +1.000          new conflict score:
4.000
step: 113       temp: 1.786     deltaE: +2.000          new conflict score:
6.000
step: 114       temp: 1.784     deltaE: -2.000          new conflict score:
4.000
step: 115       temp: 1.783     deltaE: +0.000          new conflict score:
4.000
step: 117       temp: 1.779     deltaE: +0.000          new conflict score:
4.000
step: 118       temp: 1.777     deltaE: +2.000          new conflict score:
6.000
step: 120       temp: 1.774     deltaE: +0.000          new conflict score:
6.000
step: 121       temp: 1.772     deltaE: +0.000          new conflict score:
6.000
step: 122       temp: 1.770     deltaE: -2.000          new conflict score:
4.000
```

```
step: 123        temp: 1.768     deltaE: +0.000          new conflict score:
4.000
step: 124        temp: 1.767     deltaE: -1.000          new conflict score:
3.000
step: 126        temp: 1.763     deltaE: +1.000          new conflict score:
4.000
step: 128        temp: 1.760     deltaE: +1.000          new conflict score:
5.000
step: 129        temp: 1.758     deltaE: -2.000          new conflict score:
3.000
step: 137        temp: 1.744     deltaE: -1.000          new conflict score:
2.000
step: 138        temp: 1.742     deltaE: +1.000          new conflict score:
3.000
step: 139        temp: 1.740     deltaE: +2.000          new conflict score:
5.000
step: 140        temp: 1.739     deltaE: -1.000          new conflict score:
4.000
step: 141        temp: 1.737     deltaE: +1.000          new conflict score:
5.000
step: 142        temp: 1.735     deltaE: +0.000          new conflict score:
5.000
step: 143        temp: 1.733     deltaE: +0.000          new conflict score:
5.000
step: 145        temp: 1.730     deltaE: +0.000          new conflict score:
5.000
step: 146        temp: 1.728     deltaE: +0.000          new conflict score:
5.000
step: 147        temp: 1.726     deltaE: +0.000          new conflict score:
5.000
step: 149        temp: 1.723     deltaE: +0.000          new conflict score:
5.000
step: 150        temp: 1.721     deltaE: +0.000          new conflict score:
5.000
step: 151        temp: 1.720     deltaE: +1.000          new conflict score:
6.000
step: 152        temp: 1.718     deltaE: +0.000          new conflict score:
6.000
step: 153        temp: 1.716     deltaE: +0.000          new conflict score:
6.000
step: 155        temp: 1.713     deltaE: +0.000          new conflict score:
6.000
step: 156        temp: 1.711     deltaE: +0.000          new conflict score:
6.000
step: 157        temp: 1.709     deltaE: +1.000          new conflict score:
7.000
step: 158        temp: 1.708     deltaE: +0.000          new conflict score:
7.000
```

step: 159 temp: 1.706 deltaE: -2.000 new conflict score:
5.000
step: 160 temp: 1.704 deltaE: +1.000 new conflict score:
6.000
step: 161 temp: 1.702 deltaE: -1.000 new conflict score:
5.000
step: 162 temp: 1.701 deltaE: +2.000 new conflict score:
7.000
step: 163 temp: 1.699 deltaE: -1.000 new conflict score:
6.000
step: 165 temp: 1.696 deltaE: +1.000 new conflict score:
7.000
step: 166 temp: 1.694 deltaE: -1.000 new conflict score:
6.000
step: 167 temp: 1.692 deltaE: +1.000 new conflict score:
7.000
step: 168 temp: 1.691 deltaE: -3.000 new conflict score:
4.000
step: 169 temp: 1.689 deltaE: -1.000 new conflict score:
3.000
step: 170 temp: 1.687 deltaE: +1.000 new conflict score:
4.000
step: 171 temp: 1.685 deltaE: +0.000 new conflict score:
4.000
step: 173 temp: 1.682 deltaE: +0.000 new conflict score:
4.000
step: 174 temp: 1.680 deltaE: +1.000 new conflict score:
5.000
step: 175 temp: 1.679 deltaE: +0.000 new conflict score:
5.000
step: 176 temp: 1.677 deltaE: +0.000 new conflict score:
5.000
step: 177 temp: 1.675 deltaE: -1.000 new conflict score:
4.000
step: 178 temp: 1.674 deltaE: -1.000 new conflict score:
3.000
step: 180 temp: 1.670 deltaE: +1.000 new conflict score:
4.000
step: 181 temp: 1.669 deltaE: +1.000 new conflict score:
5.000
step: 183 temp: 1.665 deltaE: +1.000 new conflict score:
6.000
step: 184 temp: 1.664 deltaE: -1.000 new conflict score:
5.000
step: 186 temp: 1.660 deltaE: -2.000 new conflict score:
3.000
step: 187 temp: 1.659 deltaE: +1.000 new conflict score:
4.000

step: 188 temp: 1.657 deltaE: -3.000 new conflict score: 1.000
step: 192 temp: 1.650 deltaE: +1.000 new conflict score: 2.000
step: 194 temp: 1.647 deltaE: +0.000 new conflict score: 2.000
step: 198 temp: 1.641 deltaE: +0.000 new conflict score: 2.000
step: 200 temp: 1.637 deltaE: +4.000 new conflict score: 6.000
step: 201 temp: 1.636 deltaE: +0.000 new conflict score: 6.000
step: 202 temp: 1.634 deltaE: +1.000 new conflict score: 7.000
step: 203 temp: 1.632 deltaE: -2.000 new conflict score: 5.000
step: 204 temp: 1.631 deltaE: -1.000 new conflict score: 4.000
step: 205 temp: 1.629 deltaE: +0.000 new conflict score: 4.000
step: 206 temp: 1.627 deltaE: +1.000 new conflict score: 5.000
step: 207 temp: 1.626 deltaE: -1.000 new conflict score: 4.000
step: 208 temp: 1.624 deltaE: +0.000 new conflict score: 4.000
step: 210 temp: 1.621 deltaE: +0.000 new conflict score: 4.000
step: 212 temp: 1.618 deltaE: +0.000 new conflict score: 4.000
step: 216 temp: 1.611 deltaE: +0.000 new conflict score: 4.000
step: 217 temp: 1.610 deltaE: -2.000 new conflict score: 2.000
step: 218 temp: 1.608 deltaE: +1.000 new conflict score: 3.000
step: 222 temp: 1.602 deltaE: +1.000 new conflict score: 4.000
step: 223 temp: 1.600 deltaE: +1.000 new conflict score: 5.000
step: 224 temp: 1.598 deltaE: -1.000 new conflict score: 4.000
step: 226 temp: 1.595 deltaE: +1.000 new conflict score: 5.000
step: 227 temp: 1.594 deltaE: -1.000 new conflict score: 4.000
step: 228 temp: 1.592 deltaE: -1.000 new conflict score: 3.000

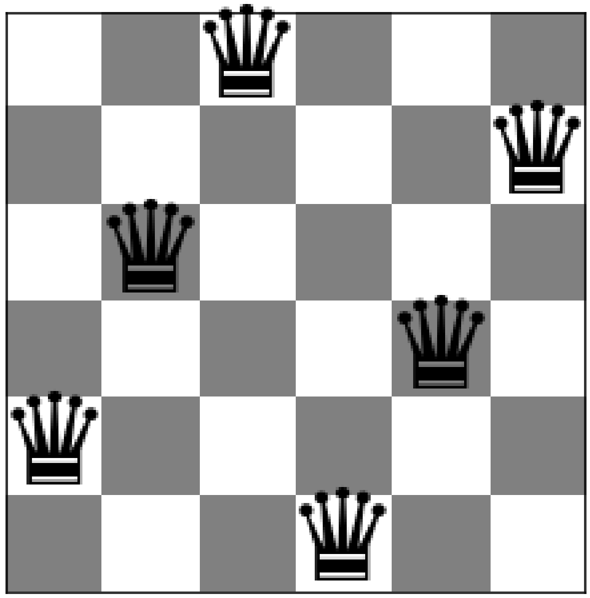
```
step: 229      temp: 1.590    deltaE: +2.000       new conflict score:
5.000
step: 231      temp: 1.587    deltaE: -1.000       new conflict score:
4.000
step: 236      temp: 1.579    deltaE: +0.000       new conflict score:
4.000
step: 237      temp: 1.578    deltaE: -1.000       new conflict score:
3.000
step: 239      temp: 1.575    deltaE: +0.000       new conflict score:
3.000
step: 243      temp: 1.568    deltaE: +1.000       new conflict score:
4.000
step: 247      temp: 1.562    deltaE: +0.000       new conflict score:
4.000
step: 248      temp: 1.561    deltaE: +1.000       new conflict score:
5.000
step: 249      temp: 1.559    deltaE: +1.000       new conflict score:
6.000
step: 250      temp: 1.557    deltaE: -2.000       new conflict score:
4.000
step: 251      temp: 1.556    deltaE: +0.000       new conflict score:
4.000
step: 252      temp: 1.554    deltaE: -1.000       new conflict score:
3.000
step: 254      temp: 1.551    deltaE: +0.000       new conflict score:
3.000
step: 255      temp: 1.550    deltaE: +1.000       new conflict score:
4.000
step: 256      temp: 1.548    deltaE: +1.000       new conflict score:
5.000
step: 259      temp: 1.543    deltaE: +0.000       new conflict score:
5.000
step: 260      temp: 1.542    deltaE: -1.000       new conflict score:
4.000
step: 261      temp: 1.540    deltaE: +0.000       new conflict score:
4.000
step: 264      temp: 1.536    deltaE: -1.000       new conflict score:
3.000
step: 265      temp: 1.534    deltaE: +0.000       new conflict score:
3.000
step: 267      temp: 1.531    deltaE: +0.000       new conflict score:
3.000
step: 268      temp: 1.530    deltaE: +0.000       new conflict score:
3.000
step: 269      temp: 1.528    deltaE: +0.000       new conflict score:
3.000
step: 271      temp: 1.525    deltaE: -1.000       new conflict score:
2.000
```

```
step: 273       temp: 1.522    deltaE: +1.000        new conflict score:
3.000
step: 274       temp: 1.520    deltaE: +0.000        new conflict score:
3.000
step: 275       temp: 1.519    deltaE: +1.000        new conflict score:
4.000
step: 276       temp: 1.517    deltaE: -1.000        new conflict score:
3.000
step: 277       temp: 1.516    deltaE: +2.000        new conflict score:
5.000
step: 278       temp: 1.514    deltaE: -1.000        new conflict score:
4.000
step: 279       temp: 1.513    deltaE: +0.000        new conflict score:
4.000
step: 281       temp: 1.510    deltaE: -1.000        new conflict score:
3.000
step: 282       temp: 1.508    deltaE: -1.000        new conflict score:
2.000
step: 285       temp: 1.504    deltaE: -1.000        new conflict score:
1.000
step: 287       temp: 1.501    deltaE: +0.000        new conflict score:
1.000
step: 289       temp: 1.498    deltaE: +1.000        new conflict score:
2.000
step: 292       temp: 1.493    deltaE: +2.000        new conflict score:
4.000
step: 293       temp: 1.492    deltaE: -2.000        new conflict score:
2.000
step: 295       temp: 1.489    deltaE: +1.000        new conflict score:
3.000
step: 296       temp: 1.487    deltaE: +1.000        new conflict score:
4.000
step: 297       temp: 1.486    deltaE: +1.000        new conflict score:
5.000
step: 299       temp: 1.483    deltaE: +1.000        new conflict score:
6.000
step: 300       temp: 1.481    deltaE: +0.000        new conflict score:
6.000
step: 301       temp: 1.480    deltaE: +0.000        new conflict score:
6.000
step: 302       temp: 1.478    deltaE: -1.000        new conflict score:
5.000
step: 303       temp: 1.477    deltaE: +0.000        new conflict score:
5.000
step: 305       temp: 1.474    deltaE: -1.000        new conflict score:
4.000
step: 307       temp: 1.471    deltaE: +0.000        new conflict score:
4.000
```

```
step: 308        temp: 1.470     deltaE: +0.000          new conflict score:
4.000
step: 309        temp: 1.468     deltaE: +0.000          new conflict score:
4.000
step: 310        temp: 1.467     deltaE: +0.000          new conflict score:
4.000
step: 313        temp: 1.462     deltaE: +1.000          new conflict score:
5.000
step: 315        temp: 1.459     deltaE: +0.000          new conflict score:
5.000
step: 316        temp: 1.458     deltaE: -1.000          new conflict score:
4.000
step: 318        temp: 1.455     deltaE: +0.000          new conflict score:
4.000
step: 319        temp: 1.454     deltaE: -1.000          new conflict score:
3.000
step: 322        temp: 1.449     deltaE: +0.000          new conflict score:
3.000
step: 323        temp: 1.448     deltaE: +2.000          new conflict score:
5.000
step: 324        temp: 1.446     deltaE: +1.000          new conflict score:
6.000
step: 326        temp: 1.443     deltaE: +0.000          new conflict score:
6.000
step: 327        temp: 1.442     deltaE: -1.000          new conflict score:
5.000
step: 328        temp: 1.440     deltaE: -1.000          new conflict score:
4.000
step: 329        temp: 1.439     deltaE: -2.000          new conflict score:
2.000
step: 330        temp: 1.438     deltaE: +0.000          new conflict score:
2.000
step: 332        temp: 1.435     deltaE: +1.000          new conflict score:
3.000
step: 335        temp: 1.430     deltaE: +0.000          new conflict score:
3.000
step: 336        temp: 1.429     deltaE: +2.000          new conflict score:
5.000
step: 337        temp: 1.428     deltaE: +1.000          new conflict score:
6.000
step: 339        temp: 1.425     deltaE: +0.000          new conflict score:
6.000
step: 340        temp: 1.423     deltaE: +0.000          new conflict score:
6.000
step: 341        temp: 1.422     deltaE: -2.000          new conflict score:
4.000
step: 342        temp: 1.420     deltaE: +1.000          new conflict score:
5.000
```

step: 343 temp: 1.419 deltaE: -1.000 new conflict score:
4.000
step: 344 temp: 1.418 deltaE: +0.000 new conflict score:
4.000
step: 345 temp: 1.416 deltaE: +0.000 new conflict score:
4.000
step: 346 temp: 1.415 deltaE: +1.000 new conflict score:
5.000
step: 347 temp: 1.413 deltaE: +1.000 new conflict score:
6.000
step: 348 temp: 1.412 deltaE: -1.000 new conflict score:
5.000
step: 349 temp: 1.411 deltaE: +0.000 new conflict score:
5.000
step: 350 temp: 1.409 deltaE: +0.000 new conflict score:
5.000
step: 351 temp: 1.408 deltaE: +0.000 new conflict score:
5.000
step: 352 temp: 1.406 deltaE: +0.000 new conflict score:
5.000
step: 354 temp: 1.404 deltaE: +1.000 new conflict score:
6.000
step: 355 temp: 1.402 deltaE: -3.000 new conflict score:
3.000
step: 356 temp: 1.401 deltaE: +1.000 new conflict score:
4.000
step: 357 temp: 1.399 deltaE: +0.000 new conflict score:
4.000
step: 358 temp: 1.398 deltaE: +0.000 new conflict score:
4.000
step: 360 temp: 1.395 deltaE: +1.000 new conflict score:
5.000
step: 362 temp: 1.392 deltaE: +0.000 new conflict score:
5.000
step: 363 temp: 1.391 deltaE: +0.000 new conflict score:
5.000
step: 365 temp: 1.388 deltaE: +1.000 new conflict score:
6.000
step: 366 temp: 1.387 deltaE: +0.000 new conflict score:
6.000
step: 368 temp: 1.384 deltaE: +0.000 new conflict score:
6.000
step: 369 temp: 1.383 deltaE: -1.000 new conflict score:
5.000
step: 371 temp: 1.380 deltaE: -2.000 new conflict score:
3.000
step: 373 temp: 1.377 deltaE: -1.000 new conflict score:
2.000

```
step: 375        temp: 1.374     deltaE: +1.000          new conflict score:
3.000
step: 376        temp: 1.373     deltaE: +1.000          new conflict score:
4.000
step: 378        temp: 1.370     deltaE: +0.000          new conflict score:
4.000
step: 380        temp: 1.367     deltaE: +1.000          new conflict score:
5.000
step: 381        temp: 1.366     deltaE: -2.000          new conflict score:
3.000
step: 383        temp: 1.363     deltaE: +0.000          new conflict score:
3.000
step: 385        temp: 1.361     deltaE: +0.000          new conflict score:
3.000
step: 386        temp: 1.359     deltaE: +0.000          new conflict score:
3.000
step: 389        temp: 1.355     deltaE: +0.000          new conflict score:
3.000
step: 392        temp: 1.351     deltaE: +0.000          new conflict score:
3.000
step: 393        temp: 1.350     deltaE: -1.000          new conflict score:
2.000
step: 396        temp: 1.346     deltaE: +0.000          new conflict score:
2.000
step: 397        temp: 1.344     deltaE: -1.000          new conflict score:
1.000
step: 403        temp: 1.336     deltaE: +1.000          new conflict score:
2.000
step: 404        temp: 1.335     deltaE: -2.000          new conflict score:
0.000
Ending board…
Board with 0 conflicts.
```



1.9 Compare Performance [2 Points]

Use runtime, scalability, and best objective function value to compare the algorithms on boards of different sizes.

For timing you can use the `time` package.

[236]:
```
# board size we wish to test
board_size = [4,5,7,10,12]
# our algos
algos = {
    steepest_ascent_hill: "Steepest Ascent",
    steepest_ascent_hill_random: "Steepest Ascent Random Restarts",
    stochastic_hill: "Stochastic",
    first_choice_hill: "First Choice",
    simulated_annealing: "Simulated Annealing"
}
# dataframe to store all data
all_data = pd.DataFrame()
# loop through algos and try board sizes
for size in board_size:
    df1 = pd.DataFrame({'Board Size': [], 'Algorithm': [],'Conflicts':␣
 ↪[],'Time': []})
    for algo in algos.items():
        time_list = []
        conflict_list = []
        # run for ten iterations
```

```
        for i in range(10):
            t0 = time.time()
            result = algo[0](size=size)
            t1 = time.time()
            time_list.append((t1-t0) * 1e3)
            conflict_list.append(result[0])
        # add to dataframe taking average
        df1 = df1.append({
            'Board Size': size,
            'Algorithm': algo[1],
            'Conflicts': (sum(conflict_list) / len(conflict_list)),
            'Time': (sum(time_list) / len(time_list))
        }, ignore_index=True)
    print("Results for board size of {}.".format(size))
    display(df1)
    all_data = all_data.append(df1)
```

Results for board size of 4.

	Board Size	Algorithm	Conflicts	Time
0	4.0	Steepest Ascent	0.6	3.851748
1	4.0	Steepest Ascent Random Restarts	0.0	3.669477
2	4.0	Stochastic	0.7	1.009274
3	4.0	First Choice	0.7	18.614650
4	4.0	Simulated Annealing	0.0	3.639936

Results for board size of 5.

	Board Size	Algorithm	Conflicts	Time
0	5.0	Steepest Ascent	0.4	2.863312
1	5.0	Steepest Ascent Random Restarts	0.0	3.422761
2	5.0	Stochastic	0.2	3.106308
3	5.0	First Choice	0.1	4.442501
4	5.0	Simulated Annealing	0.0	8.248329

Results for board size of 7.

	Board Size	Algorithm	Conflicts	Time
0	7.0	Steepest Ascent	1.0	15.247607
1	7.0	Steepest Ascent Random Restarts	0.0	25.354338
2	7.0	Stochastic	1.1	11.707926
3	7.0	First Choice	1.3	52.934623
4	7.0	Simulated Annealing	0.0	55.596018

Results for board size of 10.

	Board Size	Algorithm	Conflicts	Time
0	10.0	Steepest Ascent	1.6	46.338272

```
1         10.0  Steepest Ascent Random Restarts        0.0  328.262877
2         10.0                       Stochastic        2.0   54.265022
3         10.0                     First Choice        1.5   89.026880
4         10.0              Simulated Annealing        0.2  448.360467


Results for board size of 12.

   Board Size                        Algorithm  Conflicts         Time
0         12.0                  Steepest Ascent        1.9    89.126468
1         12.0  Steepest Ascent Random Restarts        0.0  2211.214828
2         12.0                       Stochastic        2.0   129.262757
3         12.0                     First Choice        1.8   141.669369
4         12.0              Simulated Annealing        0.0   553.148723
```

Taking a look at the above results of the algorithms, we can see that some perform much better on larger boards while others perform better on smaller boards.

For steepest ascent, we notice that the algorithm is relativively quick with smaller size boards however, it does not once end with 0 conflicts meaning it would get in a local optima quite often which is not ideal.

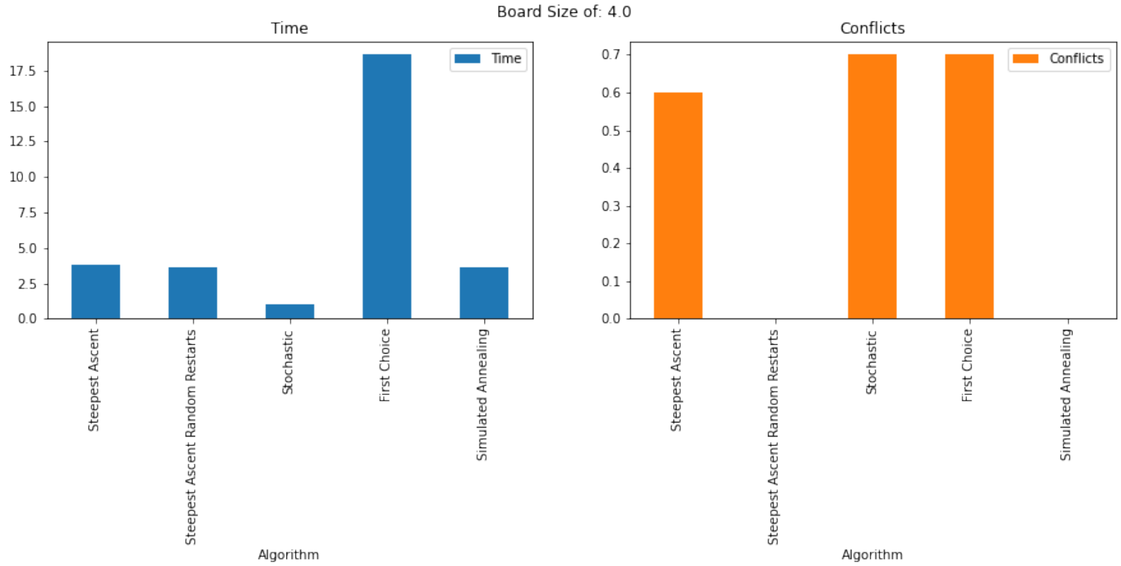
Moving on to steepest ascent with random restarts, we see that it actually always finds a solution however, the time it takes to find the solution often is quite large. This would make it un-ideal for larger sized boards. It is almost guaranteed to find the solution with enough restarts.

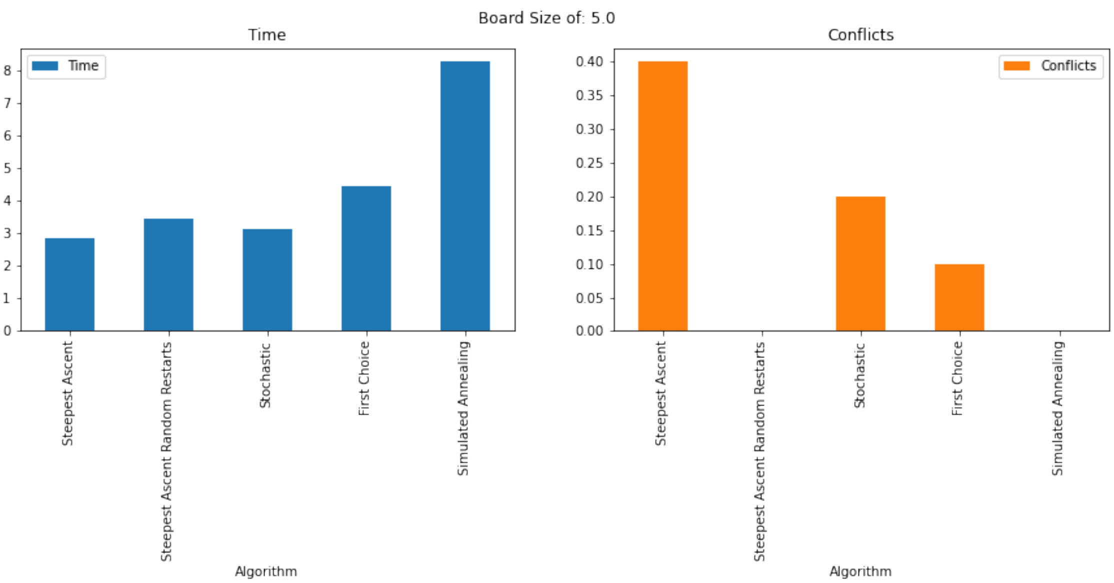
For stochastic hill climbing we see that it also does not find the optimal solution every time which implies that it also got stuck in a local optima. While its runtime was not that bad, it also tended to bump into the most conflicts out of all the algorithms.

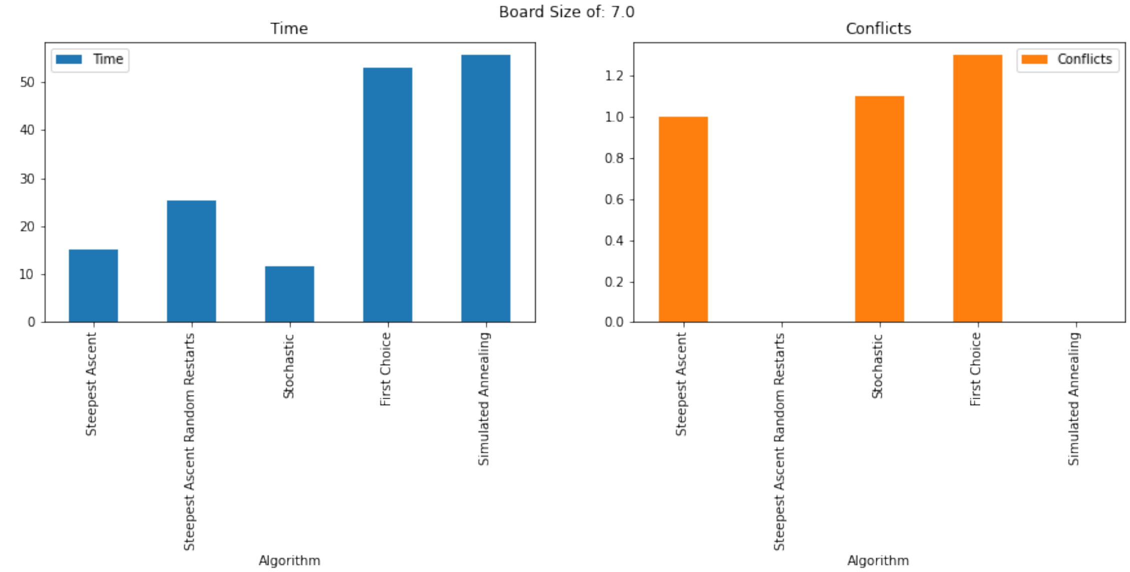
Taking a look at first choice, again we see that it typically would fall into local optima quite a bit with a high conflict score and it also wasnt the best in terms of runtime either. It was somewhat comparable to stochastic with less conflicts but a worse runtime.

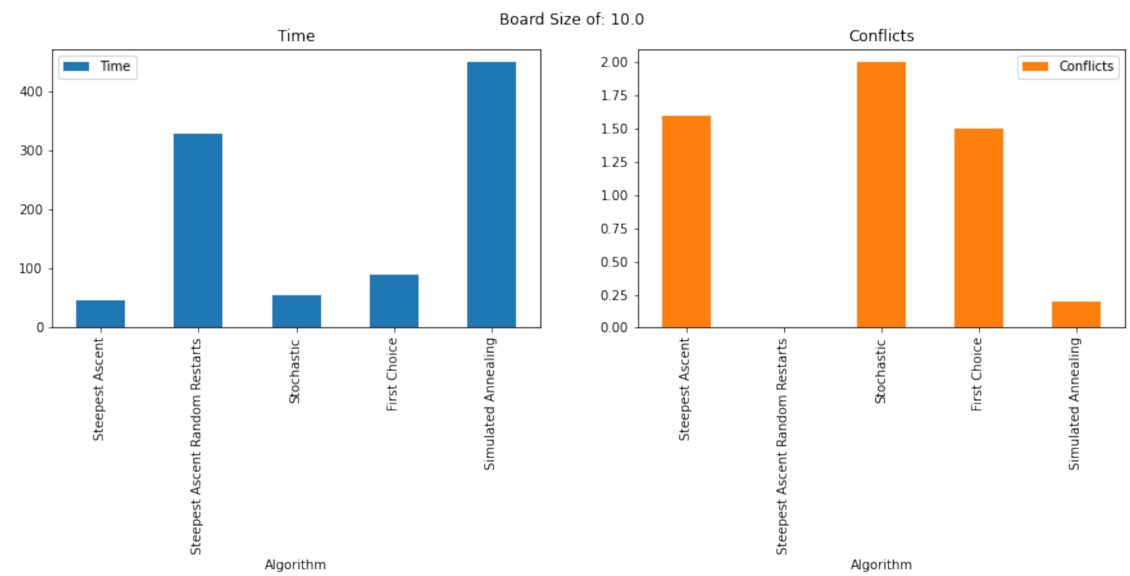
Lastly, looking at simulated annealing we see that it often would be able to find the optimal solution and also took not as long as some of the others but wasnt the best on smaller boards. When dealing with larger boards though, simulate annealing did a great job runtime and anything bigger than 10 it would probably be ideal to use it rather than the others.

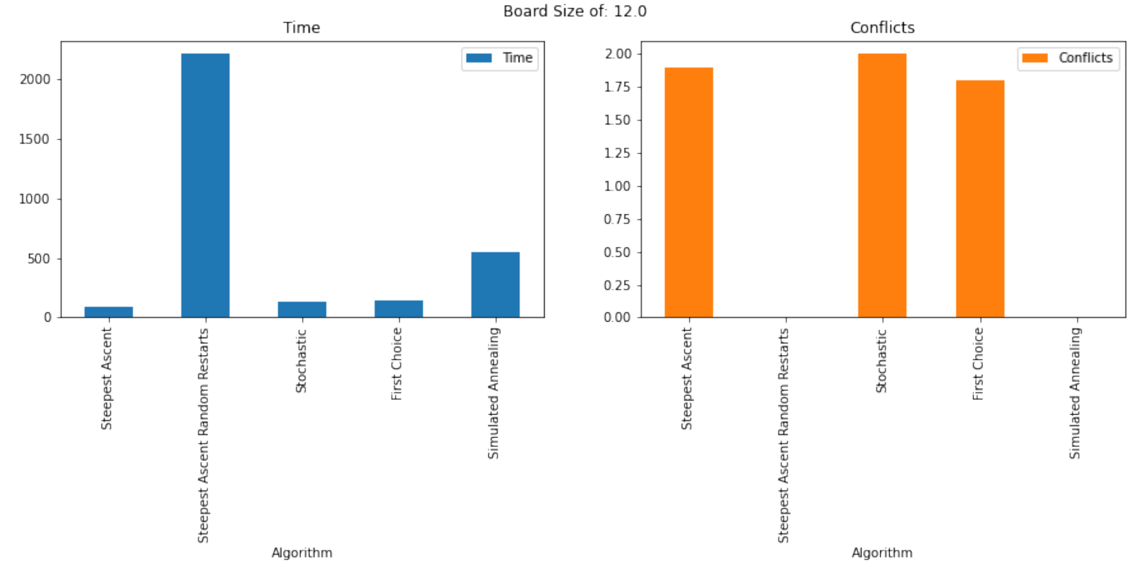
[237]:
```python
# graphs of each board size comparing algorithm time and conflicts
group_by_size = all_data.groupby('Board Size')
for i, (name, graph) in enumerate(group_by_size):
    graph.plot.bar(x="Algorithm", y=['Time','Conflicts'], subplots = True,␣
 ↪layout=(1,2),
                   figsize=(15,4), title="Board Size of: {}".format(name))
```


Board Size of: 4.0
Time
Time
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
Steepest Ascent
Steepest Ascent Random Restarts
Stochastic
First Choice
Simulated Annealing
Algorithm
Conflicts
Conflicts
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Steepest Ascent
Steepest Ascent Random Restarts
Stochastic
First Choice
Simulated Annealing
Algorithm


Board Size of: 5.0
Time
Time
8
7
6
5
4
3
2
1
0
Steepest Ascent
Steepest Ascent Random Restarts
Stochastic
First Choice
Simulated Annealing
Algorithm
Conflicts
Conflicts
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Steepest Ascent
Steepest Ascent Random Restarts
Stochastic
First Choice
Simulated Annealing
Algorithm

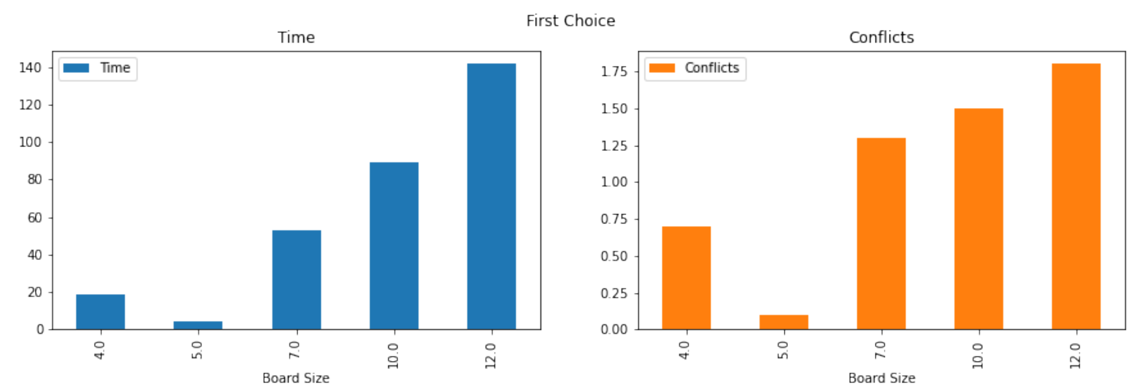

Board Size of: 7.0
Time
Time
0
10
20
30
40
50
Steepest Ascent
Steepest Ascent Random Restarts
Stochastic
First Choice
Simulated Annealing
Algorithm
Conflicts
Conflicts
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Steepest Ascent
Steepest Ascent Random Restarts
Stochastic
First Choice
Simulated Annealing
Algorithm

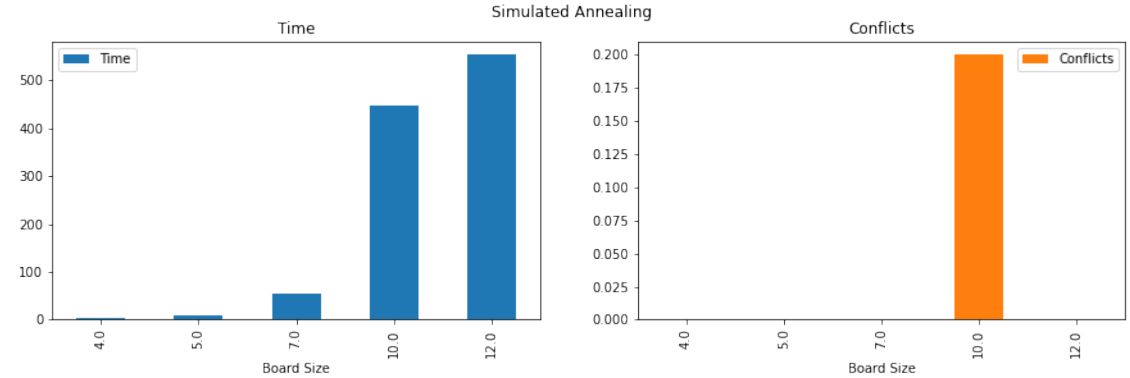

Board Size of: 10.0
Time
Time
0
100
200
300
400
Steepest Ascent
Steepest Ascent Random Restarts
Stochastic
First Choice
Simulated Annealing
Algorithm
Conflicts
Conflicts
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Steepest Ascent
Steepest Ascent Random Restarts
Stochastic
First Choice
Simulated Annealing
Algorithm

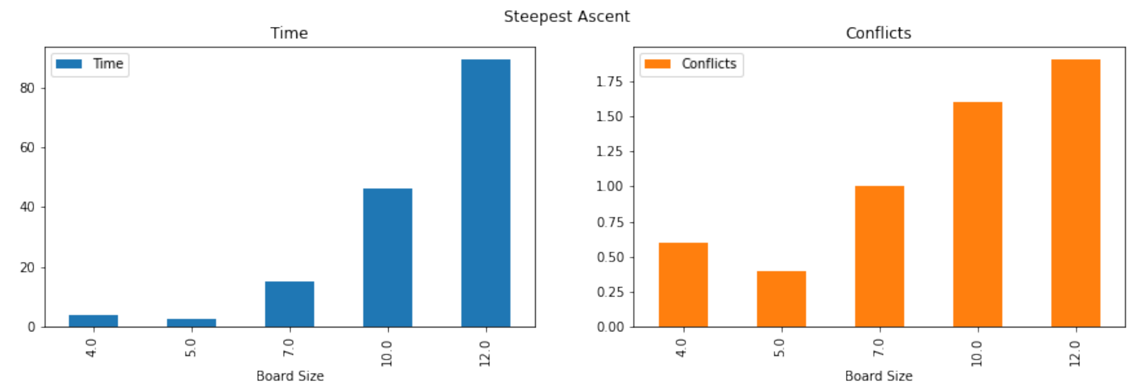


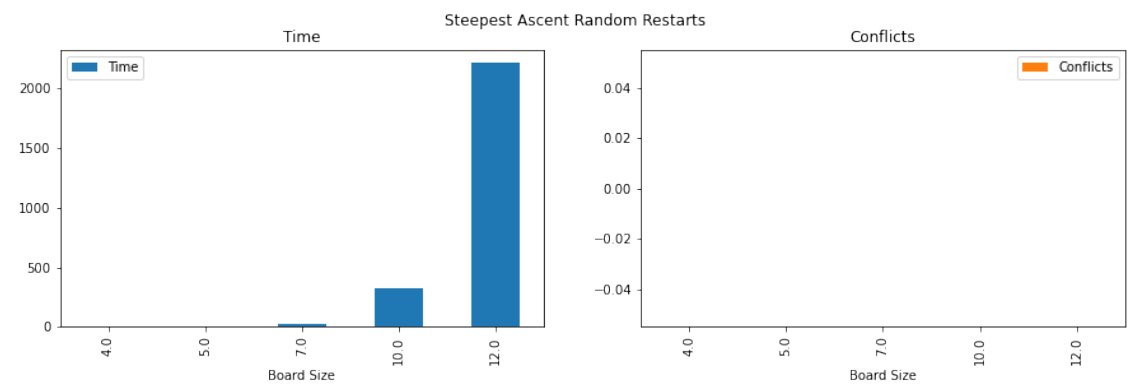
Above you can see some comparisons of the run time and conflict score of the algorithms for each board size.

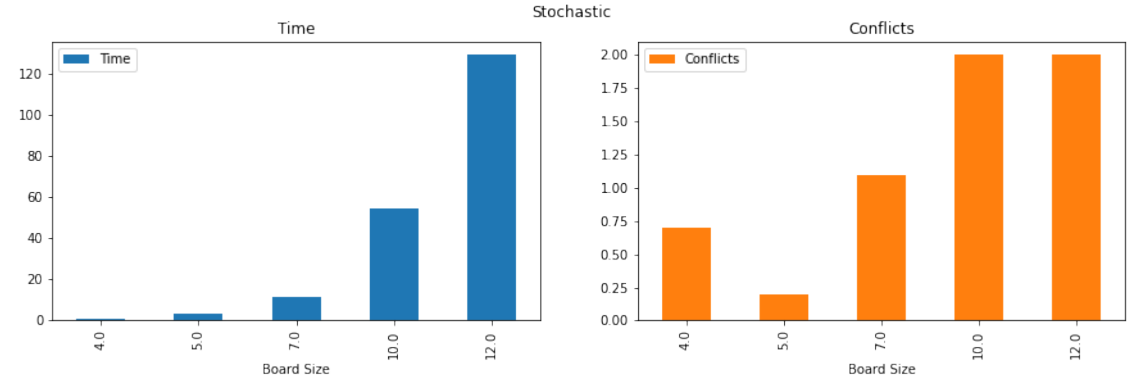
[238]:
```
# graphs of each algorithm comparing time and conflicts
group_by_algo = all_data.groupby('Algorithm')
for i, (name, graph) in enumerate(group_by_algo):
    graph.plot.bar(x="Board Size", y=['Time','Conflicts'], subplots = True,␣
↪layout=(1,2),
                   figsize=(15,4), title=name)
```




Simulated Annealing
Time
Time
0
100
200
300
400
500
4.0
5.0
7.0
10.0
12.0
Board Size
Conflicts
Conflicts
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
4.0
5.0
7.0
10.0
12.0
Board Size


Steepest Ascent
Time
Time
0
20
40
60
80
4.0
5.0
7.0
10.0
12.0
Board Size
Conflicts
Conflicts
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
4.0
5.0
7.0
10.0
12.0
Board Size

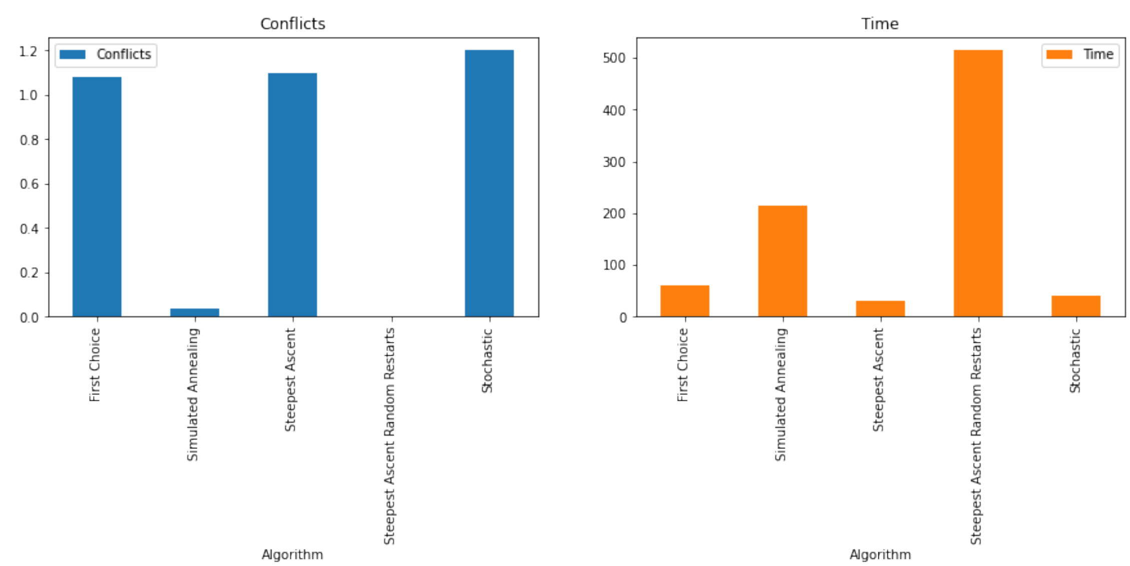

Steepest Ascent Random Restarts
Time
Time
0
500
1000
1500
2000
4.0
5.0
7.0
10.0
12.0
Board Size
Conflicts
Conflicts
−0.04
−0.02
0.00
0.02
0.04
4.0
5.0
7.0
10.0
12.0
Board Size



Above you can see some more comparison of the algorithms only this time comparing the time and conflicts for every board size of a given algorithm. We can see that simulated annealing had the most gradual shift as the board got larger while with others they seemed to not perform as well as the board grew, whether that be time of conflict score.

[239]:
```
# average time & conflicts
avg = all_data.drop(["Board Size"], axis=1).groupby('Algorithm').mean()
display(avg)
avg_plt = avg.plot.bar(subplots = True, layout=(1,2),figsize=(15,4))
```

	Conflicts	Time
Algorithm		
First Choice	1.08	61.337605
Simulated Annealing	0.04	213.798695
Steepest Ascent	1.10	31.485481
Steepest Ascent Random Restarts	0.00	514.384856
Stochastic	1.20	39.870257



The above graph shoes the average conflicts and run times of each algorithm accross all the board sizes of 4, 5, 7, 10, and 12. When we observe the above we see that simulated annealing has the best balanced results of all the algorithms, with a low average conflict score and a slightly above average run time. You can also see although steepest ascent with random restarts always finds the optimal solution, it performs terribly in regards to runtime and especially on large boards as noted above. So don't be decieved by what you see in regards to conflict scores. As for the others, they are very comparable when it comes to average run time and conflict scores with near identical results.

Given the above analysis, if I were to use one of the algorithms to perform NP Hard search, I would most likely choose simulated annealing due to the fact that the others perform poorly or take a massive amount time to complete.

1.10 Bonus: Genetic Algorithm [+1 Point]

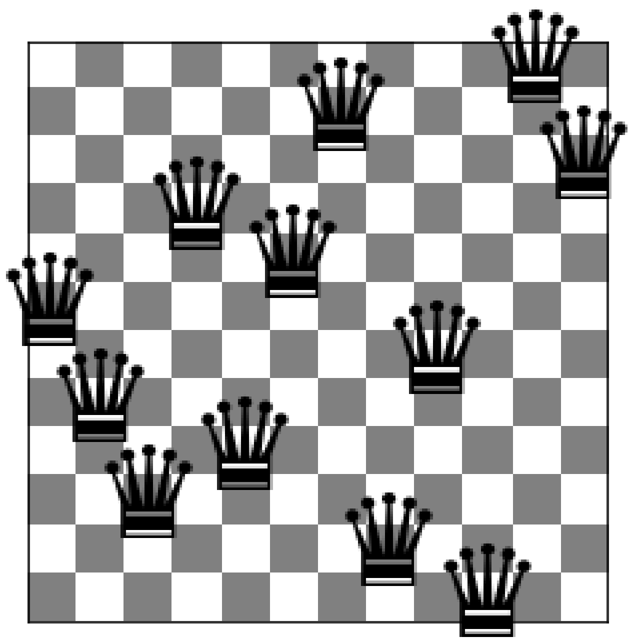
[256]:
```
def genetic_search(size, population_size=15, generations=1000, select_prob=.5,
                   mutation_prob = .2, debug=False):
    # first generate init population
    population = []
    for i in range(population_size):
        board = random_board(size)
        board_score = conflicts(board)
        population.append((board_score, board.copy()))

    best_solution = min(population, key = itemgetter(0))[1]
    best_score = min(population, key = itemgetter(0))[0]

    # sort population according to best values
    population.sort(key=lambda x:x[0])

    for generation in range(generations):
        # selection based on populcation
        selection_pop = selection(population, population_size, select_prob)
        # crossover
        crossover_pop = crossover(size, selection_pop)
        # add crossover population to selection population
        selection_pop.extend(crossover_pop)
        # mutation
        population = mutation(size, selection_pop, mutation_prob)
        # sort by score
        population.sort(key=lambda x:x[0])
        # update best solution
        if(population[0][0] < best_score):
            best_score = population[0][0]
            best_solution = population[0][1].copy()
```

```
            if debug:
                print("FOUND BETTER SOLUTION SCORE {} - {}".
↪format(best_score,best_solution))
            # if found global min, return as optimal solution
            if(best_score == 0):
                return((best_score, best_solution))

    return((best_score, best_solution))

def selection(population, population_size, prob):
    # how many of best in population choosing to take
    selected_num = population_size * prob
    # return top selected_num of solutions
    return(population[0:int(selected_num)].copy())

def crossover(size, selection_pop):
    selected = selection_pop.copy()
    crossover_pop = []
    # loop through half of solutions and peform crossover
    for i in range(int(len(selection_pop)/2)):
        # obtain two solutions to crossover and ensure we utilize as many of␣
↪the unique
        #  solutions as possible by removing them after crossing them
        p1 = selected.pop(random.randint(0, len(selected)-1))
        p2 = selected.pop(random.randint(0, len(selected)-1))
        # find crossover index that is not at the ends
        cross_index = random.randint(1, size-2)
        # obtain the crossover result
        c1 = np.concatenate(( p1[1][0:cross_index], p2[1][cross_index:size] ))
        c2 = np.concatenate(( p2[1][0:cross_index], p1[1][cross_index:size] ))
        # add the solutions to our crossover values
        crossover_pop.append((conflicts(c1), c1.copy()))
        crossover_pop.append((conflicts(c2), c2.copy()))

    return(crossover_pop)

def mutation(size, selection_pop, mutation_prob):
    # obtain number of mutations
    mutations = int(len(selection_pop) * mutation_prob)
    for i in range(mutations):
        # obtain element to mutate at random
        to_mutate_idx = random.randint(0, len(selection_pop)-1)
        to_mutate = selection_pop[to_mutate_idx][1]
        # obtain new random location for mutation
        col = random.randint(0, size-1)
        row = random.randint(0, size-1)
        # apply mutation
```

```
        to_mutate[col] = row
        # add back into population
        selection_pop[to_mutate_idx] = (conflicts(to_mutate), to_mutate.copy())

    return selection_pop.copy()
```

[260]:
```
result = genetic_search(size=12, debug=True)
show_board(result[1])
```

```
FOUND BETTER SOLUTION SCORE 9 - [11  7  9  4  8  3 10  1  6  1  9  6]
FOUND BETTER SOLUTION SCORE 8 - [ 2  7 11  1  8  4 10  1 10 10  0 11]
FOUND BETTER SOLUTION SCORE 7 - [ 5  1  9  8  8  4 10  1  6 10  0 11]
FOUND BETTER SOLUTION SCORE 5 - [ 2  7  9  4  8  4 10  1  6 10  0 11]
FOUND BETTER SOLUTION SCORE 4 - [ 5  3  9  4  8  4 10  1  6  1  0 11]
FOUND BETTER SOLUTION SCORE 3 - [ 5  7  9  3  8  4  9  1  6 10  0 11]
FOUND BETTER SOLUTION SCORE 2 - [ 5  7  9  3  8  4  9  1  6 10  0  2]
FOUND BETTER SOLUTION SCORE 1 - [ 5  7  9  3  8  4  1 11  6 10  0  2]
FOUND BETTER SOLUTION SCORE 0 - [ 5  7  9  3  8  4  1 10  6 11  0  2]
Board with 0 conflicts.
```



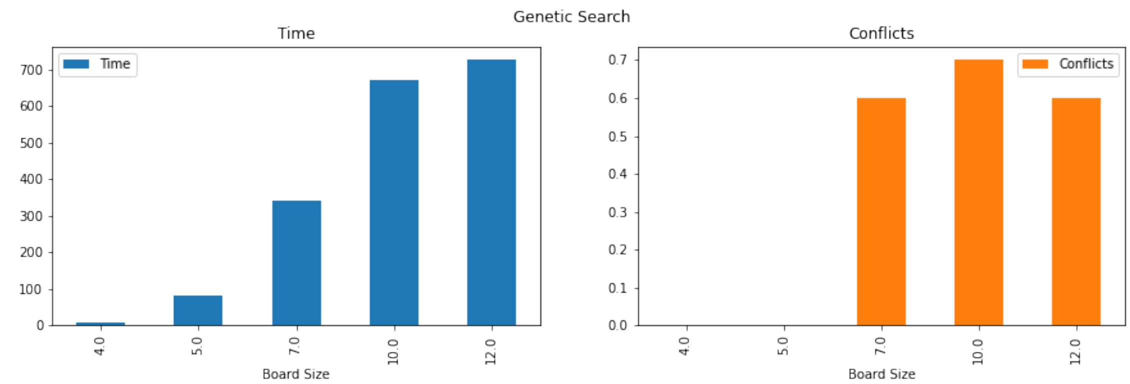
[284]:
```
# board size we wish to test
board_size = [4,5,7,10,12]
# dataframe to store all data
genetic_data = pd.DataFrame({'Board Size': [],'Conflicts': [],'Time': []})
# loop through algos and try board sizes
```

```python
for size in board_size:
    print("Running Board Size {}".format(size))
    time_list = []
    conflict_list = []
    # run for ten iterations
    for i in range(10):
        t0 = time.time()
        result = genetic_search(size=size)
        t1 = time.time()
        time_list.append((t1-t0) * 1e3)
        conflict_list.append(result[0])
    # add to dataframe taking average
    genetic_data = genetic_data.append({
        'Board Size': size,
        'Conflicts': (sum(conflict_list) / len(conflict_list)),
        'Time': (sum(time_list) / len(time_list))
    }, ignore_index=True)

display(genetic_data)
```

```
Running Board Size 4
Running Board Size 5
Running Board Size 7
Running Board Size 10
Running Board Size 12
```

	Board Size	Conflicts	Time
0	4.0	0.0	9.393954
1	5.0	0.0	81.497431
2	7.0	0.6	341.537642
3	10.0	0.7	671.277905
4	12.0	0.6	726.582718

[286]:
```python
genetic_graph = genetic_data.plot.bar(x="Board Size", y=['Time','Conflicts'],␣
 ↪subplots = True,
                                      layout=(1,2), figsize=(15,4),␣
 ↪title="Genetic Search")
```



[285]:
```
print("AVERAGE CONFLICT SCORE AND TIME: ")
genetic_avg = genetic_data.drop(["Board Size"], axis=1).mean()
display(genetic_avg)
```

```
AVERAGE CONFLICT SCORE AND TIME:

Conflicts      0.38000
Time         366.05793
dtype: float64
```

[287]:
```
new_avg = avg
new_avg.loc['Genetic'] = [genetic_avg['Conflicts'], genetic_avg['Time']]
display(new_avg)
```

	Conflicts	Time
Algorithm		
First Choice	1.08	61.337605
Simulated Annealing	0.04	213.798695
Steepest Ascent	1.10	31.485481
Steepest Ascent Random Restarts	0.00	514.384856
Stochastic	1.20	39.870257
Genetic	0.38	366.057930

In the genetic algorithm we see right off the bat that it is somewhat comparable to simulated annealing. We would expect that genetic may out perform simulated annealing but again it may perform much better on much larger boards. Above we can see the averages of the algorithm and it seems to take longer than simulated annealing while also ending in a state not that is not as good. This could be due to my implementation and fine tuning the parameters of genetic may help obtain a better solution. Regardless, it still outperformed most of the other algorithms in time and conflict score. I used a selection probability of 50% and a mutation rate of 20%.

Ultimately, it seems that my simulated annealing algorithm peformed the best by far and would be the best algorithm to choose when trying to find optimal solutions for this problem efficiently. If you have all the time in the world and dont car about long run times, then steepest ascent with random restarts is pretty good but it is unlikely you would ever find yourself in the position. Thus in the end, I would reccomend utilizing simulated annealing.

On a side note, in a class in the past I implemented a simulated annealing algorithm to solve TSP and it also was the best performing algorithm. It is interesting because it is truly one of the easier algorithms to implement and it performs so well!

[ ]:
```

```