Differential Evolution

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Differential Evolution [Feo07] I

- Differential Evolution (DE) is a popular stochastic population-based evolutionary metaheuristic method for continuous optimization problems.
- Storn and Price originally introduced DE in 1995 for minimizing non-differentiable functions [SP95].
- DE is considered to be an Evolutionary Algorithm (EA). However, it was not inspired by nature like the rest.
- An EAs ability to find the global optimum depends on a perfect balance of exploring the search space and exploiting already discovered information.

Differential Evolution [Feo07] II

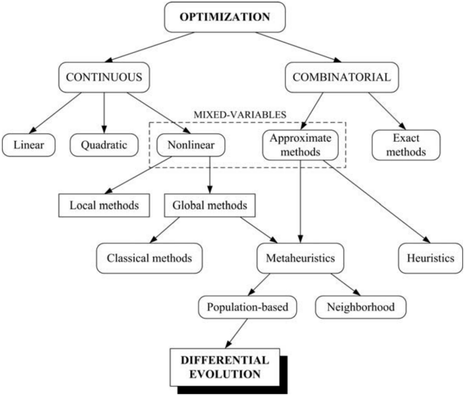

Figure 1: Optimization flow-chart from *Differential Evolution: In Search of Solutions* by Vitaliy Feoktistov [Feo07].

Differential Evolution [Feo07] III

- DE evolves a population of candidate solutions though evolutionary operations (mutations, crossover, etc) to discover favourable solutions within a search space.
- A significant advantage of vanilla DE is it only requires an adjustment of three control parameters.
- The optimization outcome of DE is heavily reliant on the choice of control parameter values and the trial vector creation scheme.
- Choosing optimal parameters is a strenuous task and becomes even more demanding as the problem becomes more advanced.

Differential Evolution [Feo07] IV

- DE along with all metaheuristics struggle as the dimensionality of the problem increases though what is known as The Curse of Dimensionality.
- Due to the dimensionality crux and the choosing of optimal control parameters, hindering DE's ability to perform on Large Scale Global Optimization (LSGO) problems, researchers have studied and developed many new state-of-the-art DE variants.

Common shorthand for DE strategies [ZSA13]: DE/a/b/c

a. A description of the chosen base vector
b. Number of random vector differences added to the base vector
c. A description of the recombination operation

Parameters [Feo07] I

Control Parameters:

- F - differentiation (or mutation) constant
- Cr - crossover constant
- NP - size of population

Other Parameters:

- D - problem dimension
- GEN - max number of generations
- L - lower bound of search space
- H - upper bound of search space

Algorithm I

Algorithm 1 Famous Differential Evolution

Require: D – problem dimension (optional)
NP, F, Cr – control parameters
GEN – stopping condition
L , H – boundary constraints

Initialize population $Pop_{ij} \leftarrow rand_{ij}[L, H]$ and Evaluate fitness $Fit_j \leftarrow f(Pop_j)$
for $g = 1$ to GEN **do**
  for $j = 1$ to NP **do**
    Choose randomly $r_{1,2,3} \in [1, \ldots, NP]$, $r_1 \neq r_2 \neq r_3 \neq j$
    Create trial individual $X \leftarrow \mathcal{S}(r, F, Cr, Pop)$
    Verify boundary constraints **if** $(x_i \notin [L, H])$ $x_i \leftarrow rand_i[L, H]$
    Select better solution (X or Pop_j), and update $iBest$ if required
  end for
end for

Figure 2: Differential Evolution algorithm summary from *Differential Evolution: In Search of Solutions* by Vitaliy Feoktistov [Feo07].

Trial Individual Creation I

$$x_i = \begin{cases} x_{i,r_3} + F \cdot (x_{i,r_1} - x_{i,r_2}) & \text{if} \quad (rand_{ij}[0,1) < Cr) \vee (Rnd = i) \\ x_{ij} & \text{otherwise} \end{cases}$$
$$i = 1, \ldots, D$$

Figure 3: The equation for creating a trial individual from *Differential Evolution: In Search of Solutions* by Vitaliy Feoktistov [Feo07].

Trial Individual Creation II

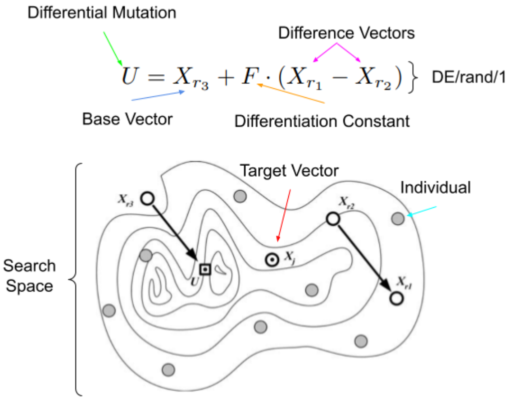

Figure 4: Creating the differential mutation vector U from *Differential Evolution: In Search of Solutions* by Vitaliy Feoktistov [Feo07].

Enhanced Adaptive DE Algorithm I

Solving large-scale global optimization problems using enhanced adaptive differential evolution algorithm was written by Ali Wagdy Mohamed in 2017 and it proposed an enhanced adaptive differential evolution (EADE) algorithm for solving LSGO problems [Moh17].

- EADE incorporates two unique modifications:
  1. A new mutation rule
     $$v_i^{G+1} = x_r^G + F1 \cdot (x_{p\_best}^G - x_r^G) + F2 \cdot (x_r^G - x_{p\_worst}^G)$$
  2. A self-adaptive crossover rate
- EADE has been shown to be highly competitive in comparison to other state-of-the-art methods through experimental tests on both the CEC2008 and CEC2010 benchmark functions.

Decentralizing & Coevolving DE I

Decentralizing and coevolving differential evolution for large-scale global optimization problems was written by Ruoli Tang in 2017 and it presented a novel decentralizing and coevolving differential evolution (DCDE) algorithm for solving LSGO problems [Tan17].

- DCDE's core ideas are as follows:
  1. Decompose population into fewer subpopulations in a ring-like fashion
  2. New mutation operation "current-to-SP-best-ring"
     $$v_i = x_i + F \cdot (SP\text{-}best\text{-}ring_i - x_i) + F \cdot (x_{r1} - x_{rsp})$$
  3. Take advantage of SP-best-rings to select a random individual for the mutation operation
- DCDE compared favourably against several state-of-the art LSGO algorithms on the CEC2008 benchmark test suite.

Micro DE With Local Directional Search I

In 2019, Yildiz and Topal wrote a paper called *Large scale continuous global optimization based on micro differential evolution with local directional search* to introduce a micro Differential Evolution with a Directional Local Search (μDSDE) algorithm for LSGO while using a small population [YT19].

- The main μDSDE technique:
    - Utilizes six individuals to solve LSGO problems
    - The best of the six stays put
    - The second-best goes though crossover and mutation
    - The remaining are dispersed randomly within the search space
- Supporting extensive empirical studies were conducted using the CEC2010 benchmark test suite up to 5000 dimensions.

Multi-population DE With Best-random Mutation Strategy I

Ma and Bai introduced a paper in 2020 titled *A multi-population differential evolution with best-random mutation strategy for large-scale global optimization* proposing a new LSGO algorithm abbreviated to mDE-brM [MB20].

- The core of mDE-brM:
  - Three sub-populations evolve in parallel
  - Sub-populations use different evolutionary strategies
  - Strategies are shared amongst individuals by migrating among sub-populations
- mDE-brM's performance was competitive compared with 5 state-of-the-art optimization technique evaluated on the CEC2013 benchmark suite.

Spark-based DE With Grouping Topology Model I

A Spark-based differential evolution with grouping topology model for large-scale global optimization was introduced in 2021 by He et al., proposing a new DE variant to solve LSGO problems, called SgtDE [HPC+21].

- The focus of SgtDE:
  - Split population evenly among three subgroups
  - Subgroups contain five sub-populations, i.e. islands
  - Utilize an individual migration strategy
  - Incorporate Intra subgroup and inter subgroup topologies
- SgtDE provided significant performance with various DE variants in contrast to other state-of-the-art approached on the CEC2010 benchmark suite.

Conclusions & Future Work I

- Differential Evolution (DE) is a famous, powerful, versatile, and well-researched evolutionary optimizer, which can find favourable solutions to all kinds of optimization problems.
- New DE variants are constantly being developed, and given the complexity of the natural world, there is still much room for improvement.
- Often, given LSGO problems' difficulty, combining methods that perform favourably for specific functions is how you can extend its usability, releasing it from a niche problem area.

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