RS-25 Engine Analysis

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This project follows and analyzes the flow of fuel (LH2) and oxidizer (LO2) from beginning to end through the RS-25 rocket engine, built by Rocketdyne for use on the United States Space Shuttle Program and the recent Space Launch System (SLS). Using the project document listed in the "References" section, the conditions of the fuel and oxidizer were systematically calculated to solve for the flow characteristics through every subsystem in the engine. Since the project involved dozens of calculations across the engine, a profound understanding of the flow pathways and fluid mechanics were foundational for success.

This project was immensely helpful for understanding the inner workings of a rocket engine. All of the components featured in this project are complex in their own right, and even the level of depth employed in this project feels like an oversimplification for the actual systems. The process of solving for conditions at every location in the engine was immensely eye-opening to the complexities of rocket propulsion. MAE565 has been the most inspiring class of undergrad, and without a doubt, there should be more rocket propulsion taught at ASU.

The following sections will describe how every calculation was conducted in detail, such that if this project is to be repeated following this report should make it exceptionally simple to do so. This report should be read in tandem with the corresponding excel spreadsheet, titled "MAE565 Project Spreadsheet", where all calculations were carried out. The sections (in order) are as follows:

APPENDIX

*All variables listed are in SI units unless specified otherwise.

a	= Local Speed of Sound
a_e	= Exit Speed of Sound (Nozzle)
A	= Area
A_C	= Combustion Chamber Area
A_e	= Exit Area (Nozzle)
A^*	= Throat Area (Nozzle)
C_p	= Constant Pressure Specific Heat
C_v	= Constant Volume Specific Heat
d_C	= Combustion Chamber Diameter
d_e	= Exit Diameter (Nozzle)
d^*	= Throat Diameter
η_{CT}	= Coefficient of Thrust Efficiency
η_P	= Pump Efficiency
η_T	= Turbine Efficiency
g_0	= Sea Level Gravitational Acceleration
γ	= Heat capacity ratio (C_p/C_v)
I_{sp}	= Specific Impulse
$\dot{m}$	= Mass Flow Rate
M_e	= Exit Mach Number (Nozzle)
MW	= Molecular Weight
P	= Pressure
P_e	= Exit Pressure (Nozzle)
P_{t2}	= Combustion Chamber Pressure (Total)
P_∞	= External Pressure
P_1	= Pressure (Inlet)
P_2	= Pressure (Outlet)
R	= Specific Gas Constant
R_U	= Universal Gas constant
ρ	= Density
T	= Temperature/Thrust (Based on Application)
T_J	= Jet Thrust
T_P	= Pressure Thrust
T_1	= Temperature (Inlet)
T_2	= Temperature (Outlet)
V_e	= Exit Velocity
$\dot{W}_P$	= Pump Power
$\dot{W}_T$	= Turbine Power
X_i	= Molar Fraction
Y_i	= Mass Fraction

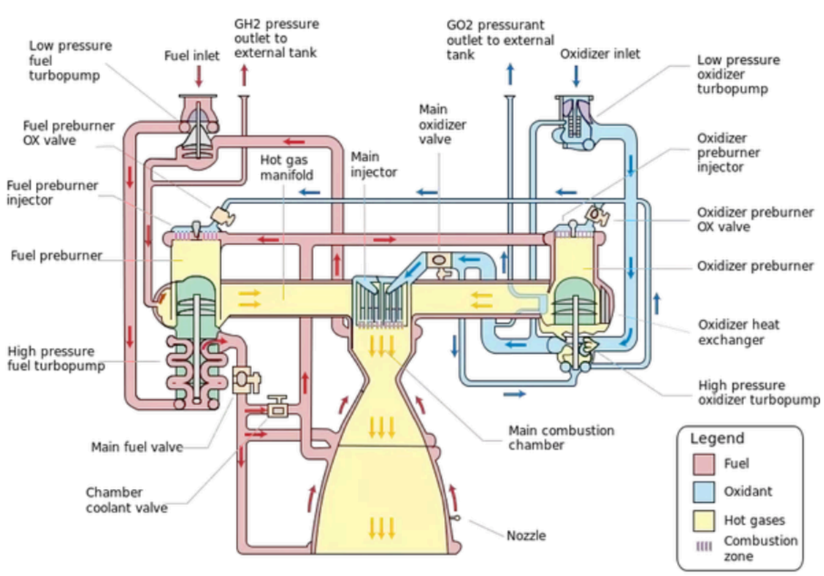
CALCULATIONS

For all cells filled in the Excel Spreadsheet, a brief statement of how every value was obtained is listed in Column G. The sections below provide more information for the calculated and carry-over values (ie blue and green cells). All math was done with excel equations to easily reference values, therefore, no calculations were done in written work. These "plug and chug" steps should be self explanatory. Thus, displayed below are *only* equations that the Spreadsheet values were directly plugged into. Many equations and justifications are repeated in the exact same format; <u>this is intentionally redundant</u>, as it makes referencing the correct process for every situation exceptionally simple, with no room for confusion. All numbered equations refer to equations taken directly from the Final Project Document, and all equations labeled with a letter are used for solutions for clarity. All calculations were done in SI units. Any page numbers referenced from the "project document" refer to Ref [1]. All final answers are listed in the spreadsheet, and are in red text.

** Indicates Green Requirement, ** Indicates Blue Requirement

Note: Many of the equations given in the project document use ΔP_t, where the lettered equations in this report simply use ΔP. The reasoning for this is specified on page 26, where the document states that since $V_2=V_1$ in most scenarios, this $\Delta P_t=\Delta P$. By using ΔP instead of the total pressures, velocity of the flow does not need to be accounted for. In fact, in most cases there is no way to solve for local flow speed since the project document often does not give information about pipe cross sectional area.

For reference: This is the total layout of the RS-25 from Ref [1]. This displays exactly where fuel and oxidizer travel through the engine, prescribing inlet and outlet conditions in various areas.



Ref [1] - "Fig. 1. Overall view of the flow paths in the RS-25 propellant feed system"

Low-Pressure Oxidizer Turbopump

The LPOTP is fed directly from the pressurized oxygen tank, thus the inlet conditions are the same as the outlet conditions of the tank. The turbopump then feeds directly to the HPOTP, so the outlet conditions of the LPOTP are the same as the inlet conditions of the HPOTP. The flow that drives the turbine comes from the output of the HPOTP, so the conditions at the inlet of the turbine are the same as the outlet conditions of the HPOTP.

PUMP SECTION

**LO2 Pressure at Pump Inlet:

Taken directly from page 2.

P_1=689K

**LO2 Temperature Entering Pump:

Taken directly from page 2.

T_1=90K

**LO2 Pressure Exiting Pump:

P_1 = 689kPa
2.1 MPa Given on page 3.

$$P_2 = P_1 + 2.1\text{MPa} \tag{a}$$

P_2=2789kPa

**LO2 Temperature Exiting Pump:

*Referenced from Eq. 11
T_1=90k, η_p=0.632,
P_2=2789kPa, P_1 = 689kPa,
ρ=1141J/kgK,
C_v = 1669kg/m^3

$$T_2 = T_1 + \frac{1-\eta_p}{\eta_p}\left(\frac{P_2-P_1}{\rho C_v}\right) \tag{b}$$

T_2=90.642K

**LPOTP Pump Power:

*Referenced from Eq. 10
$\dot{m}$=401kg/s, η_p=0.632,
P_2=2789kPa,P_1 = 689kPa,
ρ=1141J/kgK

$$\dot{W}_P = \dot{m}\frac{1}{\eta_p}\left(\frac{P_2-P_1}{\rho}\right) \tag{c}$$

$\dot{W}_P$=1167.78kW

TURBINE SECTION

Note: Flow into turbine is flow out from HPOTP, so flow conditions are the same.

**LO2 Temperature entering turbine:

Same temperature as HPOTP outlet.

T_1=97.237K

**LO2 Pressure entering turbine:

Same pressure as HPOTP outlet.

P1=29.6MPa

**LO2 Mass flow rate through turbine:

*Equation 20 Gives: $$\dot{W}_T = \dot{m}\eta_T\left(\frac{P_1-P_2}{\rho}\right) \quad \text{(d)}$$

*Which can be rearranged to solve for $\dot{m}$. $$\dot{m} = \dot{W}_T\eta_T\left(\frac{\rho}{P_1-P_2}\right) \quad \text{(e)}$$

$\dot{W}_{out}$=1167.78kW, $\eta_T = .644$,
ρ=1141J/kgK,P_1=2789kPa,
P_2=29.6MPa

$\dot{m}$=77.17kg/s

**LO2 Pressure exiting turbine:

Same as the pressure leaving the pump. The project document says on page 4, "Pressure at which LO2 leaves the impeller determines the pressure at which LO2 leaves the turbine".

P_2=2789kPa

**LO2 Temperature exiting turbine:

*Referenced from Eq. 21
T_1=97.237K, η_T=0.644,
P_1=2789kPa,P_2=29.6MPa
ρ=1141J/kgK,
C_v = 1669kg/m^3

$$T_2 = T_1 + (1 - \eta_T)\left(\frac{P_1-P_2}{\rho C_v}\right) \quad \text{(f)}$$

T_2=102.249kPa

**LPOTP Turbine power:

Same as the power from the pump section. The project document says on page 25, "The turbine that drives the pump must provide this actual pump power".

$\dot{W}_T$=1167.78kW

Low-Pressure Fuel Turbopump

The LPFTP is fed directly from the pressurized LH2 tank, so the outlet conditions given in the project document of the tank are the same as the inlet conditions for the pump. The pump outputs directly to the HPFTP, so the inlet conditions at the HPFTP are the same as the outlet conditions of the LPFTP. The turbine is driven by GH2 that was used for regenerative cooling on the thrust chamber, so the inlet conditions of the turbine can be derived from those given conditions.

PUMP SECTION

**LH2 Pressure at Pump Inlet:

Taken directly from page 2.

P_1=207kPa

**LH2 Temperature Entering Pump:

Taken directly from page 2.

T_1=20K

**LH2 Pressure Exiting Pump:

P_1 = 207kPa
1.6MPa Given on page 4.

$$P_2 = P_1 + 1.6\text{MPa} \tag{g}$$

P_2=1807kPa

**LH2 Temperature Exiting Pump:

*Referenced from Eq. 11
T_1=20K, η_p=0.674,
P_2=1807kPa, P_1=207kPa,
ρ=70.8J/kgK,
C_v = 9668kg/m^3

$$T_2 = T_1 + \frac{1-\eta_p}{\eta_p}\left(\frac{P_2-P_1}{\rho C_v}\right) \tag{h}$$

T_2=21.131K

**LPFTP Pump Power

*Referenced from Eq. 10
$\dot{m}$=67.1kg/s, η_p=0.674,
P_2=1807kPa,P_1 = 207kPa,
ρ=70.8J/kgK

$$\dot{W}_P = \dot{m}\frac{1}{\eta_p}\left(\frac{P_2-P_1}{\rho}\right) \tag{i}$$

$\dot{W}_P$=2.25MW

TURBINE SECTION

**GH2 Pressure at turbine inlet:

The flow entering the turbine of the LPFTP was used to cool the thrust chamber and turned from LH2 to GH2. The pressure going into the turbine is the pressure after it has done the cooling, and is indicated on Page 9.

P_1=32.5MPa

**GH2 Temperature at turbine inlet:

The temperature is indicated along with the pressure on Page 9.

T_1=269K

**GH2 Mass flow rate through turbine:

*Equation 20 Gives: $$\dot{W}_T = \dot{m}c_p(T_{t1} - T_{t2}) \tag{j}$$

Total temperatures can be approximated using static conditions because of low relative flow velocity (See explanation under "GH2 Temperature at turbine outlet" for more information). The above equation can then be rearranged and simplified to solve for $\dot{m}$.

$\dot{W}_{out}$=2.25MW, η_T =.536,
C_p = 14340J/kgK, P_1=32500kPa,
P_2=25000kPa

$$\dot{m} = \frac{\dot{W}_{out}}{c_p(T_1 - T_2)} \tag{k}$$

$\dot{m}$=13.282kg/s.

Alternatively, on page 9 it is specified that 20.3% of the LH2 mass flow from the HPFTP goes to drive the LPFTP turbopump. Taking 20.3% of the total mass flow through the HPFTP gives $\dot{m}$=13.62kg/s. Since approximations were made in the first method, using static temperature instead of total temperature, the second value was used in the spreadsheet. Either of these would be acceptable, though, because they are well within 10% of each other.

**GH2 Pressure at turbine outlet:

As stated on page 6, pressure drops by a factor of 1.3 across the turbine, so $P_2 = P_1\left(\frac{1}{1.3}\right)$

P_2=25MPa

**GH2 Temperature at turbine outlet:

Since gas turbines deal with compressible flow, the static conditions can no longer be related, and thus must be related through their total-static relations. So, to find the temperature relationship of the inlet and outlet of a gas turbine, the total temperatures and total pressures must be related. From there, the static temperature at the location of interest can be found. The equation for the relationship between total temperature at the inlet and total temperature at the outlet of the turbine is described in equation 22 of the project document as shown below:

$$\frac{T_{t2}}{T_{t1}} = 1 - \eta_T\left[1 - \left(\frac{p_{t2}}{p_{t1}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Unfortunately, the document does not provide any information about pipe areas or flow speed. It may be possible to solve this equation, but the local Mach number of the flow would need to be found to do so. Instead, the information given below this equation on page 28 can be used to develop a close approximation for the temperature at the outlet of the turbine. The project document states, "In a turbopump the Mach numbers M of the gas flow at the inlet and outlet of the turbine are typically low enough that there is no significant distinction between total and static states, and thes [Equation] (22) can be applied to the static states as well" (Page 28). This just means that since the flow is moving relatively slow in the turbine the static conditions are a good enough estimate for the total conditions. Equation 22. Then becomes:

$$\frac{T_2}{T_1} = 1 - \eta_T\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right] \qquad \text{(l)}$$

Solving for T_2 then gives the final equation as:

*Referenced from Equation 22
T_1=269K, η_T=0.536, P_1=32500kPa, P_2=25000kPa, γ=1.483

$$T_2 = T_1\left\{1 - \eta_T\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\} \qquad \text{(m)}$$

T_2=257.187K

**<u>LPFTP Turbine power:</u>

Same as the power from the pump section. The project document says on page 25, "The turbine that drives the pump must provide this actual pump power".

$\dot{W}_T$=2.25MW

High Pressure Fuel Turbopump

The HPFTP is fed directly by the LPFTP, so the inlet conditions are the same as the outlet conditions of the LPFTP, with the exception of the pressure drop indicated on Page 6. The turbopump outputs high pressure LH2, which goes to regenerative cooling in the thrust chamber and expansion nozzle, and an expansion valve. The flow that went to cooling the thrust chamber goes back to drive the LPFTP turbine, and the rest goes to the high pressure turbopump preburners. The HPFTP preburner is also fed by high pressure LO2 that is tapped off from the HPOTP boost pump. The hot combustion gas created in the preburner is then used to drive the HPFTP turbine, before then going to the main combustion chamber to be used as fuel. As previously mentioned, the conditions at every inlet in this system can be identified by examining the outlet conditions of where the flow came from. All conditions will be the same at the outlet and inlet unless specified otherwise.

PUMP SECTION

**LH2 Pressure at pump inlet:

Page 6 of Project. Pressure from LPFTP reduced by 398kPa due to pressure drop.

P_1=1409kPa

**LH2 Mass flow rate through pump:

All of the mass going through and leaving the LPFTP travels into the HPFTP, so mass flow rate is the same.

$\dot{m}$=67.1kg/s

**LH2 Pressure exiting pump:

Add pressure increase across the pump to initial pressure at the inlet.

$$P_2 = P_1 + P_{increase} \tag{n}$$

P_2=43109kPa

**LH2 Temperature entering pump:

No indicated temperature loss between LPFTP and HPFTP, so temperature at the exit of the LPFTP is the same as the temperature at the inlet of the HPFTP.

**LH2 Temperature exiting pump:

*Referenced from Eq. 11
T_1=21.131K, η_p=0.758,
P_2=43109kPa, P_1=1409kPa,
ρ=70.8J/kgK,
C_v = 9668kg/m^3

$$T_2 = T_1 + \frac{1-\eta_p}{\eta_p}\left(\frac{P_2-P_1}{\rho C_v}\right) \tag{o}$$

T_2=40.58K

**HPFTP Pump power:

*Referenced from Eq. 10
$\dot{m}$=67.1kg/s, η_p=0.758,
P_2=43109kPa,P_1 = 1409kPa,
ρ=70.8J/kgK

$$\dot{W}_P = \dot{m}\frac{1}{\eta_p}\left(\frac{P_2-P_1}{\rho}\right) \quad (p)$$

$\dot{W}_P$=52.138MW

FLOW SPLITS

**LH2 Mass flow rate for thrust chamber cooling:

20.3% of mass flow rate is directed to thrust chamber cooling, as indicated on page 9 of the project document. ($\dot{m}_{cooling} = \dot{m}_{pump}$*(.203))

$\dot{m}_{cooling}$=13.621kg/s

**LH2 Temperature entering thrust chamber cooling:

Flow goes directly to regenerative cooling from the pump, so it is the same as the temperature leaving the HPOTP pump section. (Page 9).

T_1=40.58K

**LH2 Pressure entering thrust chamber cooling:

Flow goes directly to regenerative cooling from the pump, so it is the same as the pressure leaving the HPOTP pump section. (Page 9).

P_1=43109kPa

**LH2 Mass flow rate for expansion nozzle cooling:

42.4% of mass flow rate is directed to expansion nozzle cooling, as indicated on page 9 of the project document. ($\dot{m}_{cooling} = \dot{m}_{pump}$*(.424)). This is also given as 28.5 kg/s on page 23.

$\dot{m}_{cooling}$=28.45kg/s

**LH2 Temperature entering expansion nozzle cooling:

Flow goes directly to regenerative cooling from the pump, so it is the same as the temperature leaving the HPOTP pump section. (Page 9). This is also given as 43.1MPa on page 23 of the project document.

T_1=40.58K

**LH2 Pressure entering expansion nozzle cooling:

Flow goes directly to regenerative cooling from the pump, so it is the same as the pressure leaving the HPOTP pump section. (Page 9).

P_1=43109kPa

**LH2 mass flow rate bypassing thrust chamber and nozzle:

37.3% of mass flow rate bypasses the thrust chamber and nozzle to be pressurized and used in the preburners, as indicated on page 9 of the project document. ($\dot{m}_{expansion} = \dot{m}_{pump}$*(.373))

$\dot{m}_{expansion}$=25.028kg/s

**GH2 Combined total mass flow rate going to preburners:

Noted on page 9, 20.3% of the mass that was initially output from the HPFTP pump is rerouted back to drive the turbine section. The two flows that rejoin to go to the preburners are the 42.4% that previously went to expansion nozzle cooling, and the 37.3% that bypassed the cooling stage. All flow that rejoins to go to the preburners has been converted to GH2. This means that the mass flow rate of the GH2 going to the preburners is simply 42.4% + 37.3% of the original mass flow rate going through the pump.

$\dot{m}$=53.479kg/s

**<u>GH2 Temperature going to preburners:</u>

42.4% of the original mass flow leaving the pump is at 265K after cooling, and the 37.3% of the original mass flow is at 28K after passing through the valve. To find the equivalent temperature of the flow when these two flows join together, a simple proportion must be found to find the average temperature of the rejoined flow. The flow amounts to 79.7% of the original flow, so the proportion is as follows:

$$T = \left(\frac{42.4}{79.7}\right)(256K) + \left(\frac{37.3}{79.7}\right)(28K) \tag{q}$$

The proportion does not involve C_p values because both gases are H2, which have the same C_p.

T=154.083K

**<u>GH2 Pressure going to preburners:</u>

Both flows that rejoin to go to the preburners are at 35.2MPa, as noted on Page 9.

P=35.2MPa

PREBURNER

**<u>GH2 Mass flow rate entering preburner:</u>

Indicated on page 9, 68% of the GH2 that joined together from thrust chamber cooling and the valve goes to HPFTP. Simply take 68% of the rejoined mass flow rate from the previous part.

$\dot{m}$=36.366kg/s

**<u>GH2 Temperature entering preburner:</u>

Same as the temperature of the rejoined GH2 flow.

T_1=154.083K

**<u>GH2 Pressure entering preburner:</u>

Same as the pressure of the rejoined GH2 flow.

P_1=35200kPa

**<u>LO2 Mass flow rate entering preburner from LO2 boost pump:</u>

All LO2 that leaves the boost pump either goes to the HPOTP or here, the HPFTP. The project document states on page 25 that 11.3kg/s goes to the HPOTP. The rest goes to this preburner. Simply subtract 11.3 from the total mass flow rate through the HPOTP boost pump.

$\dot{m}$=37kg/s

**<u>LO2 Temperature entering preburner from LO2 boost pump:</u>

Same as the temperature of the LO2 leaving the boost pump.

T_1=99.891K

**LO2 Pressure entering preburner from LO2 boost pump:

Same as the pressure of the LO2 leaving the boost pump.

P_1=50.2kPa

**Preburner product gas mass flux:

This is the sum of the two mass flow rates that enter the preburner. Mass flux in equals mass flux out, so simply add the mass flow rates of GH2 entering and LO2 entering.

$\dot{m}$=73.366kg/s

TURBINE SECTION

**Turbine inlet gas temperature:

Flow in the turbine comes directly from the preburner, so the temperature at the inlet of the turbine is the same as the temperature of the outlet of the preburner.

T_1=1117K

**Turbine inlet gas pressure:

Pressure at the inlet of the turbine is equal to the pressure at the outlet of the preburner.

P_1=35500kPa

**Turbine outlet gas pressure:

Page 12 states that the pressure drops by a factor of 1.52 across the turbine. To find pressure at the outlet, simply divide inlet pressure by 1.52.

P_2=2355.263kPa

**Turbine outlet mass flux:

Same as the inlet mass flux. Add the mass flow rates of the two gases entering, namely LO2 and GH2.

$\dot{m}$=73.366kg/s

**Turbine outlet gas temperature:

*Refer to the section for the calculation of the outlet temperature in the turbine of the LPFTP to see how to arrive at these equations.

$$\frac{T_2}{T_1} = 1 - \eta_T\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{r}$$

Solving for T_2 then gives the final equation as:

*Referenced from Equation 22
T_1=1117K,η_T=0.77,
P_2=23355.263kPa,
P_1=35500kPa,γ=1.35

$$T_2 = T_1\left\{1 - \eta_T\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\} \tag{s}$$

T_1=1028.523K

High Pressure Oxidizer Turbopump

Flow into the HPOTP comes directly from the output of the LPOTP, thus flow conditions at the inlet are the same as flow conditions at the outlet of the LPOTP. The HPOTP pump outputs to the main combustion chamber, the LPOTP turbine, and the boost pump at the bottom of the turbopump. The boost pump is used to supply high pressure LO2 to the high pressure turbopump preburners. GH2 from cooling the expansion nozzle and GH2 created via an expansion valve rejoin to supply the HPFTP and HPOTP, so the inlet conditions of the HPOTP preburner are derived from those conditions. The hot gas created from the LO2 and GH2 entering the preburner is used to drive the HPOTP turbine section, which then goes to the main combustion chamber to be used as fuel. Since no pressure or temperature changes are indicated between systems, all inlet flow conditions can be found from previous outlet flow locations.

PUMP SECTION

**LO2 Pressure at Pump Inlet:

Same as the output from LPOTP. Also given as 2.8MPa on Page 13.

P_1=2.789MPa

**Mass Flow Rate Through the Pump:

The mass flow rate into the high pressure oxygen turbopump is the combination of the flow from the LPOTP pump and turbine. Add the two mass flow rates to determine the mass flow rate into the HPOTP.

$\dot{m}$=478.17kg/s

**LO2 Temperature Entering Pump:

Same as temperature from LPOTP outlet. The temperature increase due to the LPOTP turbine flow rejoining the pump flow is negligible.

T_1=90.642K

**LO2 Temperature Exiting Pump:

*Referenced from Eq. 11
T_1=90.642K, η_p=0.681,
P_2=29.6MPa,P_1=2.789MPa
ρ=1141J/kgK,
C_v = 1669kg/m^3

$$T_2 = T_1 + \frac{1-\eta_p}{\eta_p}\left(\frac{P_2-P_1}{\rho C_v}\right) \quad \text{(t)}$$

T_2=97.237K

**HPOTP Pump Power:

*Referenced from Eq. 10
$\dot{m}$=478.17kg/s, η_p=0.681,
P_2=29.6MPa,P_1=2.789MPa,
ρ=1141J/kgK

$$\dot{W}_P = \dot{m}\frac{1}{\eta_p}\left(\frac{P_2-P_1}{\rho}\right) \quad \text{(u)}$$

$\dot{W}_P$=16.5MPa

BOOST PUMP

The flow into the boost pump is tapped off directly from the outlet of the HPOTP. Thus, inlet conditions of Boost Pump are equal to outlet conditions of HPOTP.

**LO2 Pressure Entering Boost Pump:

Same as the output pressure of HPOTP

P_1=29.6MPa

**LO2 Temperature Entering Boost Pump:

Same as the output temperature of HPOTP

T_1=97.237K

**LO2 Temperature Exiting Boost Pump:

*Referenced from Eq. 11
T_1=97.237K, η_p=0.803,
P_2=50.2MPa,P_1=29.6kPa
ρ=1141J/kgK,
C_v = 1669kg/m^3

$$T_2 = T_1 + \frac{1-\eta_p}{\eta_p}\left(\frac{P_2-P_1}{\rho C_v}\right) \quad \text{(v)}$$

T_2=99.891K

**Boost Pump Power:

*Referenced from Eq. 10
$\dot{m}$=48.3kg/s, η_p=0.803,
P_2=50.2MPa,P_1=29.6kPa
ρ=1141J/kgK

$$\dot{W}_P = \dot{m}\frac{1}{\eta_p}\left(\frac{P_2-P_1}{\rho}\right) \quad \text{(w)}$$

$\dot{W}_P$=1.086MW

PREBURNER

**GH2 Mass Flow Rate Entering Preburner:

The GH2 not used for cooling rejoins with the GH2 that was used to cool the expansion nozzle. As stated on Page 15 of the project document, 32% of the rejoined GH2 flow goes to the HPOTP preburner, where the other 68% went to the HPFTP preburner. Simply multiply the total mass flow rate of the rejoined flow by 0.32.

$\dot{m}$=17.113kg/s

**GH2 Temperature Entering Preburner:

This is the rejoined GH2 flow, so the temperature at the inlet is the same as the temperature of the rejoined flow.

T_1=154.083K

**GH2 Pressure Entering Preburner:

The pressure is the same as the rejoined flow.

P_1=35200kPa

**LO2 Temperature Entering Preburner from LO2 Boost Pump:

The LO2 entering the preburner is the same flow leaving the Boost Pump. There are no losses so the conditions of the flow are the same, and therefore temperature is the same.

$$T_1=99.891K$$

**<u>LO2 Pressure Entering Preburner From LO2 Boost Pump:</u>

The pressure is the same as the LO2 coming from the Boost Pump.

$$P_1=50.2MPa$$

**<u>Preburner Product Gas Mass Flux:</u>

The mass flux going through the HPOTP preburner is the sum of the mass flow rates of the gasses entering it. Simply sum the mass flow rates of the LO2 and GH2.

$$\dot{m}=28.413kg/s$$

TURBINE SECTION

**<u>HPOTP Turbine Power:</u>

Same as the power from the pump section. The project document says on page 25, “The turbine that drives the pump must provide this actual pump power”. However, this turbine drives both the primary pump *and* the boost pump, so the turbine power is the sum of the two. This is stated on Page 18 of the project document.

$$\dot{W}_T=17.585MW$$

**<u>Turbine Inlet Gas Temperature:</u>

Flow comes straight from the preburner, so temperature is the same as the preburner outlet temperature (ie the product gas temperature).

$$T_1=836K$$

**<u>Turbine Inlet Gas Pressure:</u>

Flow comes straight from the preburner, so pressure is the same as the preburner outlet pressure (ie the product gas pressure).

$$P_1=34400kPa$$

**<u>Turbine Outlet Gas Pressure:</u>

On Page 18 of the project document, it says that the output of the HPOTP turbine must be equal to the pressure of the fuel-rich mixture coming from the HPFTP. This is not the same as HPFTP flow at the outlet of the HPFTP turbine, since GH2 from the LPFTP joins the flow. Thus, the pressure out of the HPOTP must be equal to the pressure of the combined HPFTP turbine outlet flow and GH2 from the LPFTP turbine. This is calculated in the “Main Injectors” portion of the report.

$$P_2=23355.263$$

**<u>Turbine Pressure Ratio Pin/Pout:</u>

Inlet and outlet pressure known. Simply divide P_1 by P_2.

$$P_1/P_2=1.473$$

**<u>Turbine Outlet Mass Flux:</u>

Flow goes straight from the HPOTP preburner to the turbine, so the mass flux must also be the same.

$$\dot{m}=28.413\text{kg/s}$$

**<u>Turbine Outlet Gas Temperature:</u>

Since the turbine work is already known, Equation 24 in the project document can be used to solve for temperature at the outlet. For the reasons already stated in the other gas turbine sections, static temperature is used instead of total temperature. Equation 24 is shown below:

$$\dot{W}_T = \dot{m}c_p(T_1 - T_2)$$

This equation can then be rearranged to solve for T_2,which gives:

T_1=836K,$\dot{W}_T$=14.922MW,
$\dot{m}$=28.413kg/s,c_p=9073J/kgK

$$T_2 = T_1 - \frac{\dot{W}_T}{\dot{m}c_p} \qquad (x)$$

$$T_2=767.786\text{K}$$

**<u>Turbine Isentropic Efficiency:</u>

Since the inlet and outlet temperatures are known, the total to static relation given in equation 22 can be used. As mentioned previously, static conditions are used instead of total conditions (See LPFTP turbine section for explanation).

*Referenced Equation 22

$$\frac{T_2}{T_1} = 1 - \eta_T\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Since isentropic efficiency is needed, this equation can be rewritten as:

T_2=767.786K,T_1=836K,
P_2=23355.263kPa,
P_1=344000kPa
γ=1.37

$$\eta_T = -\left\{\left[\frac{T_2}{T_1} - 1\right] * \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]^{-1}\right\} \qquad (y)$$

$$\eta_T=0.822$$

MAIN INJECTORS

Fuel in the main injectors comes directly from the HPOTP and HPFTP turbines at equal pressure, but varying temperature and chemical composition. GH2 is also supplied from the turbine of the LPFTP. Oxidizer comes directly from the outlet of the HPOTP pump.

GH2 INJECTORS

**GH2-Rich Product Mass Flow Rate from HPFTP Supplied to Injectors:

This is the flow coming directly from the HPFTP turbine, so the mass flow rate is the same as the outlet of the HPFTP turbine.

$$\dot{m}=73.366 kg/s$$

**GH2-Rich Product Temperature from HPFTP Supplied to Injectors:

This is the flow coming directly from the HPFTP turbine, so the temperature is the same as the outlet of the HPFTP turbine.

$$T=1028.523 K$$

**GH2-Rich Product Cp From HPFTP Supplied to Injectors:

This is the flow coming directly from the HPFTP turbine, so the Cp is the same as the outlet of the HPFTP turbine. This is the same as the hot product gas from the HPFTP preburner.

$$C_p=8088 J/kgK$$

**GH2-Rich Product MW From HPFTP Supplied to Injectors:

This is the flow coming directly from the HPFTP turbine, so the MW is the same as the outlet of the HPFTP turbine. This is the same as the hot product gas from the HPFTP preburner.

$$MW=3.97 g/mol$$

**GH2-Rich Product Pressure From HPFTP Supplied to Injectors:

This is the flow coming directly from the HPFTP turbine, so the pressure is the same as the outlet of the HPFTP turbine.

$$P=23355.263 kPa$$

**GH2-Rich Product Y_H2 From HPFTP Supplied to Injectors:

This is the flow coming directly from the HPFTP turbine, so the Y_H2 is the same as the outlet of the HPFTP turbine. This is the same as the hot product gas from the HPFTP preburner.

$$Y_{H2}=0.446$$

**GH2-Rich Product Y_H2O From HPFTP Supplied to Injectors:

This is the flow coming directly from the HPFTP turbine, so the Y_H2O is the same as the outlet of the HPFTP turbine. This is the same as the hot product gas from the HPFTP preburner.

$$Y_{H2O}=0.554$$

**GH2 Mass Flow Rate From LPFTP Turbine Supplied To Injectors:

This flow comes directly from the LPFTP turbine, so the mass flow rate is the same.

$$\dot{m}=13.621 kg/s$$

**GH2 Temperature From LPFTP Turbine Supplied To Injectors:

This flow comes directly from the LPFTP turbine, so the temperature is the same.

T=257.187K

**GH2 Pressure From LPFTP Turbine Supplied To Injectors:

This flow comes directly from the LPFTP turbine, so the pressure is the same.

P=25000kPa

**GH2 C_p From LPFTP Turbine Supplied To Injectors:

This is just the Cp of GH2 (given).

C_P=14340J/kgK

**GH2 Mw From LPFTP Turbine Supplied To Injectors:

This is just the MW of H2.

MW=2.016g/mol

The gases mixing in this section are GH2 from the LPFTP turbine and the gas product from the HPFTP turbine.

**Combined GH2-Rich Mass Flow Rate From Fuel Side Going to Injectors:

Simply sum the mass flow rate of the fluids going into the injectors.

$\dot{m}$=86.987kg/s

**Combined GH2-rich temperature from fuel side going to injectors:

The temperature can be found by using the equation at the bottom of page 29 in the project document, which is based on enthalpy conservation:

$$T = Y_1\left(\frac{c_{p1}}{c_p}\right)T_1 + Y_2\left(\frac{c_{p2}}{c_p}\right)T_2 \quad \text{(z)}$$

Y_1 and Y_2 are the ratios of each gas' mass flow rate divided by the total mass flow rate.

T=837.496K

**Combined GH2-rich pressure from fuel side going to injectors:

Use the following equation since pressure is a molar property:

$$p = X_1 p_1 + X_2 p_2 \quad \text{(aa)}$$

To use the above equation, the mole fraction must be solved for. This can be done with the following equation:

$$X_1 = \frac{\frac{\dot{m}_1}{MW_1}}{\frac{\dot{m}_1}{MW_1}+\frac{\dot{m}_2}{MW_2}} \quad \text{(ab)}$$

So, in terms of mass flow rates and molecular weight, the equation is:

$$P = \left(\frac{\frac{\dot{m}_1}{MW_1}}{\frac{\dot{m}_1}{MW_1}+\frac{\dot{m}_2}{MW_2}}\right)P_1 + \left(\frac{\frac{\dot{m}_2}{MW_2}}{\frac{\dot{m}_1}{MW_1}+\frac{\dot{m}_2}{MW_2}}\right)P_2 \quad \text{(ac)}$$

P=23795.66kPa

**Combined Y_H2 in GH2-rich flow from fuel side going to injectors:

H2 mass flow rate in the GH2-rich product from the HPFTP can easily be found by multiplying Y_H2 with the GH2-rich product mass flow rate. Then, add the GH2 mass flow rate from the LPFTP to get the total H2 mass flow rate. Then, simply divide the total H2 mass flow rate by the total mixture gas flow rate.

$$Y_{H2}=05.33$$

**Combined Y_H2O in GH2-rich flow from fuel side going to injectors:

Simply divide the H2O mass flow rate by the total mass flow rate. Find the H20 mass flow rate in the GH2-rich product from the HPFTP by multiplying Y_H2O with the GH2-rich product mass flow rate.

$$Y_{H2O}=0.467$$

**Combined C_p in GH2-rich flow from fuel side going to injectors:

Use the following equation (given on Page 29) and the gas properties of the gases that will be mixed:

$$c_p = Y_1\, c_{p1} + Y_2\, c_{p2} \tag{ad}$$

$$C_P=9067.003 J/kgK$$

**Combined MW of GH2-rich flow from fuel side going to injectors:

Used the equation for mixing fluids to find their resultant molecular weight. This equation is shown below:

$$MW = X_1\, MW_1 + X_2\, MW_2$$

In terms of mass flow rate and molecular weight, this equation is then:

$$\mathrm{MW} = \left(\frac{\frac{\dot{m}_1}{MW_1}}{\frac{\dot{m}_1}{MW_1}+\frac{\dot{m}_2}{MW_2}}\right)MW_1 + \left(\frac{\frac{\dot{m}_2}{MW_2}}{\frac{\dot{m}_1}{MW_1}+\frac{\dot{m}_2}{MW_2}}\right)MW_2 \tag{ae}$$

$$MW=3.447 g/mol$$

**GH2-rich product mass flow rate from HPOTP supplied to injectors:

This flow is straight from the HPOTP turbine, so the mass flow rate is the same as the HPOTP turbine outlet.

$$\dot{m}=28.413 kg/s$$

**GH2-rich product temperature from HPOTP supplied to injectors:

This flow is straight from the HPOTP turbine, so the temperature is the same as the HPOTP turbine outlet.

$$T=767.786K$$

**GH2-rich product pressure from HPOTP supplied to injectors:

This flow is straight from the HPOTP turbine, so the pressure is the same as the HPOTP turbine outlet.

$$P=23355.263$$

**GH2-rich product Y_H2 from HPOTP supplied to injectors:

This flow is straight from the HPOTP turbine, so the Y_H2 is the same as the HPOTP turbine outlet. The gas composition is the same as the outlet of the HPOTP preburner.

$$Y_{H2}=0.549$$

**GH2-rich product Y_H2O from HPOTP supplied to injectors:

This flow is straight from the HPOTP turbine, so the Y_H2O is the same as the HPOTP turbine outlet. The gas composition is the same as the outlet of the HPOTP preburner.

$$Y_{H2O}=0.451$$

**GH2-rich product C_p from HPOTP supplied to injectors:

This flow is straight from the HPOTP turbine, so the Cp is the same as the HPOTP turbine outlet.The gas composition is the same as the outlet of the HPOTP preburner.

$$C_P=9073 J/kgK$$

**GH2-rich product MW from HPOTP supplied to injectors:

This flow is straight from the HPOTP turbine, so the MW is the same as the HPOTP turbine outlet. The gas composition is the same as the outlet of the HPOTP preburner.

$$MW=3.36 g/mol$$

The gases mixing in this section are the gas from the turbine of the HPOTP and the mixture of gases from the HPFTP turbine and LPFTP turbine.

**Total GH2-rich preburner product mass flow supplied to injectors:

This is the sum of the mass flow rates coming into the injectors. Simply add all mass flow rates that are going into the injectors.

$$\dot{m}=115.4 kg/s$$

**Y_H2 in combined GH2-rich preburner flows supplied to injectors:

Since the Y_H2 of both gasses being mixed are already known, multiply Y_H2 of the gas coming from the HPOTP with the total mass flow rate coming from the HPOTP. Then, multiply Y_H2 of the LPOTP/HPOTP mixture with the total mass flow rate of the mixture. Add these numbers. The equation should look like this:

$$Y\_H2 = Y\_H2_1\dot{m}_1+Y\_H2_2\dot{m}_2 \tag{af}$$

$$Y_{H2}=0.537$$

**Y_H2O in combined GH2-rich preburner flows supplied to injectors:

Since the Y_H2O of both gasses being mixed are already known, multiply Y_H2O of the gas coming from the HPOTP with the total mass flow rate coming from the HPOTP. Then, multiply Y_H2 of the LPOTP/HPOTP mixture with the total mass flow rate of the mixture. Add these numbers. The equation should look like this:

$$Y\_H2 = Y\_H2O_1\dot{m}_1+Y\_H2O_2\dot{m}_2 \tag{ag}$$

$$Y_{H2O}=0.463$$

**C_p of combined GH2-rich preburner flows to injectors:

Use the following equation (given on Page 29) and the gas properties of the gases that will be mixed:

$$c_p = Y_1\, c_{p1} + Y_2\, c_{p2} \tag{ah}$$

Y_1 and Y_2 are the mass flow rates of the fluids that are mixing divided by the sum of their mass flow rates.

C_p=9068.48J/kgK

Temperature of combined GH2-rich preburner flows to injectors:

The temperature can be found by using the equation at the bottom of page 29 in the project document, which is based on enthalpy conservation:

$$T = Y_1\left(\frac{c_{p1}}{c_p}\right)T_1 + Y_2\left(\frac{c_{p2}}{c_p}\right)T_2 \tag{ai}$$

Y_1 and Y_2 are the ratios of each gas' mass flow rate divided by the total mass flow rate.

T=820.324K

MW of combined GH2-rich preburner flows to injectors:

Used the equation for mixing fluids to find their resultant molecular weight. This equation is shown below:

$$MW = X_1\, MW_1 + X_2\, MW_2$$

To use the above equation, the mole fraction must be solved for. This can be done with the following equation:

$$X_1 = \frac{\frac{\dot{m}_1}{MW_1}}{\frac{\dot{m}_1}{MW_1} + \frac{\dot{m}_2}{MW_2}} \tag{aj}$$

In terms of mass flow rate and molecular weight this equation is then:

$$\mathrm{MW} = \left(\frac{\frac{\dot{m}_1}{MW_1}}{\frac{\dot{m}_1}{MW_1} + \frac{\dot{m}_2}{MW_2}}\right)MW_1 + \left(\frac{\frac{\dot{m}_2}{MW_2}}{\frac{\dot{m}_1}{MW_1} + \frac{\dot{m}_2}{MW_2}}\right)MW_2 \tag{ak}$$

MW=3.425g/mol

Pressure of combined GH2-rich preburner flows to injectors:

Use the following equation since pressure is a molar property:

$$p = X_1\, p_1 + X_2\, p_2 \tag{al}$$

To use the above equation, the mole fraction must be solved for. This can be done with the following equation:

$$X_1 = \frac{\frac{\dot{m}_1}{MW_1}}{\frac{\dot{m}_1}{MW_1} + \frac{\dot{m}_2}{MW_2}} \quad \text{(am)}$$

So, in terms of mass flow rates and molecular weight, the equation is:

$$P = \left(\frac{\frac{\dot{m}_1}{MW_1}}{\frac{\dot{m}_1}{MW_1} + \frac{\dot{m}_2}{MW_2}} \right) P_1 + \left(\frac{\frac{\dot{m}_2}{MW_2}}{\frac{\dot{m}_1}{MW_1} + \frac{\dot{m}_2}{MW_2}} \right) P_2 \quad \text{(an)}$$

P=23685.132kPa

**GH2 Pressure at exit of injectors:

This is technically the pressure of the GH2-rich fuel mixture generated by mixing all the flow from the HPOTP turbine, HPFTP turbine, and LPFTP turbine. The project document states that there is a pressure drop of 16.2MPa across the injectors, so the exit pressure of the GH2-rich fuel mixture is just 16.2MPa lower than the inlet pressure. Subtract inlet pressure by 16.2MPa.

LO2 INJECTORS

**LO2 mass flow rate from HPOTP supplied to injectors:

The LO2 flow that comes from the HPOTP is the remainder of the output after the flow has been tapped off to supply the boost pump and LPOTP turbine. So, to determine the mass flow rate going to the injectors, take the total mass flow rate from the HPOTP and subtract the LO2 mass flow rates through the boost pump and LPOTP turbine. The GO2 that pressurizes the LO2 tank is also tapped from the HPOTP, but the mass flow rate it uses is assumed to be negligible.

$\dot{m}$=352.7kg/s

**LO2 Temperature from HPOTP supplied to injectors:

Flow comes directly from the HPOTP, so the temperature of the LO2 supplied to the injectors is the same as the HPOTP outlet.

T=97.237K

**LO2 Pressure from HPOTP supplied to injectors:

Flow comes directly from the HPOTP, so the pressure of the LO2 supplied to the injectors is the same as the HPOTP outlet.

P=29.6MPa

**LO2 Pressure at exit of injectors:

Pressure of LO2 is stated to drop 9MPa across the injectors on page 19. Subtract the inlet pressure by the pressure drop to get the exit pressure.

P=20600kPa

THRUST CHAMBER

Flow in the thrust chamber comes directly from the main injectors. Thus, the input conditions of the thrust chamber are exactly the same as the output conditions of the main injectors. For this section, the combustion product gas characteristics (gamma, MW, Cp, temperature) DO NOT match those given in the project document. The project document states that the combustion product conditions are those EXITING the combustion chamber, where the spreadsheet states the conditions INSIDE the combustion chamber. For the next section, calculating flow in the expansion nozzle, the conditions exiting the combustion chamber are more applicable. **Thus, all combustion product values (yellow cells) were changed in the spreadsheet to match the given values in the project document.**

**Main combustion chamber (MCC) overall O/F mass ratio:

The overall O/F mass ratio is equal to the mass ratio that leaves the LO2 and LH2 pressurized tanks since the propellant feed system is "closed". This means that all oxygen and hydrogen that enters the system must leave through the thrust chamber, so the O/F ratio in the thrust chamber should be identical to the O/F ratio entering the system. To find the overall O/F mass ratio, divide mass flow through the LPOTP by the mass flow through the LPFTP. As given in the document on page 19, this should be about 6.0. Calculating for this value gives 5.976, which is well within 1% error of the given value, and can be explained by the several compounding approximations that were made to reach this point.

$$O/F=5.976$$

**Mass flow rate of combined GH2-rich preburner flows entering MCC:

The mass flow rate is the mass flow rate of GH2-rich flow through the injectors.

$$\dot{m}=115.4\text{kg/s}$$

**Y_H2 in combined GH2-rich preburner flows entering MCC:

Y_H2 is the same as the GH2-rich flow through the injectors.

$$Y_{H2}=0.537$$

**Y_H2O in combined GH2-rich preburner flows entering MCC:

Y_H2O is the same as the GH2-rich flow through the injectors.

$$Y_{H2O}=0.463$$

**Temperature of combined GH2-rich preburner flows entering MCC:

The temperature is the same as the GH2-rich flow through the injectors.

$$T=820.324\text{K}$$

**Mass flow rate of LO2 entering MCC:

The mass flow rate is the mass flow rate of LO2 through the injectors.

$$\dot{m}=352.7\text{kg/s}$$

**Resulting O/F mixture fraction (mass ratio) entering MCC:

To find the O/F ratio, the mass of all the oxidizer (oxygen) must be divided by the mass of all the fuel (hydrogen). This is not the same as simply dividing the mass of the LO2 in the injectors by the mass of the fuel rich mixture in the injectors, because the fuel rich mixture contains a significant amount of H2O, which will act as a partial oxidizer in the combustion chamber.

To find the mass of all the oxygen in the injectors, the mass of all the oxygen in the H2O of the fuel-rich mixture must be added to the mass of the LO2 in the injectors. This can then be divided by the remaining mass of the fuel-rich mixture, which will only be the mass of hydrogen.

On page 19 of the project document this mass ratio is given as 6.0, so to check that this result is correct, compare to the given value.

Calculating the mass of all the oxygen divided by the mass of all the hydrogen gives a O/F mass ratio of 5.891, which is exceptionally close to the given value of 6.0, so the calculation passes the logic test. The difference between this value and the overall O/F mass ratio comes from many small, but compounding, errors caused by rounding and simplification. For example, most calculations were done with static conditions, when the actual values must be determined using total conditions. This value is still within 2% error of the given value, so it is more than acceptable.

O/F=5.892

O2/H2 mass flux ratio entering MCC:

This is simply the ratio of O2 to H2. Find the mass flow rate of H2 by multiplying the mass fraction of H2 by the total mass flow rate of GH2-rich gas. Then divide LO2 mass flow rate by H2 mass flow rate.

O2/H2=5.694

Combustion product gas pressure in combustion chamber:

This value is given on page 21 of the project document as 20.6MPa.

P=20.6MPa

Combustion product gas mass flux exiting combustion chamber:

The mass flow rate out of the combustion chamber is the same as the mass flow rate into the combustion chamber. This is before the flow is choked.

$\dot{m}$=468.1kg/s

Combustion chamber cross-sectional area A_C:

The diameter of the combustion chamber is given as 45.1cm, or .451m, on page 21 of the project document. Using the area equation, A_C can easily be solved for:

$$A_C = \pi\left(\frac{d_c}{2}\right)^2 \quad \text{(ao)}$$

A_C=0.16m^2

Throat area A*:

The diameter of the throat is given on page 21 of the project document as 26.6cm, or .262m. Use the area equation to determine the throat area.

$$A^* = \pi\left(\frac{d^*}{2}\right)^2 \quad \text{(ap)}$$

A^*=0.054m^2

**Ratio of A_C over A*:

Simply divide the calculated A_C over the calculated A*. This is a given value on page 21, and should be equal to 2.96 when calculated. Calculating this value gives 2.96.

$$A_C/A^*=2.963$$

EXPANSION NOZZLE

**Nozzle exit area A_e:

The diameter of the exit is given as 230.4cm on page 22 of the project document. Using the area equation, A_e can easily be solved for:

$$A_e = \pi\left(\frac{d_e}{2}\right)^2 \tag{aq}$$

A_e=4.169m^2

**Nozzle A_e/A*:

Divide the calculated A_e by the calculated A*. This value is also given on page 22 as 77.3. The calculated result is also 77.3m^2.

**Frozen flow Combustion product gas gamma value entering nozzle:

Given on page 21 from CEA as 1.17.

γ=1.13g/mol

**Frozen flow Combustion product gas MW entering nozzle:

Given on page 21 from CEA as 10.13

MW=16.5g/mol

**Frozen flow M_e from non-isentropic nozzle flow w/ gamma entering nozzle:

To find the Mach number at the exit of the nozzle, use the Area Mach Relation for compressible flow as given:

$$\frac{A_e}{A^*} = \frac{1}{M_e}\left\{\left(\frac{2}{\gamma+1}\right)\left[1 + \left(\frac{\gamma-1}{2}\right)M_e^2\right]\right\}^{\frac{\gamma+1}{2(\gamma-1)}} * \left\{1 + \left(1 - \frac{1}{\eta_n}\right)\left(\frac{\gamma-1}{2}\right)M_e^2\right\}^{-\frac{\gamma}{\gamma-1}} \tag{ar}$$

This equation relies on nozzle efficiency, and the nozzle efficiency can be found to be:

$$\eta_n = \frac{1-\left[1+\left(\frac{\gamma-1}{2}\right)M_e^2\right]^{-1}}{1-\left(\frac{P_e}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}}} \tag{as}$$

The nozzle efficiency equation can be substituted into the non-isentropic Area Mach Relation equation to create an equation where the only unknown is M_e:

$$\frac{A_e}{A^*} = \frac{1}{M_e}\left\{\left(\frac{2}{\gamma+1}\right)\left[1 + \left(\frac{\gamma-1}{2}\right)M_e^2\right]\right\}^{\frac{\gamma+1}{2(\gamma-1)}} * \left\{1 + \left(1 - \frac{1-\left(\frac{P_e}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}}}{1-\left[1+\left(\frac{\gamma-1}{2}\right)M_e^2\right]^{-1}}\right)\left(\frac{\gamma-1}{2}\right)M_e^2\right\}^{-\frac{\gamma}{\gamma-1}} \tag{at}$$

M_e can then easily be solved by using a program such as MATLAB to iterate down to a solution using a function such as “fzero” or “fsolve”. All gamma values should be the gamma in the throat because this is solving for parameters under “frozen flow” conditions in the expansion nozzle. “Frozen flow” means that characteristic gas conditions do not change as they pass through the nozzle. Together with the "shifting equilibrium" calculations given in the spreadsheet, the actual non-isentropic flow conditions are bounded.

ALTERNATIVELY, since the design parameters of the nozzle are known, and the gas inlet conditions are known, NASA CEA can be used to calculate this. This option is simpler because this software has equations built into it and leaves much less room for math errors. Additionally, NASA CEA will give pressures and temperatures, which will be needed later. That is why in the spreadsheet the values are calculated using NASA CEA. When using CEA, input all the combustion chamber conditions, and set the frozen flow "nfz" to 2, since the spreadsheet indicates it should solve with gamma entering the nozzle (i.e. at the throat). Before running CEA, the inputs were taken from the gas conditions of the fuel and oxidizer entering the thrust chamber via the injectors. The O/F mass ratio used in the calculation is O2/H2, because the H2O present in the GH2 mixture is already reacted and will not contribute to additional energy generation in the combustion chamber. in CEA was then run with frozen flow conditions. This is the same process as used in Homework assignment 11. The output file of CEA is shown below, and it shows all manual inputs, as well as the calculated outputs:

```
         NASA-GLENN CHEMICAL EQUILIBRIUM PROGRAM CEA2, FEBRUARY 5, 2004
                   BY  BONNIE MCBRIDE AND SANFORD GORDON
      REFS: NASA RP-1311, PART I, 1994 AND NASA RP-1311, PART II, 1996

 *******************************************************************************


### CEA analysis performed on Mon 09-Dec-2024 18:03:09

# Problem Type: "Rocket" (Infinite Area Combustor)

prob case=_______________6637 ro frozen  nfz=2

# Pressure (1 value):
p,bar= 296
# Supersonic Area Ratio (1 value):
supar= 77.3

# Oxidizer/Fuel Wt. ratio (1 value):
o/f= 5.694

# You selected the following fuels and oxidizers:
reac
fuel H2                wt%=100.0000  t,k= 820.324
oxid O2(L)             wt%=100.0000  t,k=  97.237

# You selected these options for output:
# short version of output
output short
# Proportions of any products will be expressed as Mass Fractions.
output massf
# Heat will be expressed as siunits
output siunits
# Trace variable:
output trace=1e-5

# Input prepared by this script:/var/www/sites/cearun.grc.nasa.gov/cgi-bin/CEARU
N/prepareInputFile.cgi

### IMPORTANT:  The following line is the end of your CEA input file!
end
```

```
             THEORETICAL ROCKET PERFORMANCE ASSUMING FROZEN COMPOSITION
                                   AFTER POINT 2

 Pin =  4293.1 PSIA
 CASE = _______________

             REACTANT                    WT FRACTION      ENERGY      TEMP
                                          (SEE NOTE)     KJ/KG-MOL      K
 FUEL        H2                           1.0000000     15304.123    820.324
 OXIDANT     O2(L)                        1.0000000    -12979.000     90.170

 O/F=    5.69400  %FUEL= 14.938751  R,EQ.RATIO= 1.393868  PHI,EQ.RATIO= 1.393868

                 CHAMBER   THROAT     EXIT
 Pinf/P            1.0000   1.7404  1286.99
 P, BAR            296.00   170.07  0.22999
 T, K             3813.98  3586.46  1048.26
 RHO, KG/CU M    1.2021 1 7.4221 0 3.4340-2
 H, KJ/KG          789.10  -527.42 -9503.28
 U, KJ/KG        -1673.19 -2818.87 -10173.0
 G, KJ/KG        -67242.2 -64500.4 -28201.4
 S, KJ/(KG)(K)    17.8373  17.8373  17.8373

 M, (1/n)          12.879   13.013   13.013
 Cp, KJ/(KG)(K)    8.1813   7.6051   2.8908
 GAMMAs            1.1494   1.1491   1.2837
 SON VEL,M/SEC     1682.3   1622.7    927.2
 MACH NUMBER        0.000    1.000    4.893

 PERFORMANCE PARAMETERS

 Ae/At                     1.00000   77.300
 CSTAR, M/SEC               2457.8   2457.8
 CF                         0.6602   1.8460
 Ivac, M/SEC                3034.8   4684.7
 Isp, M/SEC                 1622.7   4537.0

 MASS FRACTIONS

 *H              0.00225   HO2             0.00007   *H2             0.04350
 H2O             0.90202   H2O2            0.00003   *O              0.00239
 *OH             0.04549   *O2             0.00424

  * THERMODYNAMIC PROPERTIES FITTED TO 20000.K

 NOTE. WEIGHT FRACTION OF FUEL IN TOTAL FUELS AND OF OXIDANT IN TOTAL OXIDANTS
```

Note that CEA does not use the LO2 temperature since LO2 has only been defined up to 90.17K. Since this is within 10K, the difference is negligible and CEA still runs as intended.

M_e=4.893

**<u>Frozen flow p_e/p_t2 from non-isentropic nozzle flow w/ gamma entering nozzle:</u>

Divide the exit pressure given by CEA by the pressure in the combustion chamber.

P_e/P_{t2}=.0011165

**Frozen flow p_e from non-isentropic nozzle flow w/ gamma entering nozzle:

See CEA results.

P_e=22.999kPa

**Frozen flow T_e from non-isentropic nozzle flow w/ gamma entering nozzle:

See CEA results.

T_e=1048.26K

**Frozen flow V_e from non-isentropic nozzle flow w/ gamma entering nozzle:

The definition of mach number is the speed of the flow divided by the local speed of sound. This defines that velocity is equal to the mach number times the local speed of sound. This equation is then used to find the exit velocity:

$$V_e = M_e * a_e \tag{au}$$

Where a is defined as:

$$a = \sqrt{\gamma RT}\ ,\ \mathrm{R} = \frac{R_U}{MW} \tag{av}$$

V_e=3780.176m/s

**V_e from non-isentropic nozzle flow w/ gamma exiting nozzle

Refer to the process above using equation (au) and (av), but with the shifting equilibrium conditions instead of the frozen flow conditions.

V_e=6041.875m/s

THRUST

The section above calculated for exit conditions using frozen flow calculations and shifting equilibrium calculations, these act as the upper and lower bounds for every condition. Since the frozen flow and shifting equilibrium calculations provide bounds of the actual non-isentropic flow conditions, the **average of each calculated value** was taken for a better estimate.

SEA LEVEL

P_∞ at sea level is taken to be 101.325kPa, or 1 atm.

**Resulting jet thrust (SL):

The standard thrust equation is given in the form:

$$T = \dot{m}V_e + (P_e - P_\infty)A_e \tag{aw}$$

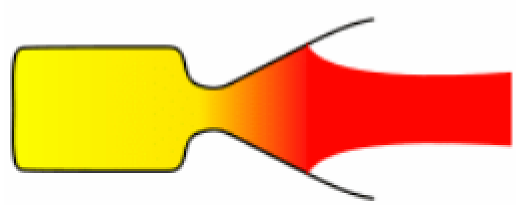
Where the jet thrust is the first part of the equation. This is the thrust as a result of conservation of momentum. Fuel is leaving the nozzle at high speeds with a characteristic mass flow, meaning the rocket must be accelerated in the opposite direction. Thus, the jet thrust is:

$$T_J = \dot{m}V_e \tag{ax}$$

T_J=2298.851kN

**Resulting pressure thrust (SL):

The pressure thrust is the other half of the thrust equation (aw) given above. Since there is a pressure difference between the gas inside the nozzle and the atmosphere at sea level, this pressure difference is applied over the surface area of the exit of the nozzle, resulting in a force. Since the gas leaves the nozzle at immensely high speeds, the pressure is actually lower than atmospheric pressure. This means that the engine is operating over-expanded and is losing thrust due to the external pressure being higher than the internal nozzle pressure. This would result in boundary layer separation, further reducing the thrust generated. This over-expanded flow with boundary layer separation would look something like the following image:



The pressure thrust equation is then:

$$T_P = (P_e - P_\infty)A_e \tag{ay}$$

T_P=-320.094kN

**Resulting nominal thrust (SL):

The resulting nominal thrust is given by the characteristic thrust equation, (aw). This is just the addition of the jet and pressure thrust values.

T=1978.757kN

**Resulting divergence-corrected thrust (SL):

The project document states that the divergence loss is 0.8%. This means that 0.8% of the flow leaving the expansion nozzle is not axially aligned, therefore 99.2% of the total flow is axially aligned. Only the axially aligned component of the flow velocity creates thrust, so to adjust for divergence loss, take 99.2% of the nominal thrust (multiply nominal thrust by 0.992).

T=1962.927kN

**Actual thrust coefficient C_T (SL):

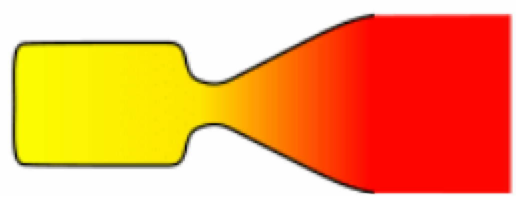
Thrust coefficient is given by the following equation:

$$C_T = \frac{T}{P_{t2}A^*} \tag{az}$$

$C_T = 1.767$

**Ideal thrust coefficient (C_T)_ideal (SL):

Ideal thrust coefficient is the thrust coefficient generated by the nozzle when the flow is isentropic and fully expanded. This means that the external pressure is equal to the nozzle exit pressure, resulting in flow that looks similar to this:



When designing a nozzle, trying to get as close as possible to the ideal coefficient of thrust is normally the goal if efficiency is important. Replicating the ideal coefficient of thrust is impossible, of course, because of non-isentropic effects, changing altitude, and unsteady flight conditions.

The $C_{T,ideal}$ equation is:

$$C_{T,ideal} = \gamma\left\{\left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{P_e}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2} \tag{ba}$$

It is important to note that this $C_{T,ideal}$ is greater than the maximum thrust that can be achieved at sea level. For this engine, flow cannot be fully expanded at sea level because the exit pressure simply is not high enough to equal the external pressure. As a result, the maximum coefficient of thrust that can be achieved at sea level is actually just the isentropic coefficient of thrust given by the equation:

$$C_{T,isentropic} = \gamma\left\{\left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{P_e}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2} + \left(\frac{P_e-P_\infty}{P_{t2}}\right)\frac{A_e}{A^*} \tag{bb}$$

The C_T ideal in the spreadsheet is the one obtained from fully expanded flow because this is more important to know for the characteristics of the nozzle. As the rocket ascends from sea level, there will be a specific altitude where the exit pressure is just about equal to the atmospheric pressure, and this will be as close as the rocket engine ever gets to $C_{T,ideal}$.

$$C_{T,ideal}=1.96$$

**Resulting nozzle C_T efficiency (SL):

$$\eta_{C_T} = \frac{(C_T)_{Actual}}{(C_T)_{Ideal}} \quad \text{(bc)}$$

$$\eta_{C_T} = 0.902$$

**Specific Impulse I_sp (SL):

Specific impulse is a measure of how efficiently the rocket uses the propellant and is given by the equation:

$$I_{sp} = \frac{T}{\dot{m}g_0} \quad \text{(be)}$$

$$I_{sp}=427.897s$$

VACUUM

P_∞ in vacuum is taken to be 0kPa.

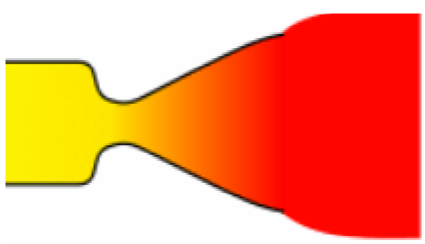
**Resulting jet thrust (vac):

Jet thrust in a vacuum is exactly the same as jet thrust produced at sea level. Review the explanation given above.

$$T_J=2298.851kN$$

**Resulting pressure thrust (vac):

Pressure thrust in a vacuum is positive, as opposed to negative at sea level, because the difference in pressure between the exit pressure of the nozzle and a vacuum results in an applied pressure that is purely outward from the nozzle. This pressure applied over the control surface (the exit area) creates an outward thrust that propels the rocket in the same direction as the jet thrust. Since the pressure is larger in the nozzle than outside, the nozzle operates under-expanded and looks like this:



Since the thrust is not able to fully expand, this flow condition creates low coefficient of thrust efficiency. Pressure thrust can be calculated using:

$$T_P = (P_e - P_\infty)A_e \quad \text{(bf)}$$

$$T_P=102.352kN$$

**Resulting nominal thrust (vac):

Add pressure thrust and jet thrust.

$$T = 2401.203kN$$

**<u>Resulting divergence-corrected thrust (vac):</u>

The project document states that the divergence loss is 0.8%. This means that 0.8% of the flow leaving the expansion nozzle is not axially aligned, therefore 99.2% of the total flow is axially aligned. Only the axially aligned component of the flow velocity creates thrust, so to adjust for divergence loss, take 99.2% of the nominal thrust (multiply nominal thrust by 0.992).

T=2381.994kN

**<u>Actual thrust coefficient C_T (vac):</u>

Thrust coefficient is given by the following equation:

$$C_T = \frac{T}{P_{t2}A^*} \tag{bg}$$

C_T=2.145

**<u>Ideal thrust coefficient (C_T)_ideal (vac):</u>

The ideal thrust coefficient is the thrust when conditions are isentropic and external pressure is equal to zero. The ideal thrust coefficient in a vacuum is the absolute upper band of performance that can be generated by a rocket engine. This can provide a reference for the maximum capabilities of an engine before it is tested.

The $C_{T,ideal,Vacuum}$ equation is:

$$C_{T,ideal,Vacuum} = \gamma\left\{\left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right\}^{1/2} \tag{bh}$$

$C_{T,ideal,vacuum}$=2.737

**<u>Resulting nozzle C_T efficiency (vac):</u>

$$\eta_{C_T} = \frac{(C_T)_{Actual,Vacuum}}{(C_T)_{Ideal,Vacuum}} \tag{bi}$$

$$\eta_{C_T} = 0.783$$

**<u>Specific Impulse I_sp (vac):</u>

Specific impulse is a measure of how efficiently the rocket uses the propellant and is given by the equation:

$$I_{sp} = \frac{T}{\dot{m}g_0} \tag{bj}$$

I_{sp}=519.249s

CONCLUSIONS

Using the cascading calculations from all of the previous parts, the thrust calculations provide insight into the performance of the RS-25 engine. Shown above in the “Thrust” section, the engine makes quite a bit more thrust in a vacuum, because the jet thrust is constant regardless of external conditions. The results are consistent with those reported in Ref [2], displaying that even though nearly every factor was taken into account, the several small approximations that were made compounded into a result that had a reasonable margin of error. Ref [2] reported that the RS-25 created about 2279kN in vacuum and about 1852kN at sea level. This means that the calculated results have about a 4.5% error for vacuum thrust and a 6% error for sea level thrust. All things considered, this is a great result, and gives an appropriate approximation to the actual thrust produced by the engine. This confirms that the work done to arrive at the thrust was accurate, and any inaccuracies along the way can be deemed negligible. It is important to note that all the calculations, gas conditions, and even the data shown in Ref [2] have their own margin of uncertainty, so the only real way to test the engine is to actually use it.

Though there was no online source to confirm the coefficient of thrust efficiency, the calculated results show that the RS-25 is extremely efficient. The thrust produced at sea level is 90% of the fully-expanded isentropic thrust. The engine produces even more thrust in a vacuum, which is expected. The calculated Isp for this engine is also higher than typically seen for other chemical propellant systems.

The RS-25 engine is an impressive feat of engineering that integrates many complex subsystems to produce a wonderfully coherent machine. Analyzing the flow as it travelled through each system provided great insight to how everything in the engine works, and how each piece interacts with the others. This project perfectly encapsulated everything that was taught in this course, embodying compressible flow, turbopumps, turbines, nozzle geometry, and aerothermochemistry. Being able to make sense of the fluid mechanics that take place in this system has been a great experience.

ACKNOWLEDGEMENTS

Great thanks to Professor Dahm for an astounding semester. He has been my favorite professor at Arizona State University, and has made me more excited to learn than anyone else ever has. Before this semester, I was one of those students who viewed much of school as an obstacle rather than a privilege. As the semester marched forward, Professor Dahm’s enthusiasm, abundance of knowledge, and extra resources have made me more curious than ever. I honestly might have more rocket propulsion questions than answers, to the point where I have gotten kind of annoying. I am disappointed that there is not more to learn about rocket propulsion at Arizona State, as I feel like even though Prof. Dahm has put together an impressively concise summary on the matter, it feels like just that. There are so many more layers to the concepts being taught that entire classes could be taught on topics we only got to spend a lecture or two on. For all things rocket propulsion, I would trust no other more than Prof. Dahm.

REFERENCES

[1] Dahm, Werner J. A., "MAE565: Rocket Propulsion Final Project."

[2] *Space launch system RS-25 core stage engine* Available: https://www.nasa.gov/wp-content/uploads/2020/09/sls_rs25_engine_fs_508.pdf.