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EEL 3850 HOMEWORK 1 PART 1

PROBLEM 1

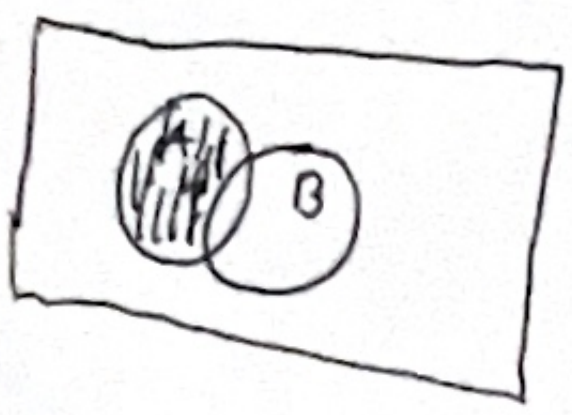
1. $P(A \cup B) = 1 - P(\overline{A \cup B}) = 1 - 0.45 = 0.55$

$\boxed{P(A \cup B) = 0.55}$

2. $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.5 + 0.2 - 0.55 = 0.15$

$\boxed{P(A \cap B) = 0.15}$

3. $P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.5 - 0.15 = 0.35$



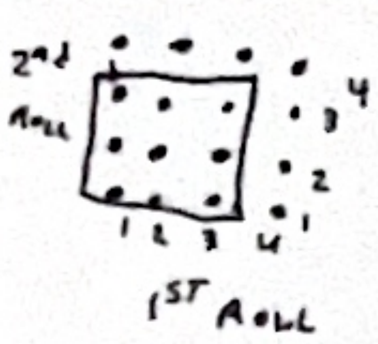
$\boxed{P(A \cap \bar{B}) = 0.35}$

4. $P(A \cap B) = 0.15 \neq 0 \therefore$ $\boxed{\text{A AND B ARE NOT MUTUALLY EXCLUSIVE}}$

PROBLEM 2

1. $A_i = \{$ ROLL A 4 ON THE i^{th} ROLL $\}$ $\bar{A_i} = \{$ NOT ROLLING A 4 ON THE i^{th} ROLL $\}$

$P(\bar{A}_1 \cap \bar{A}_2) = \frac{9}{16}$



$P(\bar{A}_1) = 1 - P(A_1) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(\bar{A}_2) = 1 - P(A_2) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(\bar{A}_1)P(\bar{A}_2) = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$

$P(\bar{A}_1 \cap \bar{A}_2) = P(\bar{A}_1)P(\bar{A}_2) = \frac{9}{16}$ $\therefore$ NOT ROLLING A 4 ON THE i^{th} ROLL IS AN INDEPENDENT EVENT

$P(A_1 \cup A_2 \cup A_3 \cup A_4) = 1 - P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \bar{A}_4) = 1 - P(\bar{A}_1)P(\bar{A}_2)P(\bar{A}_3)P(\bar{A}_4) = 1 - \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = 1 - (\frac{3}{4})^4 = \frac{175}{256} = 0.684$

$\boxed{P(A_1 \cup A_2 \cup A_3 \cup A_4) \cong 0.684}$

2. $P(A_1 \cup A_2 \cdots \cup A_{20}) = 1 - P(\bar{A}_1 \cap \bar{A}_2 \cdots \cap \bar{A}_{20}) = 1 - P(\bar{A}_1)P(\bar{A}_2) \cdots P(\bar{A}_{20}) = 1 - (\frac{3}{4})^{20} = 0.997$

$\boxed{P(A_1 \cup A_2 \cdots \cup A_{20}) = 0.997}$

3. $1 - (\frac{3}{4})^R = 0.9$

$(\frac{3}{4})^R = 1 - 0.9$

$(\frac{3}{4})^R = 0.1$

$\ln(\frac{3}{4})^R = \ln(0.1)$

$R \ln(3/4) = \ln(0.1)$

$R = \frac{\ln(0.1)}{\ln(3/4)} = 8.004 \Rightarrow$ ROUND ~~FOR~~ UP TO NEXT INTEGER $R = 9$

$\boxed{\text{9 DICE ROLLS GIVES A PROBABILITY OF AT LEAST 90\%}}$

PROBLEM 3

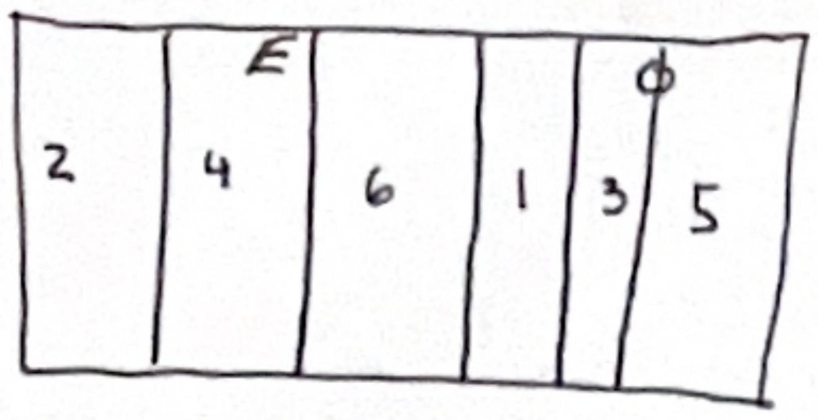
$E =$ EVEN FACE $\quad O =$ ODD FACE

$P(\Omega) = P(E) + P(O) = 1$

$P(E) = 2P(O)$

$P(E) + \frac{P(E)}{2} = 1 \Rightarrow \frac{3}{2}P(E) = 1 \Rightarrow P(E) = \frac{2}{3}$

$P(O) = 1 - P(E) = 1 - 2/3 = 1/3$



$P(6) = P(4) = P(2) = \frac{P(E)}{3} = \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$

$P(1) = P(3) = P(5) = \frac{P(O)}{3} = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

$P(1 \cup 2 \cup 3) = P(1) + P(2) + P(3) = \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}$

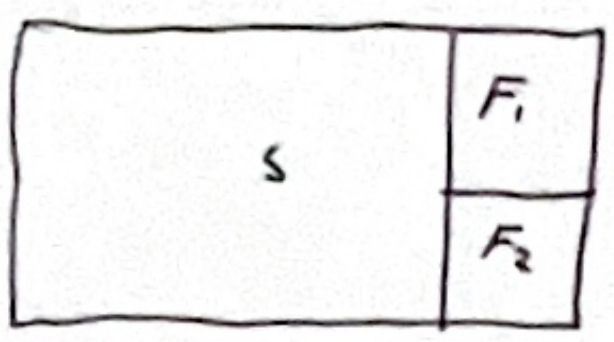
$\boxed{P(1 \cup 2 \cup 3) = \frac{4}{9}}$

PROBLEM 4

$F_1 =$ ALL PLANTS FAIL

$F_2 =$ ALL PLANTS EXCEPT 1 FAIL

$S =$ AT LEAST TWO PLANTS STAND



$P(S) = 1 - P(F_2) - P(F_1)$

$P(F_1) = P_1 P_2 \cdots P_n$

$P(F_2) = (1-P_1)P_2 \cdots P_n + P_1(1-P_2) \cdots P_n + P_1 P_2 (1-P_3) \cdots P_n + P_1 P_2 \cdots P_{n-1}(1-P_n)$

$\boxed{P(S) = 1 - P_1 P_2 \cdots P_n - [(1-P_1)P_2 \cdots P_n + P_1(1-P_2) \cdots P_n + P_1 P_2 \cdots P_{n-1}(1-P_n)]}$

PROBLEM 5

(a) 4 ACES IN DECK

$P(A) = \frac{4}{52} = 1/13 \quad \boxed{P(A) = \frac{1}{13}}$

(b) 1 JACK OF SPADES IN THE DECK

$\boxed{P(J) = \frac{1}{52}}$

(c) $J =$ JACK OF SPADES

$D =$ SIX OF DIAMONDS

$P(J \cup D) = P(J) + P(D) = \frac{1}{52} + \frac{1}{52} = \frac{2}{52} = \frac{1}{26}$

$\boxed{P(J \cup D) = \frac{1}{26}}$

(d) $C =$ CLUBS $\quad D =$ DIAMONDS

$P(C \cup D) = P(C) + P(D) = \frac{13}{52} + \frac{13}{52} = \frac{26}{52} = 1/2$

$\boxed{P(C \cup D) = 1/2}$

PROBLEM 6

1. $\boxed{P(H_1) = \frac{13}{52} = \frac{1}{4}}$

2. $P(\bar{H}_1) = 1 - \frac{13}{52} = \frac{3}{4}$

$P(H_2 | H_1) = \frac{12}{51}$

$P(H_2 | \bar{H}_1) = \frac{13}{51}$

$P(H_2) = P(H_1)P(H_2|H_1) + P(\bar{H}_1)P(H_2|\bar{H}_1)$

$P(H_2) = (\frac{1}{4})(\frac{12}{51}) + (\frac{3}{4})(\frac{13}{51}) = \frac{1}{4}$

$\boxed{P(H_2) = \frac{1}{4}}$