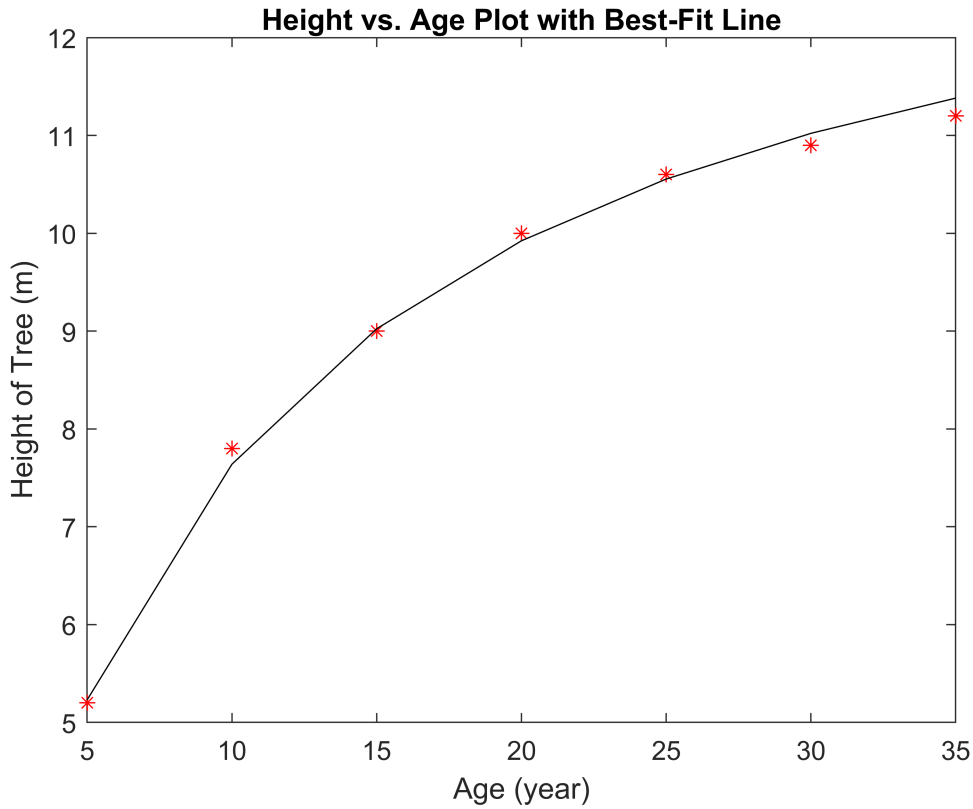
```
clc; clear; close all;
```

Problem 6.41

```
clear variables;
%The given data is initialized and plotted as asterisks here
Age = [5 10 15 20 25 30 35];
Height = [5.2 7.8 9 10 10.6 10.9 11.2];
h = plot(Age, Height,'r');
set(h,'Marker','*');
set(h,'LineStyle','none');
ylabel('Height of Tree (m)')
xlabel('Age (year)')
title('Height vs. Age Plot with Best-Fit Line')
hold on;

%y_bar is defined as 1/y
y_bar = 1./Height';
%M_mat is defined as column 1 == 1/x, and column 2 == 1
M_mat = horzcat((1./Age)', ones(length(Age),1));
%The coefficient matrix defined with pseudo-inverse
coeff_mat = M_mat \ y_bar;
%m is the reciprocal of the second coefficient as defined by the
%fit-function y=(mx)/(b+x)
m = 1/coeff_mat(2);
%b is the first coefficient multiplied by m
b = coeff_mat(1)*(1/coeff_mat(2));
%The fit values are calculated using the coefficients
y_fit = (m.*Age)./(b+Age);
%The least-squares best-fit is plotted on the same plot as data
fit_plt = plot(Age, y_fit,'k-');
```



Problem 6.42:

```
clear variables;
%Part a:

depth = [0 100 200 300 400 500 600 700 800 900 1100 1400 2000 3000];
salinity = [35.5 35.35 35.06 34.65 34.48 34.39 34.34 34.32 34.33 34.36 ...
    35.45 34.58 34.73 34.79];

%These are the x-values for part b
x=[250, 750, 1800];

%Pre-allocating some space for L_poly
L_poly = zeros(1);
%Implementing the general form for lagrange-polynomials (can take in a
%row-vector of values to evaluate at)
for x_val = 1:length(x)
    for inc1 = 1:length(depth)
        L_term = 1;
        for inc2 = 1:length(depth)
            if inc2 ~= inc1
                L_term = L_term.*(x(x_val)-depth(inc2))./(depth(inc1)-depth(inc2));
            end
        end
        L_poly(inc1) = L_term;
    end
```

```
    L_poly_vector(:,x_val) = L_poly;
end

%Part b:
y_int = zeros(length(x),1);
for interp = 1:length(x)
    y_int(interp) = sum(salinity.*L_poly_vector(:,interp)');
end

format shortG
%These are the interpolated values for part b
y_int(1) = sum(salinity.*L_poly_vector(:,1)')
```

```
y_int = 3×1
       34.819
       34.321
      -2054.8
```

```
%Part c:
YI = interp1q(depth', salinity', [250, 750, 1800]')
```

```
YI = 3×1
       34.855
       34.325
        34.68
```

```
%Part d:
% The results for my interpolation function break down for the depth of
% 1800 because the polynomial is a relatively high degree. The end behavior
% of the interpolated polynomial gives obviously incorrect values for the
% salinity. The interp1q function doesn't appear to have the same problem,
% as it gives a realistic number for the depth of 1800. The estimated salinity for
% 250ft and 750ft is very similar between the two approaches.
```

Problem 8.4

```
clear variables;
v = [12.5, 25, 37.5, 50, 62.5, 75];
d = [20, 59, 118, 197, 299, 420];
h = v(2) - v(1);  %Step size

%Part a:
dv_dt_2pt = (d(5)-d(4))/h
```

```
dv_dt_2pt =
         8.16
```

```
dv_dt_3pt = (d(3) - 4*d(4) + 3*d(5)) / (2*h)
```

```
dv_dt_3pt =
         9.08
```

```
%Part b:

stop_distance_2pt = d(5) + dv_dt_2pt * h
```

```
stop_distance_2pt =
   401
```

```
stop_distance_3pt = d(5) + dv_dt_3pt * h
```

```
stop_distance_3pt =
        412.5
```

```
%We see that the 3pt backwards estimate gives a closer estimation of the
%stopping distance based on the data, but it is still lower than the actual
%value for 75 mph
```

Problem 9.3

```
clear variables;
%Initializing data:

z = [0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36];
r = [10, 11, 11.9, 12.4, 13, 13.5, 13.8, 14.1, 13.6, 12.1, 8.9, 4.7, 4.1, 3.5, ...
    3.0, 2.4, 1.9, 1.2, 1.0];

%Part a:
h = z(2)-z(1);
SA_rect = 0;

%This is using the left-side of the interval to define the height of the
%rectangles, with intervals of h=2 because of the even division of the z.
for point = 2:length(r)
    SA_rect = SA_rect + h*r(point-1);
end
%Because surface area is 2pi*integral
SA_rect = 2*pi*SA_rect;

%An alternative implementation for the rectangular method is:
% SA_Rect = 2*pi*h*sum(r(1:end-1)

%This is following the alternative implementation template in the comment
%above, using the left-side as the height.
Vol_rect = pi*h*sum(r(1:end-1).^2);

disp('Composite rectangular method calculates surface area (cm^2):')
```

```
Composite rectangular method calculates surface area (cm^2):
```

```
disp(SA_rect)
```

```
1949
```

```
disp('and volume (cm^3):')
```

```
and volume (cm^3):
```

```
disp(Vol_rect)
```

```
10923
```

```
%Part b:

%This is the implementation of the constant-subinterval-width form (9.13 in
%textbook) of the composite trapezoid method
SA_trap = 2*pi * (h/2 * (r(1) + r(end)) + h*sum(r(2:end-1)));
Vol_trap = pi * (h/2 * (r(1)^2 + r(end)^2) + h*sum(r(2:end-1).^2));

disp('Composite trapezoid method calculates surface area (cm^2):')
```

```
Composite trapezoid method calculates surface area (cm^2):
```

```
disp(SA_trap)
```

```
1892.5
```

```
disp('and volume (cm^3):')
```

```
and volume (cm^3):
```

```
disp(Vol_trap)
```

```
10612
```

```
%Part c:

%This is the implementation of composite Simpson's 3/8 method (equal width
%version). Since there are 18 constant intervals, the 3/8 method is applicable.
SA_simp_int = 0;

%This loop is based on Eq. 9.21 and 9.22 in textbook
for inc = 2:length(r)-1
    if mod(inc,3) == 1
        SA_simp_int = SA_simp_int + r(inc)*2;
    else
        SA_simp_int = SA_simp_int + r(inc)*3;
    end
end
%This is the full value for the integral using the result of the loop and
%the start/end points of the function, with the 3/8 factor
SA_simp_int = (3*h/8) * (r(1) + r(end) + SA_simp_int);
SA_simp = 2*pi*SA_simp_int;
```

```
%This is the implementation for the volume:
Vol_simp_int = 0;
for inc = 2:length(r)-1
    if mod(inc,3) == 1
        Vol_simp_int = Vol_simp_int + r(inc)^2*2;
    else
        Vol_simp_int = Vol_simp_int + r(inc)^2*3;
    end
end
%Same thought process as for surface area above
Vol_simp_int = (3*h/8) * (r(1) + r(end) + Vol_simp_int);
Vol_simp = pi*Vol_simp_int;

disp('Composite Simpson''s 3/8 method calculates surface area (cm^2):')
```

```
Composite Simpson's 3/8 method calculates surface area (cm^2):
```

```
disp(SA_simp)
```

```
        1892
```

```
disp('and volume (cm^3):')
```

```
and volume (cm^3):
```

```
disp(Vol_simp)
```

```
       10398
```

Problem 10.1

```
clear variables;
%Part a:
h = 0.7;
x = 0:h:2.1;
y_eul = zeros(1);
y_eul(1) = 2;
deriv_func = @(a,b) (a.^2)./b;

%Calculating the true solution values here:
true_sol_func = @(a) ((2/3).*a.^3 + 4).^0.5;
y_true = true_sol_func(x);

disp('For x values of:')
```

```
For x values of:
```

```
disp(x)
```

```
           0    0.7000    1.4000    2.1000
```

```
disp('The true values for y are:')
```

```
The true values for y are:
```

```
disp(y_true)
```

```
     2       2.0564       2.4144       3.1897
```

```
%Euler's explicit method:

for inc = 2:length(x)
    y_eul(inc) = y_eul(inc-1) + h*deriv_func(x(inc-1),y_eul(inc-1));
end

disp('Euler''s explicit method calculates:')
```

```
Euler's explicit method calculates:
```

```
disp(y_eul)
```

```
     2            2       2.1715       2.8033
```

```
disp('with error between the true and predicted solution at each point:')
```

```
with error between the true and predicted solution at each point:
```

```
disp(y_true - y_eul)
```

```
     0     0.056372       0.2429      0.38635
```

```
%Part b:
y_eul_mod = zeros(1);
y_eul_mod(1) = 2;

%Implementation of the Modified Euler's Method Algorithm (pg. 402)
for inc2 = 2:length(x)
    current_deriv = deriv_func(x(inc2-1),y_eul_mod(inc2-1));
    y_est = y_eul_mod(inc2-1) + h * current_deriv;
    y_est_deriv = deriv_func(x(inc2),y_est);
    y_eul_mod(inc2) = y_eul_mod(inc2-1) + 0.5 * (current_deriv + y_est_deriv) * h;
end

disp('The modified Euler''s method calculates:')
```

```
The modified Euler's method calculates:
```

```
disp(y_eul_mod)
```

```
     2       2.0857       2.4728         3.26
```

```
disp('with error between the true and predicted solution at each point:')
```

```
with error between the true and predicted solution at each point:
```

```
disp(y_true - y_eul_mod)
```

```
     0    -0.029378    -0.058435    -0.070379
```

```
%Part c:
y_rung = zeros(1);
y_rung(1) = 2;

%Calculating the weights k1->k4 for each pass of the 4th Order Runge-Kutta
%Then, calculates the next y-value based on the classical fourth-order
%Runge-Kutta weights
for inc3 = 2:length(x)
    k1 = deriv_func(x(inc3-1), y_rung(inc3-1));
    k2 = deriv_func(x(inc3-1) + h/2, y_rung(inc3-1) + 0.5*h*k1);
    k3 = deriv_func(x(inc3-1) + h/2, y_rung(inc3-1) + 0.5*h*k2);
    k4 = deriv_func(x(inc3), y_rung(inc3-1) + k3*h);
    y_rung(inc3) = y_rung(inc3-1) + (h/6) * (k1 + 2*k2 + 2*k3 + k4);
end

disp('The classical fourth-order Runge-Kutta method calculates:')
```

```
The classical fourth-order Runge-Kutta method calculates:
```

```
disp(y_rung)
```

```
           2       2.0564       2.4147       3.1902
```

```
disp('with error between the true and predicted solution at each point:')
```

```
with error between the true and predicted solution at each point:
```

```
disp(y_true - y_rung)
```

```
           0  -4.9227e-05   -0.0002704  -0.00055593
```

Problem 6 (Custom Problem):

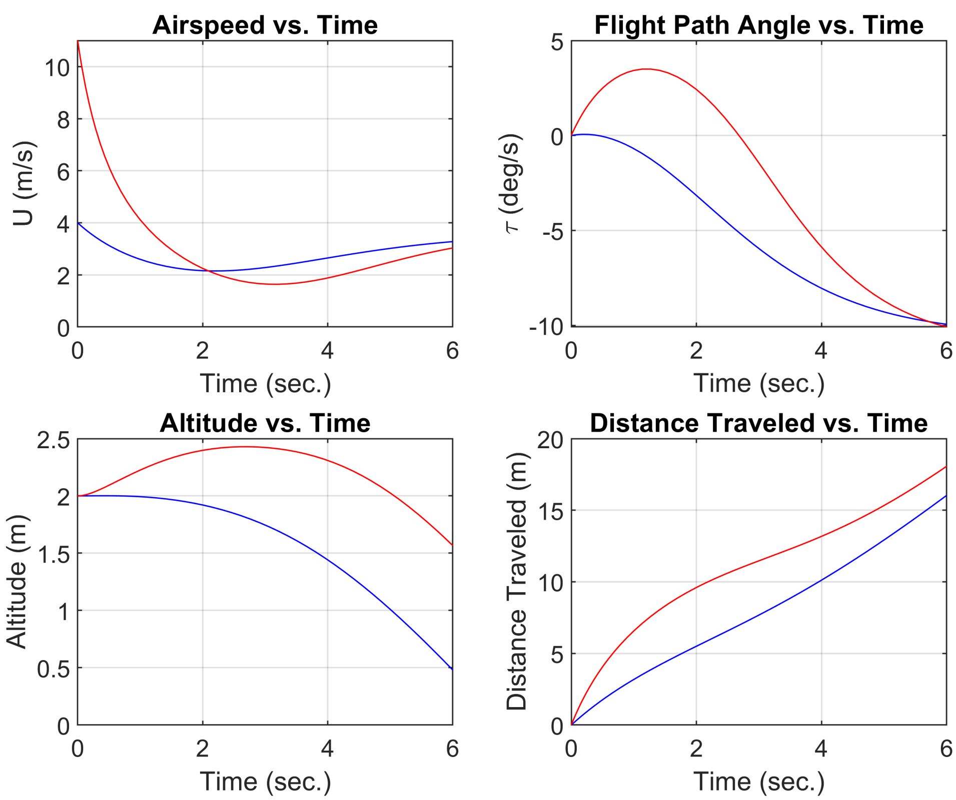
```
clear variables;
%This is where the ODE system is actually solved for both cases (Yout1 for
%first case, Yout2 for second case).
[Tout1, Yout1] = ode45(@airplane_deriv_func,[0,6],[4; 0; 2; 0]);
[Tout2,Yout2] = ode45(@airplane_deriv_func,[0,6],[11; 0; 2; 0]);

%plotting each of the value pairs now as subplots. Each plot's code is separated by a
%line of whitespace for readability. Blue is first case, red is second case
figure(1);
subplot(221);
plot(Tout1,Yout1(:,1),'b');
hold on;
grid on;
plot(Tout2,Yout2(:,1),'r');
title('Airspeed vs. Time')
xlabel('Time (sec.)')
ylabel('U (m/s)')
```

```
subplot(222);
plot(Tout1,Yout1(:,2),'b');
hold on;
grid on
plot(Tout2,Yout2(:,2),'r');
title('Flight Path Angle vs. Time')
xlabel('Time (sec.)')
ylabel('\tau (deg/s)')

subplot(223);
plot(Tout1,Yout1(:,3),'b');
hold on;
grid on;
plot(Tout2,Yout2(:,3),'r');
title('Altitude vs. Time')
xlabel('Time (sec.)')
ylabel('Altitude (m)')

subplot(224);
plot(Tout1,Yout1(:,4),'b');
hold on;
grid on;
plot(Tout2,Yout2(:,4),'r');
title('Distance Traveled vs. Time')
xlabel('Time (sec.)')
ylabel('Distance Traveled (m)')
```



```
disp('The blue line corresponds to the first set of initial conditions.')
```

```
The blue line corresponds to the first set of initial conditions.
```

```
disp('The red line corresponds to the second set of initial conditions.')
```

```
The red line corresponds to the second set of initial conditions.
```

```
function x_dot = airplane_deriv_func(t,x)
%This is the function referenced in problem 6. I define all of the given
%constants here:
    cd = .04;
    cl = .2205;
    rho = 1.225;
    s = 0.017;
    m = 0.003;
    g = 9.81;

%This is where the system of differential equations is implemented such
%that element 1 is the airspeed, element 2 is the flight path angle,
%element 3 is the altitude, and element 4 is the distance traveled.
    x_dot = [-(cd*rho*s)/(2*m)*x(1)^2 - g*sind(x(2));...
        (cl*rho*s)/(2*m)*x(1) - g/x(1)*cosd(x(2));...
        x(1)*sind(x(2));...
        x(1)*cosd(x(2))];
end
```