ESAM 447 **Homework #3**

Due 12 Noon, Chicago Time, Wednesday, June 9, 2021

Please work alone. Please clearly describe your numerical method and give all of the details of your calculations. Please give your results in tables whenever possible. All conclusions need to be supported by computational results. The report should be typed.

Consider the integro-differential equation

$$\frac{d^2h}{dx^2} + h + \frac{i}{2}\int_{-\pi/2}^{\pi/2} H_0^1(\mid x - y \mid)h(y)dy = e^{iax},$$

for $-\pi/2 \leq x \leq \pi/2$. The complex dependent variable $h(x)$ satisfies the boundary conditions $h(\pi/2) = h(-\pi/2) = 0$. This equation describes the interface displacement resulting from the scattering of an acoustic wave from an interface separating two fluids. Here H_0^1 is a Hankel function of the first kind.

1. Describe a numerical method to solve this differential equation.

2. Solve this equation numerically for $a = 0.1$. Do a numerical convergence check to show convergence of your numerical scheme and use these results to estimate the rate of convergence of your scheme. Plot the solution.

3. Solve the equation for $a = 1.0$.

4. Explain the differences between the $a = 0.1$ case and the $a = 1.0$ case. When can such differences be expected?