ESAM 447 **Homework #2**

Due 12 Noon, Chicago Time, Friday May 21, 2021

Please work alone. Please clearly describe your numerical method and give all of the details of your calculations. Please give your results in tables whenever possible. All conclusions need to be supported by computational results. The report should be typed.

Consider the singular Volterra integral equation

$$x(s) = \frac{1}{(1+s)^{1/2}} + \frac{\pi}{8} - \frac{1}{4}\sin^{-1}\left(\frac{1-s}{1+s}\right) - \frac{1}{4}\int_0^s \frac{x(t)}{(s-t)^{1/2}}dt.$$

The exact answer is $x_e(s) = 1/(1+s)^{1/2}$. Using a product integration analogue of the Trapesoid rule with constant step size h, solve the integral equation over the range $0 \le s \le 1$.

1. Determine a quadrature rule of the form,

$$\int_0^{s_i} (s_i - t)^{-1/2} x(t)dt = \sum_{j=0}^{i} w_{ij} x(t_j),$$

   where $t_i = s_i, i = 0, ..., N$. The weights w_{ij} are constructed by insisting that the rule be exact when $x(t)$ is a first degree polynomial. Identify the weights w_{ij}.

2. Solve the singular integral equation numerically and determine the N to give 6 decimal place accuracy. What is the observed order of convergence?