Spatial Derivatives: Gradient, Divergence, Laplacian

1. Consider the scalar function

$$f(x, y) = \sin\left(\frac{x}{\sqrt{y}}\right) + xy^2 + x.$$

Notice that f takes a point in 2D space and returns a single number. In math language, this can be written as

$$f : \mathbb{R}^2 \to \mathbb{R}.$$

(a) Compute f's gradient $\boldsymbol{\nabla} f$. Remember that the gradient operator acts on a scalar function, and returns a vector field.

(b) Compute f's laplacian $\boldsymbol{\nabla}^2 f$. Remember that the laplacian operator acts on a scalar function, and returns a scalar field.

(c) Compute the divergence of f's gradient of $\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} f$.