$$\boxed{\dot{y}(t) = a\,y(t)}$$

$y = e^t$

$\dot{y} = e^t$

$e^t = a e^t$

$y = e^{kt}$

$\dot{y} = e^{kt} \cdot k$

$k e^{kt} = a e^{kt}$

$a = k$

$y = b e^{at}$

$\dot{y} = b \cdot a e^{at}$

$abe^{at} = abe^{at}$

$$\boxed{y = \underline{\underline{b}}\, e^{at}}$$

$$\boxed{\begin{aligned} \dot{y}_1(t) &= a_1 y_1(t) \\ \dot{y}_2(t) &= a_2 y_2(t) \end{aligned}}$$

Easy

$\dot{y}_1(t) = a_1 y_1(t) + a_2 y_2(t)$

$\dot{y}_2(t) = a_3 y_2(t)$

$$\boxed{\begin{aligned} y_2(t) &= b_3 e^{a_3 t} \\ \dot{y}_1(t) &= a_1 y_1(t) + b_3 e^{a_3 t} \end{aligned}}$$

not to hard.

$$\dot{y}_1(t) = a_{11} y_1(t) + a_{12} y_2(t)$$

$$\dot{y}_2(t) = a_{21} y_1(t) + a_{22} y_2(t).$$

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \dot{Y}(t) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} Y(t).$$

$$Y(t) = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}.$$

$$\dot{Y}(t) = A Y(t).$$

Would like:

$$Y(t) = e^{tA}.$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Find a nice form of $A$

So we can compute $A^m$ easily.

It is always possible to find a matrix $P$.

So that

$$PAP^{-1} = \begin{bmatrix} \lambda_1 & 1 \text{ or } 0 & & & & & \\ & \lambda_2 & 1 \text{ or } 0 & & & & \\ & & \lambda_3 & 1 \text{ or } 0 & & & \\ & & & \lambda_4 & 1 & & \\ & 0 & & & \lambda_5 & & \\ & & & & & \ddots & \\ & & & & & & \lambda_n \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}.$$

$$\begin{bmatrix} \lambda_1 & 1 \\ 0 & \lambda_2 \end{bmatrix}.$$

$$\begin{bmatrix} \lambda_1 & 0 & 0 \\ & \lambda_2 & 0 \\ 0 & & \lambda_3 \end{bmatrix}$$

$$DN = ND$$

$$\begin{bmatrix} \lambda_1 & 1 & 0 \\ 0 & \lambda_2 & 0 \\ & & \lambda_3 \end{bmatrix}$$

$$(D+N)^2$$

$$= D^2 + 2DN + N^2$$

$$\begin{bmatrix} \lambda_1 & 1 & 0 \\ 0 & \lambda_2 & 1 \\ & & \lambda_3 \end{bmatrix}$$

$$(D+N)^3$$

$$= D^3 + 3D^2N + 3DN^2 + N^3$$

$$\underbrace{\begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix}}_{D} + \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_{N}$$

Nilpotent

$$(D+N)^4 =$$

$$D^4 + 4D^3N + 6D^2N^2 + \cdots$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = N^2.$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$N^3 = 0.$$

$$N = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$N^4 = 0$$

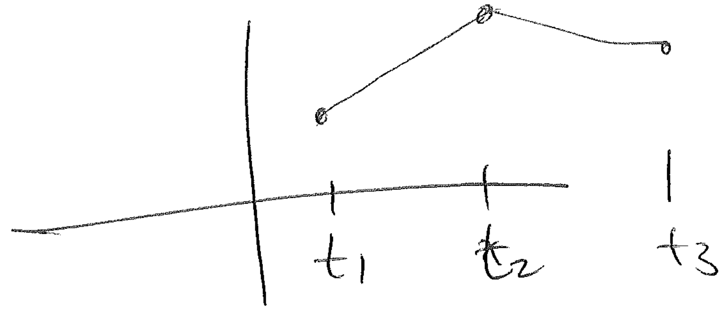
$$\left(PAP^{-1}\right)^k = (D + N)^k$$

$$PA^kP^{-1} =$$

$$PAP^{-1}PAP^{-1}PAP^{-1}P \ldots$$

$$A^k = P^{-1}(D+N)^k P$$

$[A]$




$$AX = 0$$

$$\begin{cases} \dot{x}_1 = a_{11}x_1 + a_{12}x_2 + a_{1n}x_n \\ \\ \dot{x}_n = a_{n1}x_1 + \cdots + a_{nn}x_n \end{cases}$$

$$\dot{x}_1 = a_{11}x_1(t) + a_{12}x_2(t)$$

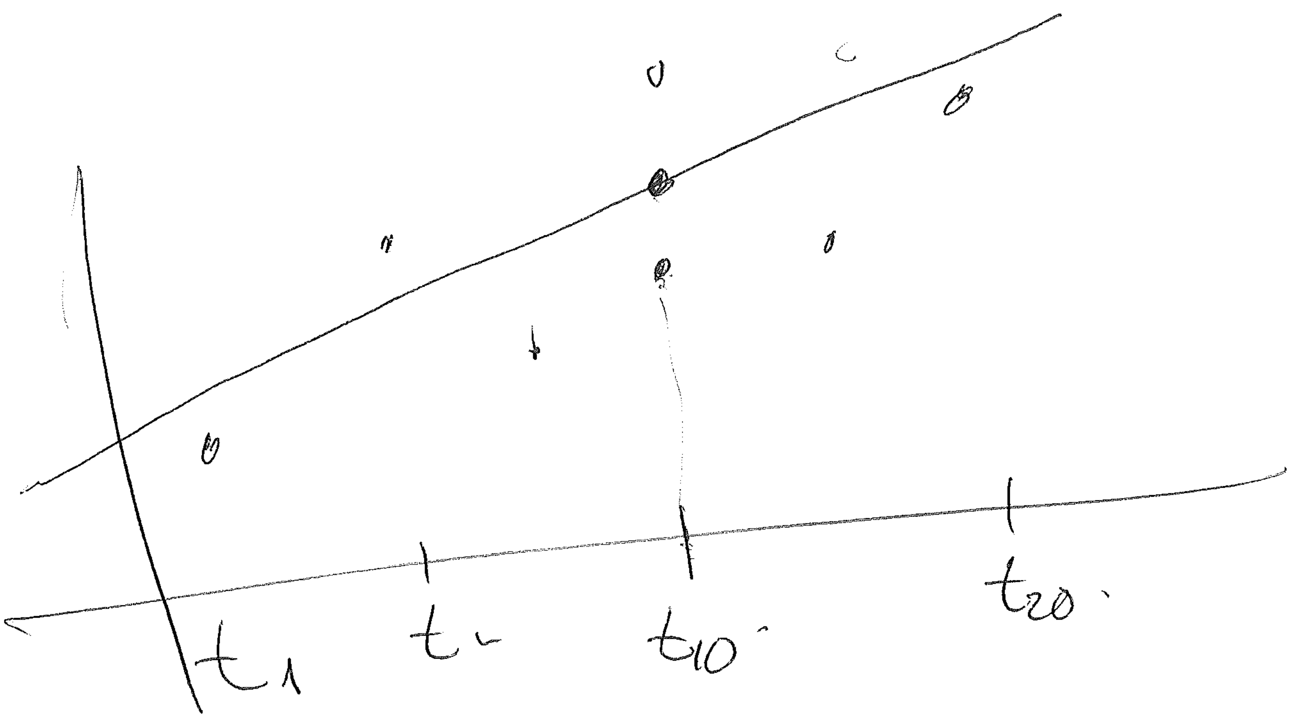
$$\dot{x}_2 = a_{21}x_1(t) + a_{22}x_2(t)$$

$x_1(t_1)$, $x_2(t_1)$

$x_1(t_2)$ $x_2(t_2)$

$x_1(t_3)$ $x_2(t_3)$

$$\begin{cases} \dot{x}_1(t_1) = a_{11}x_1(t_1) + a_{12}x_2(t_1) \\ \dot{x}_1(t_2) = a_{11}x_1(t_2) + a_{12}x_2(t_2) \end{cases}$$



t1
t2
t10
t20