

APPENDIX B

TABULAR SUMMARY OF PARAMETRIC FAMILIES OF DISTRIBUTIONS

1 INTRODUCTION

The purpose of this appendix is to provide the reader with a convenient reference to the parametric families of distributions that were introduced in Chap. III. Given are two tables, one for discrete distributions and the other for continuous distributions.

Table 1 DISCRETE DISTRIBUTIONS

Name of parametric family of distributions	Discrete density functions $f(\cdot)$	Parameter space	Mean $\mu = E[X]$
Discrete uniform	$f(x) = \frac{1}{N} I_{\{1, \dots, N\}}(x)$	$N = 1, 2, \dots$	$\frac{N+1}{2}$
Bernoulli	$f(x) = p^x q^{1-x} I_{\{0, 1\}}(x)$	$0 \leq p \leq 1$ ($q = 1 - p$)	p
Binomial	$f(x) = \binom{n}{x} p^x q^{n-x} I_{\{0, 1, \dots, n\}}(x)$	$0 \leq p \leq 1$ $n = 1, 2, 3, \dots$ ($q = 1 - p$)	np
Hypergeometric	$f(x) = \frac{\binom{K}{x} \binom{M-K}{n-x}}{\binom{M}{n}} I_{\{0, 1, \dots, n\}}(x)$	$M = 1, 2, \dots$ $K = 0, 1, \dots, M$ $n = 1, 2, \dots, M$	$n \frac{K}{M}$
Poisson	$f(x) = \frac{e^{-\lambda} \lambda^x}{x!} I_{\{0, 1, \dots\}}(x)$	$\lambda > 0$	λ
Geometric	$f(x) = pq^x I_{\{0, 1, \dots\}}(x)$	$0 < p \leq 1$ ($q = 1 - p$)	$\frac{q}{p}$
Negative binomial	$f(x) = \binom{r+x-1}{x} p^r q^x I_{\{0, 1, \dots\}}(x)$	$0 < p \leq 1$ $r > 0$ ($q = 1 - p$)	$\frac{rq}{p}$

	Moments $\mu'_r = \mathcal{E}[X^r]$ or $\mu_r = \mathcal{E}[(X - \mu)^r]$ and/or cumulants κ_r	Moment generating function $\mathcal{E}[e^{tX}]$
Variance $\sigma^2 = \mathcal{E}[(X - \mu)^2]$		
$\frac{N^2 - 1}{12}$	$\mu'_3 = \frac{N(N + 1)^2}{4}$ $\mu'_4 = \frac{(N + 1)(2N + 1)(3N^2 + 3N - 1)}{30}$	$\sum_{j=1}^N \frac{1}{N} e^{jt}$
pq	$\mu'_r = p$ for all r	$q + pe^t$
npq	$\mu_3 = npq(q - p)$ $\mu_4 = 3n^2p^2q^2 + npq(1 - 6pq)$	$(q + pe^t)^n$
$n \frac{K}{M} \frac{M - K}{M} \frac{M - n}{M - 1}$	$\mathcal{E}[X(X - 1) \cdots (X - r + 1)] = r! \frac{\binom{K}{r} \binom{n}{r}}{\binom{M}{r}}$	not useful
λ	$\kappa_r = \lambda$ for $r = 1, 2, \dots$ $\mu_3 = \lambda$ $\mu_4 = \lambda + 3\lambda^2$	$\exp[\lambda(e^t - 1)]$
$\frac{q}{p^2}$	$\mu_3 = \frac{q + q^2}{p^2}$ $\mu_4 = \frac{q + 7q^2 + q^3}{p^4}$	$\frac{p}{1 - qe^t}$
$\frac{rq}{p^2}$	$\mu_3 = \frac{r(q + q^2)}{p^3}$ $\mu_4 = \frac{r[q + (3r + 4)q^2 + q^3]}{p^4}$	$\left(\frac{p}{1 - qe^t}\right)^r$

Table 2 CONTINUOUS DISTRIBUTIONS

Name of parametric family of distributions	Cumulative distribution function $F(\cdot)$ or probability density function $f(\cdot)$	Parameter space	Mean $\mu = E[X]$
Uniform or rectangular	$f(x) = \frac{1}{b-a} I_{[a,b]}(x)$	$-\infty < a < b < \infty$	$\frac{a+b}{2}$
Normal	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp[-(x-\mu)^2/2\sigma^2]$	$-\infty < \mu < \infty$ $\sigma > 0$	μ
Exponential	$f(x) = \lambda e^{-\lambda x} I_{(0,\infty)}(x)$	$\lambda > 0$	$\frac{1}{\lambda}$
Gamma	$f(x) = \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x} I_{(0,\infty)}(x)$	$\lambda > 0$ $r > 0$	$\frac{r}{\lambda}$
Beta	$f(x) = \frac{1}{B(a, b)} x^{a-1} (1-x)^{b-1} I_{(0,1)}(x)$	$a > 0$ $b > 0$	$\frac{a}{a+b}$
Cauchy	$f(x) = \frac{1}{\pi\beta\{1 + [(x-\alpha)/\beta]^2\}}$	$-\infty < \alpha < \infty$ $\beta > 0$	Does not exist
Lognormal	$f(x) = \frac{1}{x\sqrt{2\pi}\sigma} \exp[-(\log_e x - \mu)^2/2\sigma^2] I_{(0,\infty)}(x)$	$-\infty < \mu < \infty$ $\sigma > 0$	$\exp[\mu + \frac{1}{2}\sigma^2]$
Double exponential	$f(x) = \frac{1}{2\beta} \exp\left(-\frac{ x-\alpha }{\beta}\right)$	$-\infty < \alpha < \infty$ $\beta > 0$	α

Variance $\sigma^2 = \mathcal{E}[(X - \mu)^2]$	Moments $\mu'_r = \mathcal{E}[X^r]$ or $\mu_r = \mathcal{E}[(X - \mu)^r]$ and/or cumulants κ_r	Moment generating function $\mathcal{E}[e^{tx}]$
$\frac{(b-a)^2}{12}$	$\mu_r = 0$ for r odd $\mu_r = \frac{(b-a)^r}{2^r(r+1)}$ for r even	$\frac{e^{bt} - e^{at}}{(b-a)t}$
σ^2	$\mu_r = 0$, r odd; $\mu_r = \frac{r!}{(r/2)!} \frac{\sigma^r}{2^{r/2}}$, r even; $\kappa_r = 0$, $r > 2$	$\exp[\mu t + \frac{1}{2} \sigma^2 t^2]$
$\frac{1}{\lambda^2}$	$\mu'_r = \frac{\Gamma(r+1)}{\lambda^r}$	$\frac{\lambda}{\lambda-t}$ for $t < \lambda$
$\frac{r}{\lambda^2}$	$\mu'_r = \frac{\Gamma(r+j)}{\lambda^j \Gamma(r)}$	$\left(\frac{\lambda}{\lambda-t}\right)^r$ for $t < \lambda$
$\frac{ab}{(a+b+1)(a+b)^2}$	$\mu_r = \frac{B(r+a, b)}{B(a, b)}$	not useful
Does not exist	Do not exist	Characteristic function is $e^{iat - \beta t }$
$\exp[2\mu + 2\sigma^2]$ $- \exp[2\mu + \sigma^2]$	$\mu'_r = \exp[r\mu + \frac{1}{2} r^2 \sigma^2]$	not useful
$2\beta^2$	$\mu_r = 0$ for r odd; $\mu_r = r! \beta^r$ for r even	$\frac{e^{\alpha t}}{1 - (\beta t)^2}$

(continued)

Table 2 CONTINUOUS DISTRIBUTIONS (continued)

Name of parametric family of distributions	Cumulative distribution function $F(\cdot)$ or probability density function $f(\cdot)$	Parameter space	Mean $\mu = E[X]$
Weibull	$f(x) = abx^{b-1} \exp[-ax^b] I_{(0, \infty)}(x)$	$a > 0$ $b > 0$	$a^{-1/b} \Gamma(1 + b^{-1})$
Logistic	$F(x) = [1 + e^{-(x-\alpha)/\beta}]^{-1}$	$-\infty < \alpha < \infty$ $\beta > 0$	α
Pareto	$f(x) = \frac{\theta x_0^\theta}{x^{\theta+1}} I_{(x_0, \infty)}(x)$	$x_0 > 0$ $\theta > 0$	$\frac{\theta x_0}{\theta - 1}$ for $\theta > 1$
Gumbel or extreme value	$F(x) = \exp(-e^{-(x-\alpha)/\beta})$	$-\infty < \alpha < \infty$ $\beta > 0$	$\alpha + \beta\gamma$, $\gamma \approx .577216$
<i>t</i> distribution	$f(x) = \frac{\Gamma[(k+1)/2]}{\Gamma(k/2)} \frac{1}{\sqrt{k\pi}} \frac{1}{(1+x^2/k)^{(k+1)/2}}$	$k > 0$	$\mu = 0$ for $k > 1$
<i>F</i> distribution	$f(x) = \frac{\Gamma[(m+n)/2]}{\Gamma(m/2)\Gamma(n/2)} \left(\frac{m}{n}\right)^{m/2} \times \frac{x^{(m-2)/2}}{[1+(m/n)x]^{(m+n)/2}} I_{(0, \infty)}(x)$	$m, n = 1, 2, \dots$	$\frac{n}{n-2}$ for $n > 2$
Chi-square distribution	$f(x) = \frac{1}{\Gamma(k/2)} \left(\frac{1}{2}\right)^{k/2} x^{k/2-1} e^{-(1/2)x} I_{(0, \infty)}(x)$	$k = 1, 2, \dots$	k

Variance $\sigma^2 = \mathcal{E}[(X - \mu)^2]$	Moments $\mu'_r = \mathcal{E}[X^r]$ or $\mu_r = \mathcal{E}[(X - \mu)^r]$ and/or cumulants κ_r	Moment generating function $\mathcal{E}[e^{tX}]$
$a^{-2/b}[\Gamma(1 + 2b^{-1}) - \Gamma^2(1 + b^{-1})]$	$\mu'_r = a^{-r/b}\Gamma\left(1 + \frac{r}{b}\right)$	$\mathcal{E}[X^t] = a^{-t/b}\Gamma\left(1 + \frac{t}{b}\right)$
$\frac{\beta^2\pi^2}{3}$		$e^{\alpha t}\pi\beta t \csc(\pi\beta t)$
$\frac{\theta x_0^2}{(\theta - 1)^2(\theta - 2)}$ for $\theta > 2$	$\mu'_r = \frac{\theta x_0^r}{\theta - r}$ for $\theta > r$	does not exist
$\frac{\pi^2\beta^2}{6}$	$\kappa_r = (-\beta)^r \psi^{(r-1)}(1)$ for $r \geq 2$, where $\psi(\cdot)$ is digamma function	$e^{\alpha t}\Gamma(1 - \beta t)$ for $t < 1/\beta$
$\frac{k}{k-2}$ for $k > 2$	$\mu_r = 0$ for $k > r$ and r odd $\mu_r = \frac{k^{r/2}B((r+1)/2, (k-r)/2)}{B(\frac{1}{2}, k/2)}$ for $k > r$ and r even	does not exist
$\frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$ for $n > 4$	$\mu'_r = \left(\frac{n}{m}\right)^r \frac{\Gamma(m/2+r)\Gamma(n/2-r)}{\Gamma(m/2)\Gamma(n/2)}$ for $r < \frac{n}{2}$	does not exist
$2k$	$\mu'_j = \frac{2^j \Gamma(k/2+j)}{\Gamma(k/2)}$	$\left(\frac{1}{1-2t}\right)^{k/2}$ for $t < 1/2$